A MULTIFRACTAL DETRENDED FLUCTUATION ANALYSIS OF GOLD PRICE FLUCTUATIONS

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It is a well-established fact that gold has many economic functions including hedging against inflation and providing economic and physical safety. For this it is very important to know the nature of fluctuations in gold prices. In this paper, applying the Multifractal Detrended Fluctuation Analysis (MF-DFA) for the world gold price data for over 40-year period from 1968 to 2010, the multifractal properties and scaling behavior of gold price time series is numerically investigated. The scaling exponents, generalized Hurst exponents, generalized fractal dimensions and singularity spectrum are derived. Furthermore, impact of two major sources of multifractality, *i.e.* fat-tailed probability distributions and nonlinear temporal correlations are also examined. Our findings suggest that multifractality in gold price is mainly due to the temporal correlation.

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1. Introduction

It has been a long history that physicists show interests on financial markets, which can be at least traced back to 1900 when Bachelier modeled stock prices with Brownian motions [1]. In the middle of last century, Mandelbrot proposed the concept of "fractal world" which was based on scale-invariant statistics with power law correlations [2]. This new theory has been progressively developed in recent years and finally it brought a more general concept of multiscaling. It allows one to study the global and local behavior of a singular measure or in other words, the mono- and multifractal properties of a system. In economy, multifractality is a one of the well known stylized facts which characterize non-trivial properties of financial time series [3].

In recent years the detrended fluctuation analysis (DFA) method [4] has become a widely used technique for the determination of (mono-) fractal scaling properties and the detection of long-range correlations in noisy, nonstationary time series [5–8]. It has successfully been applied to a variety of fields such as DNA sequences, heart rate dynamics, neuron spiking, human gait, long-time weather data, studying the cloud structure, geology, ethnology, economics time series, and solid state physics [9–19]. One reason to employ the DFA method is to avoid spurious detection of correlations that are artifacts of nonstationarities in the time series. Many records do not exhibit a simple monofractal scaling behavior, which can be accounted for by a single scaling exponent.

The simplest type of the multifractal analysis is based upon the standard partition function multifractal formalism, which has been developed for the multifractal characterization of normalized, stationary measurements. Unfortunately, this standard formalism does not give correct results for nonstationary time series that are affected by trends or that cannot be normalized. Thus, in the early 1990s an improved multifractal formalism has been developed, the wavelet transform modulus maxima (WTMM) method [20], which is based on the wavelet analysis and involves tracing the maxima lines in the continuous wavelet transform over all scales. The other method, the multifractal detrended fluctuation analysis, is based on the identification of scaling of the qth-order moments depending on the signal length and is generalization of the standard DFA using only the second moment $q = 2$.

The MF-DFA does not require the modulus maxima procedure in contrast to the WTMM method, and hence does not require more effort in programming and computing than the conventional DFA. On the other hand, experimental data are often affected by non-stationarities like trends, which have to be well distinguished from the intrinsic fluctuations of the system in order to find the correct scaling behavior of the fluctuations. In addition, very often we do not know the reasons for underlying trends in collected data and even worse, we do not know the scales of the underlying trends, also, usually the available record data is small. For the reliable detection of correlations, it is essential to distinguish trends from the fluctuations intrinsic in the data. Hurst rescaled-range analysis and other non-detrending methods work well if the records are long and do not involve trends. But if trends are present in the data, they might give wrong results. DFA is a well-established method for determining the scaling behavior of noisy data in the presence of trends without knowing their origin and shape [21].

Since the introduction of MF-DFA method there has been a vigorous continuing investigation aimed at discovering multifractal nature of financial markets [22–24]. But to best of our knowledge there are very rare work studying the multifractal nature of gold price fluctuations. In recent years gold has been controversial because of sharp increase in its prices. Price of gold exceeded 1092 per ounce in New York Stock Exchange in November 2009. Recent global economic crisis has given rise to uncertainty in the global economy including developed and developing countries. Gold is not only used in jewelery but also in industrial and medical applications. Moreover, gold is used for investment purposes by governments, households, institutional and private equity investors. If there is an economic uncertainty then gold becomes an insurance. In such cases, gold can protect us against the inflation and deflation. In addition, according to the World Gold Council (2006) Central Banks hold gold reserves because gold provides economic and physical safety, sustains world wide confidence and offers diversification benefits.

The main purpose of this paper is to characterize the complex behavior of gold price time series through the computation of the signal parameters — scaling exponents — which quantifies the correlation exponents and multifractality of the signal.

The rest of paper is organized as follows. In Sec. 2, MF-DFA method is described and other related theoretical concepts are reviewed. Data are provided in Sec. 3. In Sec. 4 numerical results are presented and finally, conclusions are given in Sec. 5.

2. Method description

The generalized Multifractal Detrended Fluctuation Analysis (MF-DFA) procedure consists of five steps [21]. The first three steps are essentially identical to the conventional DFA Procedure. Let assume that x_k is a series of length N and this series is of compact support, *i.e.* $x_k = 0$ for an insignificant fraction of values only:

Step 1: Determine the "profile"

$$Y(i) \equiv \sum_{k=1}^{i} \left[x_k - \langle x \rangle \right] . \tag{1}$$

Subtraction of $\langle x \rangle$ is not compulsory because it would be eliminated by the later detrending in the third step.

Step 2: Divide the profile $Y(i)$ into $N_s \equiv N/s$ non-overlapping segments of equal length s. Since the length N of the series is not often a multiple of considered time scale s, short part at the end of profile may remain. In order to disregard this part of the series, the same procedure is repeated starting from the end of series. Thus, $2N_s$ segments are obtained eventually.

Step 3: calculate the local trend for each of the $2N_s$ segments by a least-square fit of the series. Then determine the variance

$$F^2(s,\nu) \equiv \frac{1}{s}\sum_{i=1}^{s}\{Y[(\nu-1)s+i]-y_\nu(i)\}^2 \tag{2}$$

for each segment ν, $\nu = 1,2,\ldots,N_s$ and

$$F^2(s,\nu) \equiv \frac{1}{s}\sum_{i=1}^{s}\{Y[N-(\nu-N_s)s+i]-y_\nu(i)\}^2 \tag{3}$$

for $\nu = N_s,\ldots,2N_s$. Here $y_\nu(i)$ is the fitting polynomial in segment ν. Linear, quadratic, cubic or higher order polynomials can be used for fitting.

Step 4: Average over all segments to obtain the qth order fluctuation function

$$F_q(s) \equiv \left\{\frac{1}{2N_s}\sum_{\nu=1}^{2N_s}\left[F^2(s,\nu)\right]^{q/2}\right\}^{1/q}, \tag{4}$$

where in general, the index variable q can take any real value except zero. For $q = 2$, the standard DFA procedure is retrieved. We are interested in how the generalized q dependent fluctuation function $F_q(s)$ depends on the time scale s for different values of q. Hence we must repeat steps 2 to 4 for several time scales s. It is apparent that $F_q(s)$ will increase with increasing s.

Step 5: Determine the scaling behavior of the fluctuation functions by analyzing log–log plots of $F_q(s)$ *versus* s for each value of q. If the series x_i are long range power-law correlated, $F_q(s)$ increases, for large values of s, as a power-law

$$F_q(s) \sim s^{h(q)}. \tag{5}$$

In general, the exponent $h(q)$ may depend on q. For stationary time series, $h(2)$ is identical to the well-known Hurst exponent H. Thus we call the function $h(q)$ generalized Hurst exponent. For monofractal times series with compact support, $h(q)$ is independent of q, since the scaling behavior of the variances $F^2(s,\nu)$ is identical for all segments ν, and the averaging procedure in Eq. (4) will give just this identical scaling behavior for all values of q. The family of the exponents $h(q)$ describe the scaling of the qth order fluctuation function. For positive values of q, $h(q)$ exponents describe the scaling behavior of boxes with large fluctuations while those of negative values of q, describe scaling behavior of boxes with small fluctuations [21].

However, the MF-DFA method can only determine positive generalized Hurst exponents $h(q)$, and it already becomes inaccurate for strongly anti-correlated signals when $h(q)$ is close to zero. In such cases, a modified (MF-)DFA technique has to be used. The most simple way to analyze such data is to integrate the time series before the MF-DFA procedure. Hence, we replace the single summation in Eq. (1), which is describing the profile from the original data x_k, by a double summation [21]

$$\tilde{Y}(i) \equiv \sum_{k=1}^{i} [Y_k - \langle Y \rangle] \,. \tag{6}$$

The $h(q)$ obtained from MF-DFA is related to the Renyi exponent $\tau(q)$ by

$$qh(q) = \tau(q) + 1 \,. \tag{7}$$

Therefore, another way to characterize a multifractal series is the singularity spectrum $f(\alpha)$ defined by [25]

$$\alpha = h(q) + qh'(q) \tag{8}$$

and

$$f(\alpha) = q[\alpha - h(q)] + 1 \,, \tag{9}$$

where $h'(q)$ stands for the derivative of $h(q)$ with respect to q. α is the Holder exponent or singularity strength which characterizes the singularities in a time series. The singularity spectrum $f(\alpha)$ describes the singularity content of the time series. Finally, it must be noted that $h(q)$ is different from the generalized multifractal dimensions

$$D(q) \equiv \frac{\tau(q)}{q-1} = \frac{qh(q)-1}{q-1} \,, \tag{10}$$

that are used instead of $\tau(q)$ in some papers. While $h(q)$ is independent of q for a monofractal time series with compact support, $D(q)$ depends on q in that case [26].

3. Data analysis

The data which is analyzed in this study include the time series of the Gold price in London Metal Exchange (LME) logarithmic variations (*i.e.* $\ln(P(t+1)/P(t))$ from the time period 1968–2010. In Fig. 1 gold price, logarithmic return as a function of time and also Cumulative Distribution Function (CDF) of its normalized returns in log–log plot are presented. Power-law

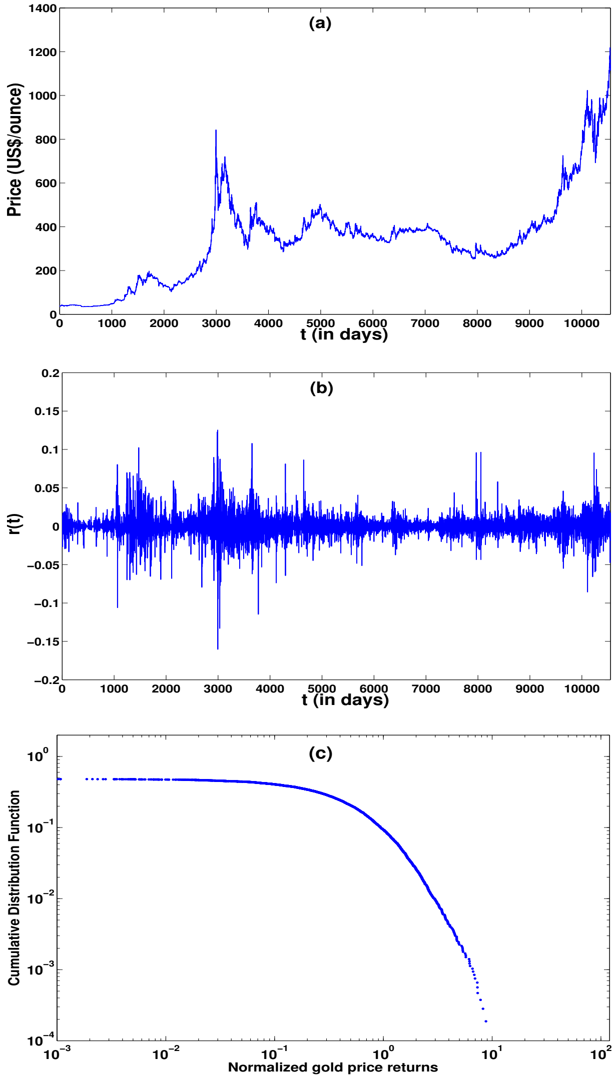

Fig. 1. (a) Gold price, (b) returns and (c) cumulative distribution function (CDF) of its normalized returns with the scaling exponent $\alpha \simeq -3$ for the period 1968–2010.

regression fit yields estimate of the tail exponent $\alpha \simeq -3$. This scaling index is consistent with the inverse cubic power law identified for many other markets [27–31]. By putting our result in the context of previous studies, one can see that all of these markets share the same scaling functions and characteristic exponents and therefore belong to one universality class. As we see significant increase in the price of gold in the end of period is clear which is unprecedented in the whole period.

4. Results

The fluctuation functions $F_q(s)$ for gold price time series with the scaling parameter ranging from $s = 50$ to $s = N/5$ are calculated, where N is the total length of the time series. Since the tail scaling exponent determines the range of q-parameter, q varies between -4 and 4, with a step of 1. It also must be mentioned that q values larger than 4 would lead to the divergent moments. Fig. 2(a) shows the MF-DFA2 fluctuations $F_q(s)$

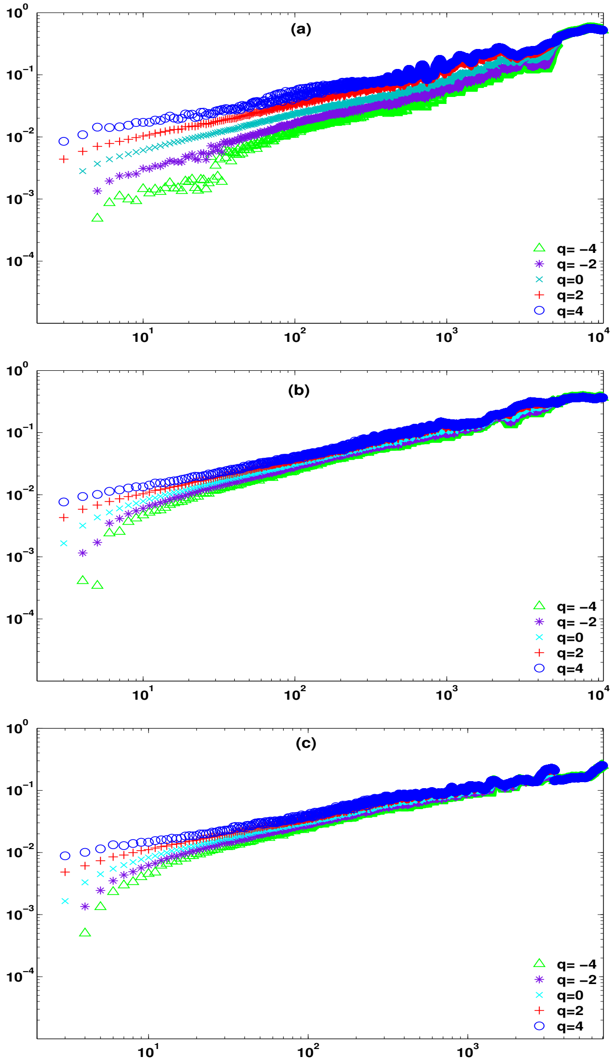

Fig. 2. The MF-DFA2 functions $F_q(s)$ of gold price time series *versus* the time scale s in log–log plot for (a) original, (b) shuffled and (c) surrogate data.

for various qs for the time series of gold price time series logarithmic variations. One can clearly observe that above the crossover region, the $F_q(s)$ functions are straight lines and the slopes increase slightly when going from high positive moments towards high negative moments.

For better understanding of the source of multifractality in time series we have analyzed the modified times series including shuffled and surrogated times series. This is because, generally, two different types of sources for multifractality in time series are identified: *(i)* multifractality due to different long-range temporal correlations for small and large fluctuations, and *(ii)* multifractality related to the fat-tailed probability distributions of variations. Shuffling, and phase randomization (surrogated data) are main procedure to find the contributions of two sources of multifractality and to indicate the multifractality strength. Shuffling preserves the distribution of the variations but destroys any temporal correlations. In fact, one can destroy the temporal correlations by randomly shuffling the corresponding time series of variations. What then remains are data with exactly the same fluctuation distributions but without any correlation. On the other hand, surrogate data is a method for testing the Gaussianity and one can eliminate the any sort of nonlinearity. In Fig. 2(b), 2(c) the MF-DFA2 fluctuations $F_q(s)$ for various qs for the shuffled and surrogated time series are shown respectively. It can be seen that in each case the slope of straight lines after the crossover area have significantly increased in comparison with the original time series.

The $h(q)$ spectra have been shown for original, reshuffled and surrogate series in Fig. 3. One can see that the q dependence of $h(q)$ for the original time series is higher than the two other randomized time series. But main feature of these two plots is that for the shuffled time series q dependence of $h(q)$ is lowest. These findings suggest that multifractality nature of gold price is due to the long-range correlation.

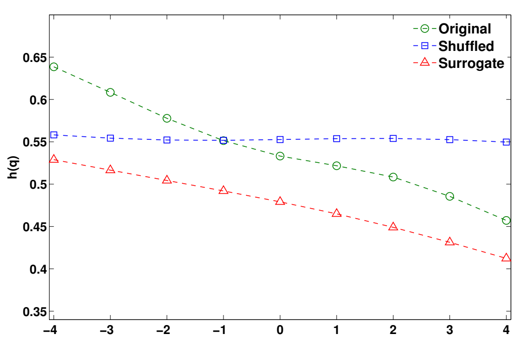

Fig. 3. Generalized Hurst exponent, $h(q)$ as a function of q for gold price time series.

In order to better analyzing the strength of multifractality for original, reshuffled and surrogate data, the singularity spectra are shown in Fig. 4. There is a clear difference between spectra for the original and the modified time series. The width of the singularity spectrum for the original data is $\Delta\alpha_{\rm org} \simeq 0.293$ while for shuffled and surrogate time series this value is $\Delta\alpha_{\rm shuf} \simeq 0.018$ and $\Delta\alpha_{\rm surr} \simeq 0.1778$, respectively. This also indicates that the main multifractality source in gold price fluctuations is long-rang correlations.

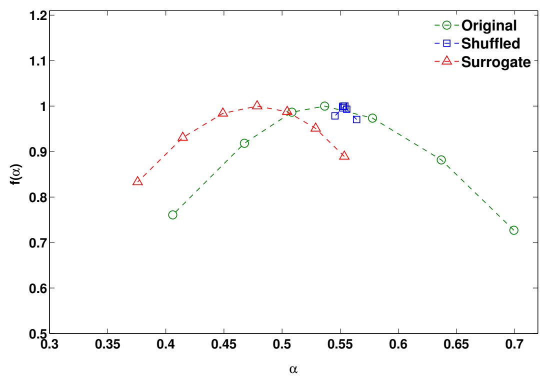

Fig. 4. Singularity spectrum $f(\alpha)$ for gold price time series.

In order to study the scaling character of the data, in Fig. 5, the multifractal scaling spectra $\tau(q)$ for gold price time series are shown. It is the well-established fact that monofractal time series are associated with a linear plot $\tau(q)$ while multifractal ones possess the spectra nonlinear in q. The more the nonlinearity of the spectrum, the stronger the multifractality nature in time series. It can be seen nonlinearity of $\tau(q)$ is much stronger for the original time series in comparison with two other modified time series. Again interestingly, the lowest nonlinearity of $\tau(q)$ is for the shuffled data.

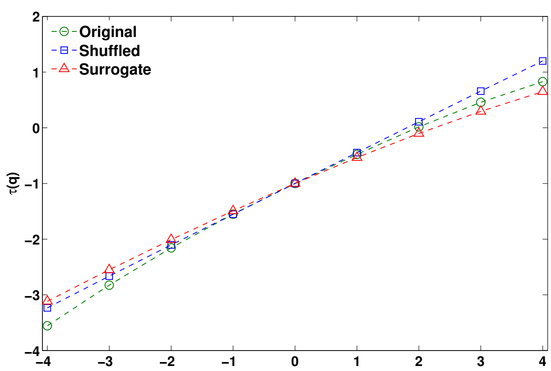

Fig. 5. Renyi exponent $\tau(q)$ for gold price time series.

5. Conclusion

We applied the MF-DFA technique to gold price time series in order to examine the fractal properties of the gold price fluctuations. By relating the singularity spectra of the original gold price series and of their shuffled counterparts one can conclude that the main source of multifractality in the gold price dynamics are the temporal correlations. Even more, based on the recent paper [32] one may expect that for sufficiently longer gold price time series (of the order of 10^5–10^6) the shuffled data singularity spectrum shrinks to a point (monofractal) which would provide an indication that the gold price multifractality originates entirely from temporal correlations.

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