RADIAL EXCITATION MASS SPECTRUM OF TENSOR MESON NONET

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Based on the relations derived from the Regge phenomenology, we investigate the mass spectrum of radial excitation of tensor meson nonet. The results suggest that the states $f_2(1810)$, $f_2(2010)$ and $K_2^*(1980)$ should be assigned as the first radial excitation of tensor meson. Our prediction can be useful for the assignment of tensor meson nonet in the future.

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1. Introduction

Quantum Chromodynamics (QCD) is widely accepted as a successful theory of the strong interactions in particle physics. However, the understanding of the strong interactions is far from complete and it is difficult to interpret the particle properties for the experimental data from the first principles up to now. To be able to interpret the nature of a new resonance, it is important to create a template to compare observed states with theoretical predictions. Therefore, different models, *e.g.* quenched lattice gauge theory [1], the Dyson–Schwinger formalism [2], constituent quark model [3], are built to explain the properties of experimental data.

In recent years, with the development of relative experiment and theories, the assignment of meson state becomes more explicit. In the new edition of Particle Data Group (PDG) [4], the ground mesons have been assigned explicitly. However, there are many mesons absent from the meson Summary Table [4], especially for the radial excitations of mesons. In order to complete and identify the meson spectrum, there are still a lot of work to be done both theoretically and experimentally.

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Regge theory is concerned with particle spectrum, the forces and the high energy behavior of scattering amplitudes. In the last decade, the Regge trajectories are reconsidered with the great quantity of new resonances appearance. On the one hand, people attempt to identify the problematic meson states by using the Regge trajectories. On the other hand, abundance of candidate states which are unobserved need to be estimated and predicted.

In the paper, we concentrate on the tensor meson and its radial excited state. According to the PDG, the states $a_2(1325)$, $f_2(1270)$, $f_2(1525)$ and $K_2(1430)$ are accepted as the ground state tensor meson nonet. The heavy–light and heavy–heavy meson of tensor meson have also been identified recently. However, the assignment of radial excitation of tensor meson nonet remains interesting and received much less attention. For the light meson sector, several additional isoscalar states have been observed, the states $f_2(1430)$, $f_2(1565)$, $f_2(1640)$, $f_2(1810)$, $f_2(1910)$, $f_2(2150)$, $f_2(1950)$, $f_2(2010)$, $f_2(2300)$, $f_2(2340)$ need to be confirmed in the experiments and one of which is a candidate for the tensor glueball [5, 6]. So far, none of the ten reported isoscalars can be definitely assigned to the expected radial excitation of tensor meson. In this work, we calculate the masses of radial excitation of 1^3P_2 meson nonet, and the results are compared with other references from different approaches.

2. Regge phenomenology

According to the hadron with a set of given quantum number belonging to a quasi-linear trajectory, we will have the following relation

$$J = \alpha_{i\bar{j}'N}(0) + \alpha'_{i\bar{j}'N} M^2_{i\bar{j}'N} \,, \tag{1}$$

where $i\bar{j}'$ refers to the quark (antiquark) flavor, J and $M_{i\bar{j}'}$ are respectively the spin and mass of the $i\bar{j}'$ meson, N is the radial quantum number. The parameters $\alpha'_{i\bar{i}'N}$ and $\alpha_{i\bar{i}'N}(0)$ are respectively the slope and intercept of the trajectory. The intercepts and slopes can be parameterized by [7, 8, 9, 10]

$$\alpha_{i\bar{i}N}(0) + \alpha_{j\bar{j}N}(0) \; = \; 2\alpha_{i\bar{j}N}(0) \,, \tag{2}$$

$$\frac{1}{\alpha'_{i\bar{i}'N}} + \frac{1}{\alpha'_{j\bar{j}'N}} \; = \; \frac{2}{\alpha'_{i\bar{j}'N}} \,. \tag{3}$$

The relation (2) is satisfied in two-dimensional QCD [11], the dual-analytic model [12], and the quark bremsstrahlung model [13]. The relation (3) is deprived from the topological and the $q\bar{q}$-string picture of hadrons [14]. In recent years, in view of available data for mesonic resonances of light, medium and heavy flavors, Filipponi has constructed a slope of linear Regge trajectories for all quark flavors [15]

$$\alpha'_{i\bar{j}} = \frac{0.9\ \mathrm{GeV}^{-2}}{1 + 0.22\left(\frac{m_i+m_j}{\mathrm{GeV}}\right)^{3/2}}\,. \tag{4}$$

In Ref. [16], Burakovsky and Goldman indicate that the relation (4) is not only a form of Regge slope, they construct a slope formula (5) also for all quark flavors

$$\frac{\pi}{4}\alpha'_{ji} + \frac{\pi}{4}\sqrt{\alpha'}\frac{m_i+m_j}{2}\alpha'_{ji} = \alpha'\,, \tag{5}$$

where m_i, m_j are the corresponding constituent quark masses, and the $\alpha' = 0.88\ \mathrm{GeV}^{-2}$ is the standard Regge slope. According to the relations (4) and (5), we find the slope of Regge trajectory depend on the constituent quark masses through the combination $(m_i + m_j)$.

As the slopes of the parity partners trajectories coincide, and the ground and the radial excitation have the same slopes, we obtain from the relation (1)

$$M^2_{i\bar{j}N}\alpha'_{i\bar{j}1} - M^2_{i\bar{j}1}\alpha'_{i\bar{j}1} = \alpha_{i\bar{j}1}(0) - \alpha_{i\bar{j}N}(0)\,. \tag{6}$$

Based on the fact that the values of $\alpha_{i\bar{j}1}(0) - \alpha_{i\bar{j}N}(0)$ depend on the constituent quarks masses, we introduce two parameters $f_{i\bar{j}}(m_i + m_j)$ and $C_{i\bar{j}}$ into Eq. (6), so we have the following relation [9, 17]

$$M^2_{i\bar{j}N}\alpha'_{i\bar{j}1} = N + Nf_{i\bar{j}}(m_i + m_j) + \alpha'_{i\bar{j}1}C_{i\bar{j}}\,. \tag{7}$$

And the parameter $f_{i\bar{j}}(m_i + m_j)$ is described as

$$2f_{i\bar{j}}(m_i + m_j) = f_{i\bar{i}}(m_i + m_i) + f_{j\bar{j}}(m_j + m_j)\,. \tag{8}$$

From the relations (7) and (8), the following relations (where and below n denotes u and d quark) are obtained

$$M^2_{n\bar{n}1}\alpha'_{n\bar{n}1} = 1 + f_{n\bar{n}}(m_n + m_n) + \alpha'_{n\bar{n}1}C_{n\bar{n}}\,, \tag{9}$$

$$M^2_{n\bar{s}1}\alpha'_{n\bar{s}1} = 1 + f_{n\bar{s}}(m_n + m_s) + \alpha'_{n\bar{s}1}C_{n\bar{s}}\,, \tag{10}$$

$$M^2_{s\bar{s}1}\alpha'_{s\bar{s}1} = 1 + f_{s\bar{s}}(m_s + m_s) + \alpha'_{s\bar{s}1}C_{s\bar{s}}\,, \tag{11}$$

$$M^2_{n\bar{n}2}\alpha'_{n\bar{n}1} = 2 + 2f_{n\bar{n}}(m_n + m_n) + \alpha'_{n\bar{n}1}C_{n\bar{n}}\,, \tag{12}$$

$$M^2_{n\bar{s}2}\alpha'_{n\bar{s}1} = 2 + 2f_{n\bar{s}}(m_n + m_s) + \alpha'_{n\bar{s}1}C_{n\bar{s}}\,, \tag{13}$$

$$M^2_{s\bar{s}2}\alpha'_{s\bar{s}1} = 2 + 2f_{s\bar{s}}(m_s + m_s) + \alpha'_{s\bar{s}1}C_{s\bar{s}}\,, \tag{14}$$

$$2f_{n\bar{s}}(m_n + m_s) = f_{n\bar{n}}(m_n + m_n) + f_{s\bar{s}}(m_s + m_s)\,. \tag{15}$$

In addition to Regge phenomenology, we can also use mass matrix for the assignment of isoscalar state. In the quark model, the two isoscalar states with the same J^{PC} will mix to form the physical states. Therefore, we can establish the mass-squared matrix in the $s\bar{s}$ and $N = (u\bar{u} + d\bar{d})/\sqrt{2}$ basis [10]

$$M^2 = \begin{pmatrix} M^2_{n\bar{n}N} + 2A_{nn} & \sqrt{2}A_{ns} \\ \sqrt{2}A_{ns} & 2M^2_{n\bar{s}N} - M^2_{n\bar{n}N} + A_{ss} \end{pmatrix}, \tag{16}$$

where $M_{n\bar{n}N}$ and $M_{n\bar{s}N}$ are the masses of isovector and isodoublet states of the N^3P_2 meson nonet, respectively; A_{nn}, A_{ns} and A_{ss} are the mixing parameter which describe the $q\bar{q} \leftrightarrow q'\bar{q}'$ transition amplitudes. In order to reduce the number of parameters, we adopt the similar expression of the transition amplitudes in the $q\bar{q} \leftrightarrow q'\bar{q}'$ process which is widely used in Refs. [18, 19, 20]

$$\left\{ \begin{array}{c} A_{nn} = \frac{\Lambda}{m_n m_n} \\ A_{ss} = \frac{\Lambda}{m_s m_s} \\ A_{ns} = \frac{\Lambda}{m_n m_s} \end{array} \right\}, \tag{17}$$

where Λ is a phenomenological parameter. Based on the isospin symmetry, we have $m_u = m_{\bar{u}} = m_d = m_{\bar{u}}$, $m_s = m_{\bar{s}}$ (m_n, m_s denote the mass of light quark u, d and s).

In the N^3P_2 meson nonet, we assume the physical states are the eigenstates of mass squared matrix and the masses are the eigenvalues, respectively

$$UM^2U^\dagger = \begin{pmatrix} M^2_{f_2} & 0 \\ 0 & M^2_{s\bar{s}N} \end{pmatrix}. \tag{18}$$

According to the relations (16), (17) and (18), we will obtain

$$2\frac{\Lambda}{m_n^2} + 2M^2_{n\bar{s}N} + \frac{\Lambda}{m_s^2} = M^2_{f_2} + M^2_{s\bar{s}N}, \tag{19}$$

$$\left(M^2_{n\bar{n}N} + 2\frac{\Lambda}{m_n^2}\right)\left(2M^2_{n\bar{s}N} - M^2_{n\bar{n}N} + \frac{\Lambda}{m_s^2}\right) - 2\frac{\Lambda^2}{m_s^2 m_n^2} = M^2_{f_2} M^2_{s\bar{s}N}. \tag{20}$$

In this work, we use the values $f_{i\bar{j}}$ and $\alpha_{i\bar{j}}$ in Refs. [9, 17] as input parameters

$$\left\{ \begin{array}{c} f_{n\bar{n}} = 0.6277 \\ f_{s\bar{s}} = 0.5644 \\ f_{n\bar{s}} = 0.5889 \end{array} \right\} \tag{21}$$

and

$$\left\{ \begin{array}{c} \alpha'_{n\bar{n}} = 0.8830 \\ \alpha'_{s\bar{s}} = 0.8181 \\ \alpha'_{n\bar{s}} = 0.8493 \end{array} \right\}. \quad (22)$$

Inserting the masses of ground tensor meson, $M_{a_2(1320)}$, $M_{f_2(1525)}$, $M_{K_2^*(1430)}$ and the constituent quark masses (see Table I), into relations (9)–(15), we obtain the values of $C_{i\bar{j}}$ for the tensor meson

$$\left\{ \begin{array}{c} C_{n\bar{n}} = 0.1490 \pm 0.0016 \\ C_{s\bar{s}} = 0.4255 \pm 0.0152 \\ C_{n\bar{s}} = 0.3015 \pm 0.0043 \end{array} \right\}. \quad (23)$$

TABLE I

Constituent quark masses (in MeV) in different phenological models.

Mass	Ref. [21]	Ref. [22]	Ref. [23]	Ref. [24]	Ref. [25]	Average
$m_n(n = u, d)$	290	360	337.5	311	310	321
m_s	460	540	486	487	483	491.2

By using the relations (12)–(14), (19) and (20), we obtain the masses of radial excitation of tensor meson. The results are shown in Table II and Fig. 1. Considering the errors of parameters $C_{n\bar{n}}$, $C_{n\bar{s}}$, $C_{s\bar{s}}$ and ground meson nonet state of tensor meson, we estimate the errors of calculated masses of resonances in Table II. All the masses used as input for our calculation are taken from PDG.

TABLE II

Predicted masses (in GeV) of radial excitations of N^3P_2.

	1^3P_2	2^3P_2			3^3P_2	
	Ref. [4]	Present work	Ref. [19]	Ref. [26]	Present work	Ref. [26]
$M_{n\bar{n}}$	1.318	1.8239±0.0004	1.82	1.779	2.2171±0.0004	2.049
$M_{n\bar{s}}$	1.429	1.9446±0.0011	1.94	1.896	2.3497±0.0009	2.206
$M_{s\bar{s}}$	1.525	2.0557±0.0037	2.04	2.030	2.4750±0.0031	2.412
M_{f_2}	1.275	1.8073±0.0266	1.82		2.2124±0.0221	

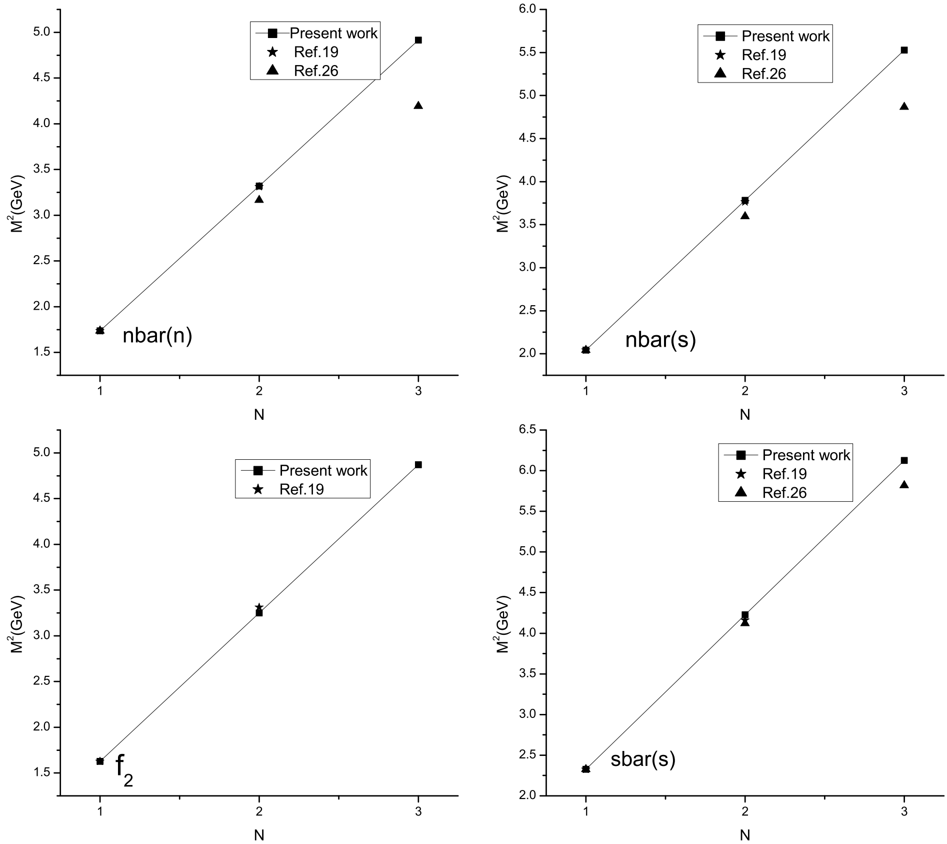

Fig. 1. The quasi-linear trajectory for N^3P_2 meson nonet.

3. Conclusion

In this paper, combining the Regge and meson mass matrix, we predict the radial excited mass spectrum of tensor meson nonet. According to the predictions in Table II, we suggest that the states $f_2(1810)$, $f_2(2010)$ and $K_2^*(1980)$ should be assigned as the first radial excited of tensor meson. This results are in good agreement with the values which were predicted in a relativized quark model [19]. For the isovector sector, the $a_2(1320)$ is assigned as the ground state of tensor meson. The first radial excited state of $a_2(1320)$ remains interesting, $a_2(1660)$ is a possible candidate for this state. But this arrangement is still a great uncertainty, the masses are not consistent from different experiments. The L3 Collaboration reported the $a_2(1700)$ in $\rho\pi$ and $f_2\pi$ decay modes mainly with mass 1767 ± 14 MeV and width 187 ± 60 MeV [27]. The Belle Collaboration observed a resonance in $\gamma\gamma \to K^+K^-$ at $1737 \pm 5 \pm 7$ MeV [28]. The Crystal Barrel Collaboration reported the $a_2(1700)$ in the $\pi^0\eta$ mode in $\bar{p}p$ collisions with mass 1600 ± 40 MeV and width 208 ± 70 MeV [29,30]. Moreover, if the state $f_2(1810)$ and $K_2^*(1980)$ are assigned as 2^3P_2 members, with the help of the relation derived from the Gell-Mann–Okubo formula $M^2_{n\bar{n}2} = 4M^2_{K^*(1980)} - 2M^2_{f_2(2010)} - M^2_{f_2(1810)}$, we have $M_{n\bar{n}2} = 1851$ MeV. The value is consistent with our previous prediction 1824 ± 21 MeV. Therefore, we suggest further testing of the assignment of this state in the new experiments. In view of Fig. 1, we also conclude that the quasi-linear trajectory should give a reasonable description for the radial excitation of meson. The errors of resonances in Table II are too small to have little effect in Fig. 1. Our results would be useful for the assignment of tensor meson in the future.

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