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A SINGULARITY-FREE ROBERTSON–WALKER UNIVERSE IN THE GRAVITATION WITH VACUUM POLARIZATION

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The equations of the new theory of gravitation with vacuum polarization (TGVP) are offered. The case of the flat Robertson–Walker universe is considered. The variant of singularity-free model is constructed.

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It is known, that at present time the theories with varying "gravitational constant" κ are being widely discussed (see, for example, [1–6]). They can be "effective" as some generalizations of GR for the research of the astrophysical and cosmological problems with nonzero energy-momentum tensor. However, it is obvious, that in the framework of such theories the singularity (or collapse) problem of the spherical symmetric vacuum solutions remains open.

Therefore, it is interesting to consider of the more general variants of theories with varying "gravitational constant". We use for this purpose the formal analogy between gravitational equations and the correlations for the induction and intensity vectors of the electric and magnetic fields, which can be expressed in the following way

$$D_i = \varepsilon_{ik} E_k \,, \tag{1a}$$

$$B_i = \mu_{ik} H_k \,, \tag{1b}$$

where ε_{ik} is the dielectric permeability tensor, μ_{ik} is the magnetic permeability tensor.

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For the isotropic medium expressions (1a) has a form

$$D_i = \varepsilon E_i\,, \tag{2a}$$
$$B_i = \mu H_i\,. \tag{2b}$$

In analogic way, the Einstein's tensor $G_{\mu\nu}$ in the GR equations

$$G^{\mu}{}_{\nu} = \kappa\, T^{\mu}{}_{\nu} \tag{3}$$

can be considered (formally) as "gravitational intensity". Accordingly, the energy-momentum tensor can be interpreted as "induction of gravitational field". Then κ^{-1} play a role of "the gravitational permeability", which is constant in equation (3) (by analogy with (2b)). It is obvious, that for this interpretation the above mentioned theories with varying κ correspond to the variants of theories with "the inhomogeneous isotropic continuous medium", which is (in accordance with "classical" papers [7–9]) the physical vacuum. Thus, by analogy with equations (1a), the anisotropic continuous medium can be characterized by tensor of "gravitational permeability" $K^{\alpha}{}_{\beta}$. As in the case of the electromagnetic field, here the anisotropic $K_{\alpha\beta}$ describe the polarization of medium (physical vacuum).

We will consider a simpler variant of the new gravitational theory with vacuum polarization (TGVP), where "gravitational intensity" is defined by terms, which are linear with respect to the curvature tensor and his convolutions

$$\begin{aligned} T^{\mu}{}_{\nu} \;=\; & f_1 K^{\mu}{}_{\nu} R^{\mu\beta}{}_{\nu\alpha} + f_2 (K^{\mu}{}_{\alpha} R^{\alpha}{}_{\nu} + K^{\alpha}{}_{\nu} R^{\mu}{}_{\alpha}) \\ & + f_3 K^{\mu}{}_{\nu} R + f_4 K R^{\mu}{}_{\nu} + f_5 K R \delta^{\mu}{}_{\nu} \end{aligned} \tag{4}$$

(here f_i are arbitrary constants, $K = K^{\alpha}{}_{\alpha}$). For conditions

$$K^{\alpha}{}_{\beta} = \frac{1}{4\kappa}\,\delta^{\alpha}{}_{\beta}\,, \tag{5}$$

and

$$-\tfrac{1}{4} f_1 + \tfrac{1}{2} f_2 + f_4 \;=\; 1\,, \tag{6a}$$
$$\tfrac{1}{4} f_3 + f_5 \;=\; -\tfrac{1}{2} \tag{6b}$$

the equation (4) coincide with GR equation (3).

We will consider the system (4) for the flat Robertson–Walker cosmological model

$$ds^2 = -dt^2 + a^2(t)\left(dr^2 + r^2\left[d\theta^2 + \sin^2\theta d\phi^2\right]\right)\,. \tag{7}$$

It is known, that for the metrics (7) the GR equations have the form (see, for example, [10]):

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\varepsilon + 3p)\,, \tag{8a}$$

$$\frac{\dot{a}^2}{a^2} = \frac{\kappa}{3}\,\varepsilon\,, \tag{8b}$$

where $\varepsilon = -T^0{}_0$ and $T^1{}_1 = T^2{}_2 = T^3{}_3 = p$ (the dot denotes differentiation with respect to t).

It is obvious that the present functions ε and p cannot satisfy the "standard" conditions for "usual" matter $\varepsilon \geq p \geq 0$ (at least near the initial point of the cosmological evolution t_0, where $\dot{a}(t_0) = 0$; $\ddot{a}(t_0) > 0$) even for a variable "gravitational constant" κ.

Substituting now (7) into the system (4), we obtain (in the assumption of symmetry of $K^\alpha{}_\beta$)

$$\begin{aligned} T^0{}_0 = {} & \frac{1}{4a^2}\big(6\big(-2K^0{}_0 - 2K^1{}_1 - 2K^2{}_2 - 2K^3{}_3 \\ & + 3K^0{}_0 f_3 - K^1{}_1 f_3 - K^2{}_2 f_3 - K^3{}_3 f_3\big)\dot{a}^2 \\ & + a\ddot{a}\left(3K^0{}_0 - K^1{}_1 - K^2{}_2 - K^3{}_3\right)(f_1 + 6f_2 + 6f_3)\big)\,, \end{aligned} \tag{9a}$$

$$T^0{}_1 = \frac{K^0{}_1}{a^2}\left(2(f_2 + 3f_3)\dot{a}^2 + (f_1 + 4f_2 + 6f_3)a\ddot{a}\right)\,, \tag{9b}$$

$$T^0{}_2 = \frac{K^0{}_2}{a^2}\left(2(f_2 + 3f_3)\dot{a}^2 + (f_1 + 4f_2 + 6f_3)a\ddot{a}\right)\,, \tag{9c}$$

$$T^0{}_3 = \frac{K^0{}_3}{a^2}\left(2(f_2 + 3f_3)\dot{a}^2 + (f_1 + 4f_2 + 6f_3)a\ddot{a}\right)\,, \tag{9d}$$

$$\begin{aligned} T^1{}_1 = {} & \frac{1}{4a^2}\big(-4K^2{}_2 f_1\dot{a}^2 - 4K^3{}_3 f_1 a\ddot{a} + 24K^1{}_1 f_3\left(\dot{a}^2 + a\ddot{a}\right) \\ & - 6\left(K^0{}_0 + K^1{}_1 + K^2{}_2 + K^3{}_3\right)(2 + f_3)(\dot{a}^2 + a\ddot{a}) \\ & + \left(K^0{}_0 + K^1{}_1 + K^2{}_2 + K^3{}_3\right)(4 + f_1 - 2f_2)(2\dot{a}^2 + a\ddot{a}) \\ & + 8K^1{}_1 f_2(2\dot{a}^2 + a\ddot{a})\big)\,, \end{aligned} \tag{9e}$$

$$T^1{}_2 = \frac{K^1{}_2}{a^2}\left((f_1 + 4f_2 + 6f_3)\dot{a}^2 + 2(f_2 + 3f_3)a\ddot{a}\right)\,, \tag{9f}$$

$$T^1{}_3 = \frac{K^1{}_3}{a^2}\left((f_1 + 4f_2 + 6f_3)\dot{a}^2 + 2(f_2 + 3f_3)a\ddot{a}\right)\,, \tag{9g}$$

$$\begin{aligned} T^2{}_2 = {} & \frac{1}{4a^2}\big(-4K^1{}_1 f_1\dot{a}^2 - 4K^3{}_3 f_1 a\ddot{a} + 24K^2{}_2 f_3\left(\dot{a}^2 + a\ddot{a}\right) \\ & - 6\left(K^0{}_0 + K^1{}_1 + K^2{}_2 + K^3{}_3\right)(2 + f_3)(\dot{a}^2 + a\ddot{a}) \\ & + \left(K^0{}_0 + K^1{}_1 + K^2{}_2 + K^3{}_3\right)(4 + f_1 - 2f_2)(2\dot{a}^2 + a\ddot{a})\big)\,, \end{aligned} \tag{9h}$$

$$
\begin{aligned}
T^2{}_3 &= \frac{K^2{}_3}{a^2}\left((f_1+4f_2+6f_3)\dot a^2+2(f_2+3f_3)a\ddot a\right) , &(9i)\\
T^3{}_3 &= \frac{1}{4a^2}\Big(-4K^1{}_1f_1\dot a^2-4K^2{}_2f_1a\ddot a+24K^3{}_3f_3(\dot a^2+a\ddot a)\\
&\quad -6\left(K^0{}_0+K^1{}_1+K^2{}_2+K^3{}_3\right)(2+f_3)(\dot a^2+a\ddot a)\\
&\quad +\left(K^0{}_0+K^1{}_1+K^2{}_2+K^3{}_3\right)(4+f_1-2f_2)\left(2\dot a^2+a\ddot a\right)\\
&\quad +8K^3{}_3f_2\left(2\dot a^2+a\ddot a\right)\Big) . &(9j)
\end{aligned}
$$

We research a possibility for construction of cosmological model with initial Big Bang at “the minimal deviation” from GR

$$K^1{}_1 = K^2{}_2=K^3{}_3=\frac{1}{4\kappa} \tag{10a}$$

and

$$K^0{}_0 = \frac{1}{4\kappa}+\mathit{\Delta}\,, \tag{10b}$$

where $\mathit{\Delta}$ is a small positive parameter.

Then the system (9a) is reduced to the equations

$$
\begin{aligned}
\kappa\,\varepsilon &= -\frac{3}{4}\kappa\,\mathit{\Delta}(f_1+6f_2+6f_3)\dot h\\
&\quad +\frac{3}{4}\,h^2(4+4\kappa\mathit{\Delta}-\kappa\mathit{\Delta}(f_1+6f_2+12f_3))\,, &(11a)\\
\kappa\,p &= -\frac{1}{4}\dot h(8+8\kappa\,\mathit{\Delta}+\kappa\,\mathit{\Delta}(3f_1+2f_2+6f_3))\\
&\quad -\frac{1}{4}\,h^2(12+12\kappa\,\mathit{\Delta}+\kappa\,\mathit{\Delta}(f_1+6f_2+12f_3))\,, &(11b)
\end{aligned}
$$

where $h=\dot a/a$.

Thus, the equations (11) contain 3 functions and 4 parameters. We shall choose the following additional condition

$$a=\left(a_0+\alpha t^2\right)^{\frac{1}{2}}\;, \tag{12}$$

here α and a_0 — the arbitrary positive constants.

Expression (12) corresponds to model of unlimited expansion (“asymptotically inertial”) without initial singularity.

Substituting function (12) in the equations (11), we obtain

$$\begin{aligned}
\kappa\,\varepsilon \;&=\; \frac{3\alpha^2 t^2}{(1+\alpha t^2)^2}\left[1+\left(1+\tfrac{1}{4}(f_1+6f_2)\right)\kappa\,\mathit{\Delta}\right] \\
&\quad -\frac{3}{4}\frac{\alpha}{(1+\alpha t^2)}(f_1+6f_2+6f_3)\kappa\,\mathit{\Delta}\,, \qquad (13a)\\
\kappa\,p \;&=\; -\frac{\alpha}{(1+\alpha t^2)}\left(2+\left(2+\tfrac{3}{4}f_1+\tfrac{1}{2}f_2+\tfrac{3}{2}f_3\right)\kappa\,\mathit{\Delta}\right) \\
&\quad +\frac{\alpha^2 t^2}{(1+\alpha t^2)^2}\left(1+\left(1+\tfrac{5}{4}f_1-\tfrac{1}{2}f_2\right)\kappa\,\mathit{\Delta}\right)\,. \qquad (13b)
\end{aligned}$$

We research applicability for expressions (13) sets of the physical inequalities corresponding to “usual” matter:

$$\varepsilon \;\geq\; p \geq 0\,, \qquad (14a)$$

$$0 \;\leq\; \frac{\dot{p}}{\dot{\varepsilon}} \leq 1\,. \qquad (14b)$$

It is easy to examine that the sufficient conditions for performance of requirement (14a) are

$$\begin{aligned}
(2+\tfrac{3}{4}f_1+\tfrac{1}{2}f_2+\tfrac{3}{2}f_3)\,\kappa\mathit{\Delta} \;&\leq\; -2\,, \qquad (15a)\\
(1+\tfrac{5}{4}f_1-\tfrac{1}{2}f_2)\,\kappa\mathit{\Delta} \;&\geq\; -1\,, \qquad (15b)\\
(2-4f_2-3f_3)\,\kappa\mathit{\Delta} \;&\geq\; -2\,, \qquad (15c)\\
(2-\tfrac{1}{2}f_1+5f_3)\,\kappa\mathit{\Delta} \;&\geq\; -2\,. \qquad (15d)
\end{aligned}$$

The principle of causality (14b) gives the following constraints:

$$\begin{aligned}
(2+f_1+6f_2+3f_3)\,\kappa\mathit{\Delta} \;&\leq\; -2\,, \qquad (16a)\\
\left(-\tfrac{1}{3}f_1+6f_2+2f_3\right)\,\kappa\mathit{\Delta} \;&\geq\; 0\,, \qquad (16b)\\
2+(2-3f_3)\,\kappa\mathit{\Delta} \;&\geq\; 0\,, \qquad (16c)\\
-8+(-8+f_1-2f_2+6f_3)\,\kappa\mathit{\Delta} \;&\geq\; 0\,. \qquad (16d)
\end{aligned}$$

It is easy to examine, that all requirements (15), (16) are satisfied at $\mathit{\Delta} > 0$ and

$$f_3 \;\leq\; -\frac{10}{3}\,, \qquad (17a)$$

$$f_2 \;\geq\; \frac{1}{18}-\frac{1}{3}f_3\,, \qquad (17b)$$

$$f_1 \;\geq\; 12f_2+\frac{41}{4}f_3\,. \qquad (17c)$$

Hence, the performance of standard physical inequalities for a "usual" matter appears possible at a wide set of parameters. In result, we conclude that the present model "with minimal deviation from GR" is realistic. Besides it possesses non-negative cosmological acceleration $\ddot{a}$ at all values t according to "modern wishes" based on observations I_a supernova. However, speed of expansion $\dot{a}$ remains finite.

Thus, the question about the formulation of tests for check availability of vacuum polarization is interesting.

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