THE NEUTRAL HEAVY SCALAR PRODUCTIONS ASSOCIATED WITH Z_L IN THE LITTLEST HIGGS MODEL AT ILC AND CLIC

A. Çağil[†], M.T. Zeyrek[‡]

Physics Department, Middle East Technical University
06531 Ankara, Turkey

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In this work, the production processes of heavy neutral scalar and pseudoscalar associated with standard model gauge boson Z_L at future e^+e^- colliders (ILC and CLIC) are examined. The total and differential cross-sections are calculated for the processes in the context of the littlest Higgs model. Dependence of production processes to littlest Higgs model parameters in the range of compatibility with electroweak precision measurements and decays to lepton-flavor violating final states are also analyzed. We have found that both heavy scalar and pseudoscalar will be produced in e^+e^- colliders. Depending on the model parameters, the neutral heavy scalar can be reconstructed or lepton-flavor violating signals can be observed.

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1. Introduction

Standard Model (SM) is an effective theory with a cut-off scale around electroweak symmetry breaking (EWSB) scale. However in SM, Higgs scalar giving mass to fermions and gauge bosons gets loop corrections to its mass up to cut off scale, which is called the hierarchy problem. The little Higgs models [1, 2, 3, 4] are introduced to solve the hierarchy problem among the alternative solutions such as supersymmetry, extra dimensions and dynamical symmetry breaking models. The little Higgs models propose a solution by enlarging the symmetry group of the SM. The constraints on little Higgs

† `ayse.cagil@cern.ch`
‡ `zeyrek@metu.edu.tr`

models are studied [5, 6, 7, 8, 9, 10, 11], and the phenomenology of the little Higgs models are reviewed [12, 13, 14]. The little Higgs models are also expected to give significant signatures in future high energy colliders and studied [15, 16, 17, 18, 19, 20].

In the littlest Higgs model [1], as a result of enlarged symmetry group, there appears new vector gauge bosons and also a new heavy scalar triplet. The appearance of new scalars in the littlest Higgs model result in lepton-flavor violation when a 5D operator is implemented in the Yukawa Lagrangian [21, 22, 23, 24].

In this work, we examined the production of neutral scalar (ϕ^0) and pseudoscalar ($\phi^{\rm P}$) associated with Z_L boson in the littlest Higgs model at e^+e^- colliders, namely, International Linear Collider (ILC) [25] and Compact Linear Collider (CLIC) [26]. To analyze the production rates, firstly the most promising channel $e^+e^- \to Z_L\phi^0$ is analyzed. Since the final signals of ϕ^0 and $\phi^{\rm P}$ is the same, to analyze the behavior of the final states for the energies ($\sqrt{s} > 1$ TeV), the higher order production processes: $e^+e^- \to Z_L\phi^0\phi^0$, $e^+e^- \to Z_L\phi^{\rm P}\phi^{\rm P}$ and $e^+e^- \to Z_L\phi^0\phi^{\rm P}$ are also examined. Since the process $e^+e^- \to Z_L\phi^{\rm P}$ is not allowed in the littlest Higgs model, the latter two processes involving $\phi^{\rm P}$ are also important for the $\phi^{\rm P}$ production. Finally, lepton-flavor violating signals of neutral scalars as "Z_L+ missing energy" which characterizes the new neutral scalar and pseudoscalar to be littlest Higgs are analyzed [21].

In this paper, we present the relevant formulas and calculations in Section 2. In Section 3, the results and discussions are presented.

2. Theoretical framework

In the littlest Higgs model global symmetry SU(5) is broken spontaneously to SO(5) at an energy scale $f \sim 1$ TeV leaving 14 Nambu–Goldstone bosons (NGB) corresponding to broken symmetries. In the model SU(5) contains the gauged subgroup $[{\rm SU}(2)_1 \otimes {\rm U}(1)_1] \otimes [{\rm SU}(2)_2 \otimes {\rm U}(1)_2]$. As a consequence symmetry breaking, gauge bosons gain mass by eating the four of the NGBs. The mixing angles between the SU(2) subgroups and between the U(1) subgroups are defined respectively as

$$s \equiv \sin\theta = \frac{g_2}{\sqrt{g_1^2+g_2^2}}\,, \qquad s' \equiv \sin\theta' = \frac{g'_2}{\sqrt{g_1'^2+g_2'^2}}\,, \tag{1}$$

where g_i and g'_i are the gauge couplings of ${\rm SU}(2)_i$ and ${\rm U}(1)_i$ subgroups, respectively. By EWSB vector bosons get extra mixings due to vacuum expectation values of h doublet and ϕ triplet resulting the final masses to the order of $\frac{v^2}{f^2}$ such as [13]

$$
\begin{aligned}
M_{A_L}^2 &= 0\,, \\
M_{Z_L}^2 &= m_z^2\left[1-\frac{v^2}{f^2}\left(\frac{1}{6}+\frac{1}{4}\left(c^2-s^2\right)^2+\frac{5}{4}\left(c'^2-s'^2\right)^2\right)+8\frac{v'^2}{v^2}\right]\,, \\
M_{A_H}^2 &= \frac{f^2g'^2}{20s'^2c'^2}-\frac{1}{4}g'^2v^2+g^2v^2\frac{x_H}{4s^2c^2} \\
&= m_z^2s_{\mathrm{w}}^2\left(\frac{f^2}{5s'^2c'^2v^2}-1+\frac{x_Hc_{\mathrm{w}}^2}{4s^2c^2s_{\mathrm{w}}^2}\right)\,, \qquad (2) \\
M_{Z_H}^2 &= \frac{f^2g^2}{4s^2c^2}-\frac{1}{4}g^2v^2-g'^2v^2\frac{x_H}{4s'^2c'^2} \\
&= m_w^2\left(\frac{f^2}{s^2c^2v^2}-1-\frac{x_Hs_{\mathrm{w}}^2}{s'^2c'^2c_{\mathrm{w}}^2}\right)\,, \qquad (3)
\end{aligned}
$$

where $m_z \equiv gv/(2c_{\mathrm{w}})$ and $x_H = \frac{5}{2}gg'\frac{scs'c'(c^2s'^2+s^2c'^2)}{(5g^2s'^2c'^2-g'^2s^2c^2)}$ and s_{w} and c_{w} are the usual weak mixing angles. The parameters v and v' are the v.e.v. of scalar doublet and triplet given as [13]

$$\langle h^0\rangle = v/\sqrt{2}\,, \qquad \langle i\phi^0\rangle = v' \le \frac{v^2}{4f} \tag{4}$$

bounded by electroweak precision data, where $v = 246$ GeV. Moreover, diagonalizing the mass matrix for scalars the physical states are found to be the SM Higgs scalar H, the neutral scalar ϕ^0, the neutral pseudoscalar ϕ^{P}, and the charged scalars ϕ^+ and ϕ^{++}. The masses of the heavy scalars are degenerate, and in terms of Higgs mass expressed as [13]

$$M_\phi = \frac{\sqrt{2}f}{v\sqrt{1-\left(\frac{4v'f}{v^2}\right)^2}}M_H\,. \tag{5}$$

The scalar fermion interactions in the model are written in Yukawa Lagrangian preserving gauge symmetries of the model for SM leptons and quarks, including the third generation having an extra singlet, the T quark. The fermions in the littlest Higgs model can be charged under both $\mathrm{U}(1)_1$ and $\mathrm{U}(1)_2$ subgroups [10, 13]. Besides for light fermions, a lepton number violating coupling can be implemented in Yukawa Lagrangian [21,22] which results in lepton-flavor violation by unit two, such as

$$\mathcal{L}_{\rm LFV} = iY_{ij}L_i^T\ \phi C^{-1}L_j + {\rm h.c.}\,, \tag{6}$$

where L_i are the lepton doublets ($l\ \nu_l$), and Y_{ij} are the elements of the mixing matrix with $Y_{ii} = Y$ and $Y_{ij(i\neq j)} = Y'$. The values of Yukawa couplings Y and Y' are restricted by the current constraints on the neutrino masses [27] given as: $M_{ij} = Y_{ij}v' \simeq 10^{-10}$ GeV [21]. Since the v.e.v. v' has only an upper bound (Eq. (4)), Y_{ij} can be taken up to the order of unity without making v' unnaturally small.

The parameters f, the symmetry breaking scale, and s, s', the mixing angles of the littlest Higgs model, are not restricted by the model. These parameters are constrained by observables of electroweak precision data and the direct search for a heavy gauge bosons at Tevatron [5,6,7,8,9,10]. In the case when fermions are charged under both U(1) groups, the allowed parameter space is listed as follows. For the values of the symmetry breaking scale 1 TeV $\leq f \leq$ 2 TeV, mixing angles are in the range $0.75 \leq s \leq 0.99$ and $0.6 \leq s' \leq 0.75$, for 2 TeV $\leq f \leq$ 3 TeV they have acceptable values in the range $0.6 \leq s \leq 0.99$ and $0.6 \leq s' \leq 0.8$, for 3 TeV $\leq f \leq$ 4 TeV they are in the range $0.4 \leq s \leq 0.99$ and $0.6 \leq s' \leq 0.85$, and for the higher values of the symmetry breaking scale, *i.e.* $f \geq 4$ TeV, the mixing angles are less restricted and they are in the range $0.15 \leq s \leq 0.99$ and $0.4 \leq s' \leq 0.9$ [10].

In the model, the couplings of vector bosons to fermions are written as $i\gamma_\mu(g_{V_i} + g_{A_i}\gamma_5)$, where $i = 1,2,3,4$ corresponds to Z_L, Z_H, A_H and A_L, respectively. The couplings of gauge vector to e^+e^- pairs are given in Table I, where $y_e = \frac{3}{5}$, $e = \sqrt{4\pi\alpha}$, $x_Z^{W'} = -\frac{1}{2c_{\rm w}}sc(c^2 - s^2)$ and $x_Z^{B'} = -\frac{5}{2s_{\rm w}}s'c'(c'^2 - s'^2)$.

The total decay widths of SM vector bosons also get corrections of the order of $\frac{v^2}{f^2}$, since the decay widths of vectors to fermion couples are written as: $\Gamma(V_i \to f\bar{f}) = \frac{N}{24\pi}(g_V^2 + g_A^2)M_{V_i}$, where $N = 3$ for quarks, and $N = 1$ for leptons. The total decay widths of the new vector bosons are given as [18]

$$\begin{aligned}
\Gamma_{A_H} &\approx \frac{g'^2 M_{A_H}\left(21 - 70s'^2 + 59s'^4\right)}{48\pi s'^2\left(1 - s'^2\right)}\,, \\
\Gamma_{Z_H} &\approx \frac{g^2\left(193 - 388s^2 + 196s^4\right)}{768\pi s^2\left(1 - s^2\right)}M_{Z_H}\,.
\end{aligned} \tag{7}$$

The final decays and also the decay widths of ϕ^0 and $\phi^{\rm P}$ are studied in detail in Ref. [21], and they are strongly dependent on the v.e.v. of the

TABLE I

The vector and axial vector couplings of e^+e^- with vector bosons. Feynman rules for $e^+e^-V_i$ vertices are given as $i\gamma_\mu(g_{V_i}+g_{A_i}\gamma_5)$.

i	Vertices	g_{V_i}	g_{A_i}
1	$e^+e^-Z_L$	$\frac{-g}{2c_{\rm w}}\left\{-\frac{1}{2}+2s_{\rm w}^2-\frac{v^2}{f^2}\left[\frac{-cc_{\rm w}x_Z^{W'}}{2s}+\frac{s_{\rm w}x_Z^{B'}}{s'c'}\left(2y_e-\frac{9}{5}+\frac{3}{2}c'^2\right)\right]\right\}$	$\frac{-g}{2c_{\rm w}}\left\{\frac{1}{2}-\frac{v^2}{f^2}\left[\frac{cc_{\rm w}x_Z^{W'}}{2s}+\frac{s_{\rm w}x_Z^{B'}}{s'c'}\left(-\frac{1}{5}+\frac{1}{2}c'^2\right)\right]\right\}$
2	$e^+e^-Z_H$	$-gc/4s$	$gc/4s$
3	$e^+e^-A_H$	$\frac{g'}{2s'c'}\left(2y_e-\frac{9}{5}+\frac{3}{2}c'^2\right)$	$\frac{g'}{2s'c'}\left(-\frac{1}{5}+\frac{1}{2}c'^2\right)$
4	$e^+e^-A_L$	$-e$	0

scalar triplet, v'. For $v' \gtrsim 1$ GeV, the decay modes of ϕ^0 include decays into quark pairs, $t\bar{t}$, $b\bar{b}$ and $t\bar{T}+T\bar{t}$, and also decays into SM pairs, Z_LZ_L and HH. In this case, the decays of $\phi^{\rm P}$ are similar to ϕ^0 as the decays into quark pairs: $t\bar{t}$, $b\bar{b}$ and $t\bar{T}+T\bar{t}$, and to SM Z_LH couples different from ϕ^0. For $v' \sim 10^{-10}$ GeV, the non leptonic decays are suppressed by a factor of $\frac{v'}{v}$ for both ϕ^0 and $\phi^{\rm P}$, and the final states contain only lepton-flavor violating decays to $\nu_i\nu_j+\bar{\nu}_i\bar{\nu}_j$. In this work, we analyze the cases $v' \sim 1$ GeV ($Y \ll 1$) and $v' = 10^{-10}$ GeV ($Y \sim 1$). The decay widths of scalars in these cases can be written as [21]

$$\begin{aligned} \Gamma_{\phi(v'\sim 1)} &\simeq \frac{N_cM_\phi}{32\pi f^2}\left(M_b^2+M_t^2\right)+\frac{v'^2M_\phi^3}{2\pi v^4}\,, \\ \Gamma_{\phi(v'\sim 10^{-10})} &\simeq \Gamma_{\phi({\rm LFV})}=\frac{|Y|^2}{8\pi}M_\phi\,. \end{aligned} \tag{8}$$

The properties of new neutral scalar ϕ^0, its couplings to SM and new neutral vector bosons can be examined in single production of ϕ^0 associated with Z_L events. The couplings of ϕ^0 to Z_L and vectors are in the form $ig_{\mu\nu}B_i$, where $i=1,2,3$ corresponds to Z_L, Z_H, A_H, respectively, and given in Table II, where $s_0 \simeq 2\sqrt{2}\frac{v'}{v}$. The Feynman diagrams contributing this process are given in Fig. 1.

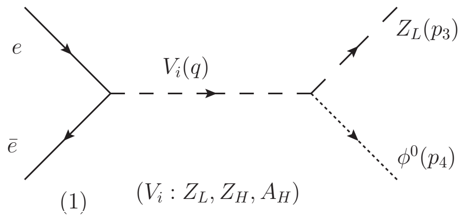

Fig. 1. Feynman diagrams contributing to $e^+e^- \to Z_L\phi^0$ in littlest Higgs model.

TABLE II

The Feynman rules for $\phi^0 V_i V_j$ vertices.

i/j	Vertices	$ig_{\mu\nu}B_{ij}$
1/1	$\phi^0 Z_L Z_L$	$-\frac{i}{2}\frac{g^2}{c_{\rm w}^2}\left(vs_0 - 4\sqrt{2}v'\right) g_{\mu\nu}$
2/2	$\phi^0 Z_H Z_H$	$\frac{i}{2}g^2\left(vs_0 + \frac{\left(c^2-s^2\right)^2}{s^2c^2}\sqrt{2}v'\right) g_{\mu\nu}$
1/2	$\phi^0 Z_L Z_H$	$\frac{i}{2}\frac{g^2}{c_{\rm w}}\frac{\left(c^2-s^2\right)}{2sc}\left(vs_0 - 4\sqrt{2}v'\right) g_{\mu\nu}$
2/3	$\phi^0 Z_H A_H$	$\frac{i}{4}gg'\frac{1}{scs'c'}\left(\left(c^2s'^2 + s^2c'^2\right) vs_0\right)$
1/3	$\phi^0 Z_L A_H$	$\frac{i}{2}\frac{gg'}{c_{\rm w}}\frac{\left(c'^2-s'^2\right)}{2s'c'}\left(vs_0 - 4\sqrt{2}v'\right) g_{\mu\nu}$
3/3	$\phi^0 A_H A_H$	$\frac{i}{2}g'^2\left(vs_0 + \frac{(c'^2-s'^2)^2}{s'^2c'^2}\sqrt{2}v'\right) g_{\mu\nu}$

The pair productions of neutral heavy scalar and pseudoscalar associated with Z_L via $e^+e^- \to Z_L\phi^0\phi^0$, $e^+e^- \to Z_L\phi^{\rm P}\phi^{\rm P}$ and $e^+e^- \to Z_L\phi^0\phi^{\rm P}$ are also examined in this work. The Feynman rules for scalar–vector couplings are given in Table II, the Feynman rules for four point scalar(pseudoscalar)–vector couplings are given in Table III and the Feynman rules for pseudoscalar–vector–scalar couplings are given in Table IV, where $s_{\rm P} = \frac{2\sqrt{2}v'}{\sqrt{v^2+8v'^2}} \simeq 2\sqrt{2}\frac{v'}{v}$. The Feynman diagrams for the processes $e^+e^- \to Z_L\phi^0\phi^0$, $e^+e^- \to Z_L\phi^{\rm P}\phi^{\rm P}$ and $e^+e^- \to Z_L\phi^0\phi^{\rm P}$ are presented in Figs. 2, 3 and 4, respectively.

TABLE III

The Feynman rules for four-point interaction vertices between scalars and vectors. Their couplings are given in the form $iC_{ij}g_{\mu\nu}$ and $iC^{\rm P}_{ij}g_{\mu\nu}$ respectively for $\phi^0\phi^0V_iV_j$ and $\phi^{\rm P}\phi^{\rm P}V_iV_j$.

i/j	Vertices	$iC_{ij}g_{\mu\nu}$	Vertices	$iC^{\rm P}_{ij}g_{\mu\nu}$
1/1	$\phi^0\phi^0Z_LZ_L$	$2i\frac{g^2}{c_{\rm w}^2}g_{\mu\nu}$	$\phi^{\rm P}\phi^{\rm P}Z_LZ_L$	$2i\frac{g^2}{c_{\rm w}^2}g_{\mu\nu}$
1/2	$\phi^0\phi^0Z_LZ_H$	$-2i\frac{g^2}{c_{\rm w}}\frac{(c^2-s^2)}{2sc}g_{\mu\nu}$	$\phi^{\rm P}\phi^{\rm P}Z_LZ_H$	$-2i\frac{g^2}{c_{\rm w}}\frac{(c^2-s^2)}{2sc}g_{\mu\nu}$
1/3	$\phi^0\phi^0Z_LA_H$	$-2i\frac{gg'}{c_{\rm w}}\frac{(c'^2-s'^2)}{2s'c'}g_{\mu\nu}$	$\phi^{\rm P}\phi^{\rm P}Z_LA_H$	$-2i\frac{gg'}{c_{\rm w}}\frac{(c'^2-s'^2)}{2s'c'}g_{\mu\nu}$

TABLE IV

The Feynman rules for $\phi^{\rm P}V_iS_j$ vertices.

i/j	Vertices	$-iE^{\rm P}_{ij}\left(p_j-p_{\phi^{\rm P}}\right)$
1/1	$\phi^{\rm P}HZ_L$	$\frac{1}{2}\frac{g}{c_{\rm w}}\left(s_{\rm P}-2s_0\right)\left(p_{\phi^{\rm P}}-p_H\right)_\mu$
1/2	$\phi^{\rm P}\phi^0Z_L$	$-\frac{g}{c_{\rm w}}\left(p_{\phi^{\rm P}}-p_{\phi^0}\right)_\mu$
2/1	$\phi^{\rm P}HZ_H$	$-\frac{1}{2}g\frac{(c^2-s^2)}{2sc}\left(s_{\rm P}-2s_0\right)\left(p_{\phi^{\rm P}}-p_H\right)_\mu$
2/2	$\phi^{\rm P}\phi^0Z_H$	$g\frac{(c^2-s^2)}{2sc}\left(p_{\phi^{\rm P}}-p_{\phi^0}\right)_\mu$
3/1	$\phi^{\rm P}HA_H$	$-\frac{1}{2}g'\frac{(c'^2-s'^2)}{2s'c'}\left(s_{\rm P}-2s_0\right)\left(p_{\phi^{\rm P}}-p_H\right)_\mu$
3/2	$\phi^{\rm P}\phi^0A_H$	$g'\frac{(c'^2-s'^2)}{2s'c'}\left(p_{\phi^{\rm P}}-p_{\phi^0}\right)_\mu$

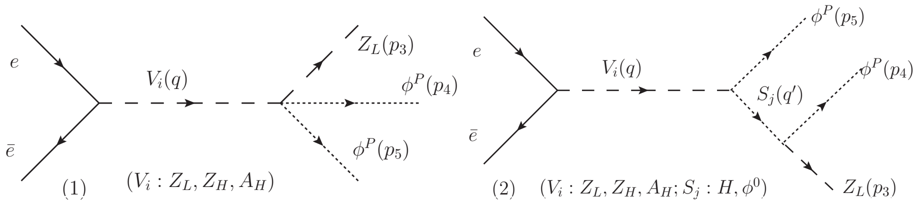

Fig. 2. Feynman diagrams contributing to $e^+e^-\to Z_L\phi^{\rm P}\phi^{\rm P}$ in littlest Higgs model.

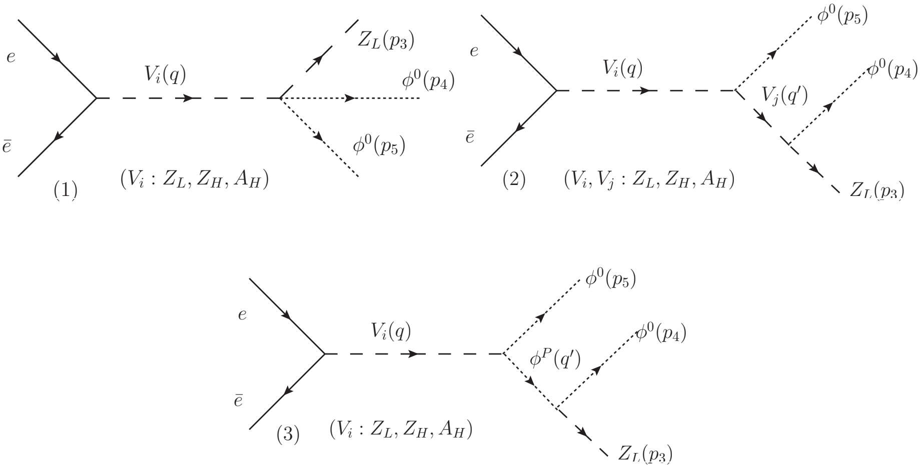

Fig. 3. Feynman diagrams contributing to $e^+e^- \to Z_L\phi^0\phi^0$ in littlest Higgs model.

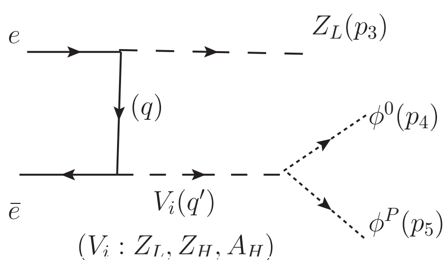

Fig. 4. Feynman diagrams contributing to $e^+e^- \to Z_L\phi^0\phi^{\rm P}$ in littlest Higgs model.

3. Results and discussions

In this section the results for the processes $e^+e^- \to Z_L\phi^0$, $e^+e^- \to Z_L\phi^0\phi^0$, $e^+e^- \to Z_L\phi^{\rm P}\phi^{\rm P}$ and $e^+e^- \to Z_L\phi^0\phi^{\rm P}$ are presented. The numerical values of the input parameters are taken to be: the Higgs mass $M_H = 120$ GeV and the masses of standard model bosons $M_{Z_L} = 91$ GeV, $M_{W_L} = 80$ GeV, and the fine structure constant $\alpha = 1/137.036$, consistent with recent data [29]. The numerical calculations of cross-sections of the production processes are performed by `CalcHep` [30] generator after implementing necessary vertices.

For the examination of the production of heavy neutral scalar ϕ^0 at linear colliders, the single production of ϕ^0 associated with Z_L is the most dominant channel. For this process, total cross-section is plotted with respect to center of mass energy in Fig. 5 for different values of symmetry breaking scale f and mixing angles s and s' allowed by recent constraints. In these calculations the v.e.v. of the scalar triplet is taken to be $v' = 1$ GeV allowed by the limit given in Eq. (4). It is seen from Fig. 5 that, for symmetry breaking scale $f = 1$ TeV, and $s/s' = 0.80/0.60$, the total cross-section is of the order of 10^{-2} pb for $\sqrt{S} \sim 1000$–3000 GeV. For parameters $f = 1$ TeV and $s/s' = 0.90/0.60(0.95/0.60)$, the total cross-section is of the order of 10^{-2} pb for $\sqrt{S} \geq 1200(1500)$ GeV. Also for these parameter sets, a resonance corresponding to heavy gauge boson Z_H exist at $\sqrt{S} \sim 900(1100)$ GeV increasing the total cross-section up to 0.1(1) pb. Unfortunately, these resonances can have significance only if ϕ^0 can be reconstructed. For $f = 2.5$ TeV, the cross-section *versus* $\sqrt{S}$ graphs are also plotted in Fig. 5, for mixing angles $s/s' = 0.80/0.60$ and $s/s' = 0.90/0.60$. In both sets of parameters total cross-section is about 5×10^{-3} pb for $\sqrt{S} \gtrsim 2$ TeV. In addition, for $s/s' = 0.90/0.60$, total cross-section receives a peak up to 0.1 pb about $\sqrt{S} \sim 1.8$ TeV corresponding to the resonance of Z_H. Finally, the production of ϕ^0 via $e^+e^- \to Z_L\phi^0$ process is possible for low values of symmetry breaking scale $f = 1$ TeV, for both ILC($\sqrt{S} = 1$ TeV) and CLIC($\sqrt{S} = 3$ TeV). However, for higher values of f, this channel is not promising.

For $v' \sim 1$ GeV, the neutral scalar ϕ^0 dominantly decays into quark pairs $t\bar{t}$ and $t\bar{T} + \bar{t}T$ with branching ratios of 0.8 and 0.2, respectively [21]. Thus, the channel $e^+e^- \to Z_L t\bar{t}$ is promising for ϕ^0 observation. In this channel, there will be more than thousands events which are observable as a contribution of $e^+e^- \to Z_L\phi^0$ process. In this channel, the SM background is of the order of 10^{-2} pb at $\sqrt{S} = 1$ TeV, and reduces to 10^{-4} pb for $\sqrt{S} \sim 2$ TeV. So, for $\sqrt{S} \geq 1$ TeV, the collider signal $Z_L t\bar{t}$ is dominated by the

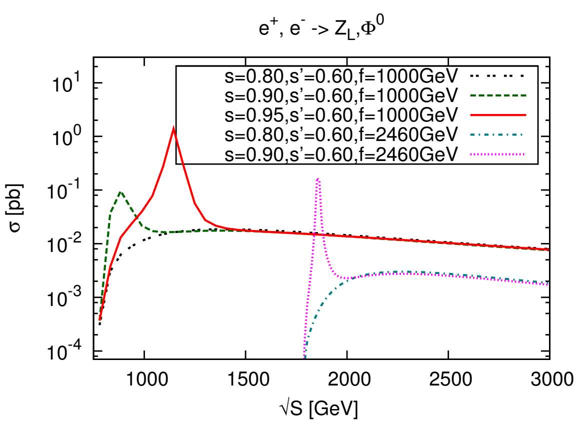

Fig. 5. Total cross-section *versus* center of mass energy graphs of the process $e^+e^- \to Z_L\phi^0$ for some selected values of littlest Higgs model parameters when $v' = 1$ GeV.

decays of neutral scalars produced via $e^+e^- \to Z_L\phi^0$ process. Moreover, by applying a cut on the energy of final state $t\bar{t}$ pair, *i.e.* $E_{t\bar{t}} \geq M_\phi$, will suppress the background contribution from SM. Thus in this channel, ϕ^0 can be observed and reconstructed from $t\bar{t}$ jets for $\sqrt{S} \geq 1$ TeV.

For the double production of neutral scalar and pseudoscalar via $e^+e^- \to Z_L\phi^0\phi^0$, $e^+e^- \to Z_L\phi^{\rm P}\phi^{\rm P}$ and $e^+e^- \to Z_L\phi^0\phi^{\rm P}$ processes, differential cross-section *versus* energy of Z_L graphs are plotted in Figs. 6, 7 and 8, respectively, for $f = 1$ TeV, and $s/s' = 0.80/0.60, 0.90/0.60, 0.95/0.60$ at $\sqrt{S} = 3$ TeV. In these calculations the v.e.v. of the scalar triplet is taken to be $v' = 3$ GeV. It is seen from the figures that the production rates are not strongly dependent on mixing angles s/s' in the parameter region allowed by electroweak and experimental constraints.

For the production process $e^+e^- \to Z_L\phi^0\phi^0$, the differential cross-section is of the order of 10^{-4} pb/GeV. The corresponding total cross-section is calculated by integrating over E_Z and found to be 0.25 pb. At CLIC, the expected luminosity is 100 fb^{-1}, which will result in more than few thousands of production events in this channel. At ILC the expected center of mass energy is about 0.5–1 TeV, hence this production channel is out of reach, due to kinematical limits from high values of scalar mass M_ϕ.

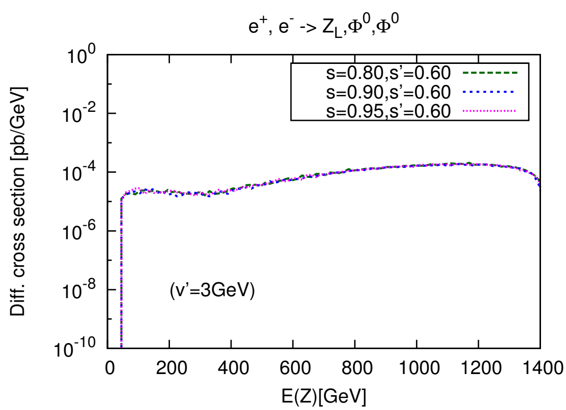

Fig. 6. Differential cross-section *versus* energy of the Z_L boson graphs of the process $e^+e^- \to Z_L\phi^0\phi^0$ for some selected values of mixing angles when $f = 1$ TeV and $v' = 3$ GeV at $\sqrt{S} = 3$ TeV.

In the littlest Higgs model, the single production of pseudoscalar $\phi^{\rm P}$ associated with Z_L is not allowed. The most promising channel for $\phi^{\rm P}$ production is therefore $e^+e^- \to Z_L\phi^{\rm P}\phi^{\rm P}$. In this channel, the differential cross-section is calculated at the order of 10^{-6} pb/GeV for all allowed values of mixing angles when $f = 1$ TeV (Fig. 7). The maximum value of cross-

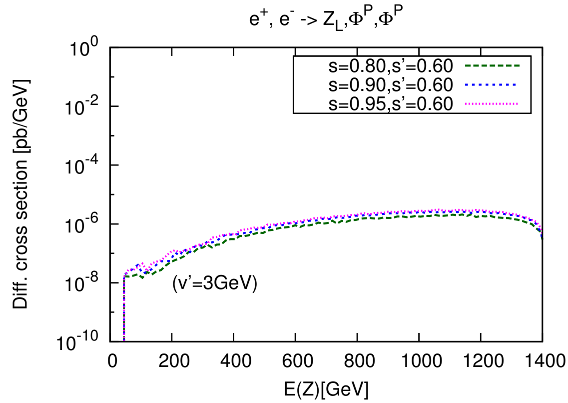

Fig. 7. Differential cross-section *versus* energy of the Z_L boson graphs of the process $e^+e^- \to Z_L\phi^{\mathrm{P}}\phi^{\mathrm{P}}$ for some selected values of mixing angles when $f = 1$ TeV and $v' = 3$ GeV at $\sqrt{S} = 3$ TeV.

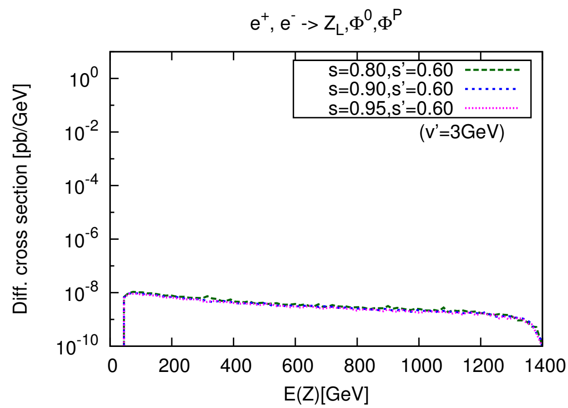

Fig. 8. Differential cross-section *versus* energy of the Z_L boson graphs of the process $e^+e^- \to Z_L\phi^0\phi^{\mathrm{P}}$ for some selected values of mixing angles when $f = 1$ TeV and $v' = 3$ GeV at $\sqrt{S} = 3$ TeV.

section for this process is calculated as 3×10^{-3} pb at $\sqrt{S} = 3$ TeV. For an integrated luminosity of $100\,\mathrm{fb}^{-1}$, up to a few hundreds of ϕ^{P} will be produced within Z_L.

For the process $e^+e^- \to Z_L\phi^0\phi^{\mathrm{P}}$ the maximum value of differential cross-section is about 10^{-8} pb/GeV. The corresponding total cross-section at $\sqrt{S} = 3$ TeV is calculated as 10^{-5} pb. Thus in this channel the production rate is not promising.

A distinguishing feature of neutral scalars in littlest Higgs model is their lepton-flavor violating decay modes. For lepton-flavor violation to be dom-

TABLE V

The total cross-sections in pb for production of neutral scalars for $f = 1$ TeV and at $\sqrt{s} = 3$ TeV when $v' = 3$ GeV and $v' = 10^{-10}$ GeV.

Process	σ(pb)[$v' = 3$GeV]	σ(pb)[$v' = 10^{10}$GeV]
$Z_L\phi^0$	10^{-2}	10^{-23}
$Z_L\phi^0\phi^0$	0.25	2.8×10^{-3}
$Z_L\phi^{\rm P}\phi^{\rm P}$	2.8×10^{-3}	2.7×10^{-3}
$Z_L\phi^0\phi^{\rm P}$	1.0×10^{-5}	1.0×10^{-5}

inant, the v.e.v. of the triplet should be at the order of $v' = 10^{-10}$ GeV. For this value, all other decays of the neutral scalar and pseudoscalar are suppressed. In Table V, the total cross-sections of the Z_L associated productions of the neutral scalar and pseudoscalar are given for $s/s' = 0.8/0.6$, $f = 1$ TeV at $\sqrt{S} = 3$ TeV, for $v' = 3$ GeV and $v' = 10^{-10}$ GeV.

For the process $e^+e^- \to Z_L\phi^0$ at $v' = 10^{-10}$ GeV the production cross-section is at the order of 10^{-23} pb. This is due to the explicit dependence of scalar vector vector couplings on the triplet v.e.v. Thus, for this channel observation of any lepton-flavor violation is not possible. For double production of neutral scalars within Z_L dependence of cross-section on v' is plotted in Fig. 9, for $f = 1$ TeV at $\sqrt{S} = 3$ GeV. It is seen that for $v' < 0.1$ GeV total cross-section is not dependent on v'. This is due to mass of the heavy scalars, which is steady with respect to variations of v' in this region (Fig. 9). In Fig. 10, the total cross-sections of the processes $e^+e^- \to Z_L\phi^0\phi^0$, $e^+e^- \to Z_L\phi^{\rm P}\phi^{\rm P}$ and $e^+e^- \to Z_L\phi^{\rm P}\phi^0$ are plotted with respect to center of mass energy, for $f = 1$ TeV, $s/s' = 0.80/0.60$ and $v' = 10^{-10}$ GeV. For $v' = 10^{-10}$, the total cross-section of the processes $e^+e^- \to Z_L\phi^0\phi^0$ and $e^+e^- \to Z_L\phi^{\rm P}\phi^{\rm P}$ are at the order of 10^{-4} pb at $\sqrt{S} \sim 2$ TeV, and increases smoothly to 2.8×10^{-3} pb as center of mass energy approaches to 3 TeV. Since the value of the Yukawa coupling is $Y \sim 1$ in this scenario, for an integrated luminosity of $100\,{\rm fb}^{-1}$, the number of lepton-flavor violating events per year will be close to a thousand. For the process $e^+e^- \to Z_L\phi^{\rm P}\phi^0$ the total cross-section is not sufficient to produce lepton-flavor violating events.

In Fig. 11, we have plotted the number of lepton-flavor violating final states with respect to v', for a linear collider with an integrated luminosity of $100\,{\rm fb}^{-1}$ at $\sqrt{S} = 3$ TeV. For these events the collider signature will be "Z_L+missing energy". The SM background in this channel is mostly produced via $e^+e^- \to Z_L\nu\bar{\nu}$ processes which has a total cross-section of

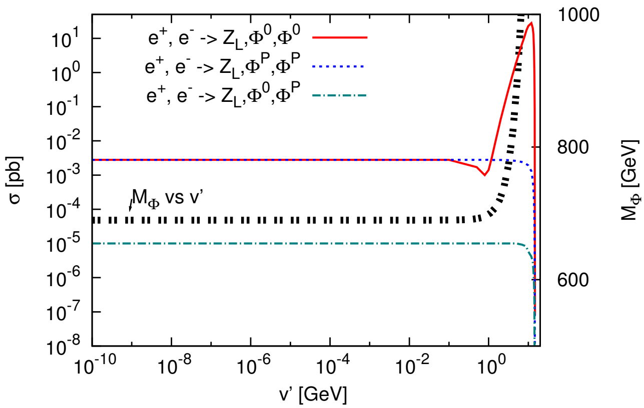

Fig. 9. Thin lines: Dependence of total cross-section on the v.e.v. of the scalar triplet v' when $f = 1$ TeV and $s/s' = 0.80/0.60$ at $\sqrt{S} = 3$ TeV for Z_L associated pair production of neutral scalar and pseudoscalar. Thick line: Dependence of heavy scalar mass M_ϕ on v'.

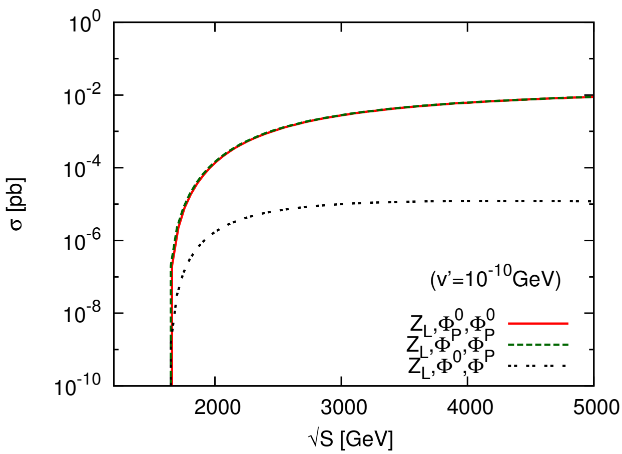

Fig. 10. Dependence of total cross-section on center of mass energy when $f = 1$ TeV, $v' = 10^{-10}$ and $s/s' = 0.80/0.60$ for Z_L associated pair production of neutral scalar and pseudoscalar.

5 pb. For this channel, only applying a constraint on the energy of the Z_L boson can improve the signal background ratio. By choosing Z_L bosons carrying the recoil momentum of the scalar pair, *i.e.* $E_{Z_L} \geq 2M_\phi$, SM contributions suppressed to 3000 events at $\sqrt{S} \sim 3$ TeV. In this case the final state analysis will give important results since this signature makes ϕ^0 and $\phi^{\rm P}$ indistinguishable but quite different in appearance from their counter

partners in either SM or two Higgs doublet model.

In conclusion, the new heavy scalar ϕ^0 and pseudoscalar $\phi^{\rm P}$ of the littlest Higgs model will be produced in e^+e^- colliders associated with Z_L. The production rates significantly depend on the symmetry braking scale parameter f and the v.e.v. of the scalar triplet v'. For $f = 1$ TeV and $v' \sim 1$ GeV highest production rates are achieved in both channels. For these parameter set, the productions are quite detectable in the channel $e^+e^- \to Z_L\phi^0$ when $\sqrt{S} > 0.8$ TeV and in the channels $e^+e^- \to Z_L\phi^0\phi^0$ and $e^+e^- \to Z_L\phi^{\rm P}\phi^{\rm P}$ when $\sqrt{S} > 1.7$ TeV. However, in the channel $e^+e^- \to Z_L\phi^0\phi^{\rm P}$ there is no significant production rate. For higher values of symmetry breaking scale $f \sim 2.5$ TeV the production is achieved only in the channel $e^+e^- \to Z_L\phi^0$ for $\sqrt{S} \gtrsim 1.8$ TeV. For $v' \sim 1$ GeV, the channel $e^+e^- \to Z_L\phi^0$ is the most promising channel for reconstruction of ϕ^0 from $t\bar{t}$ pairs. The effects of the littlest Higgs model heavy scalar can be observed in $Z_Lt\bar{t}$ final states in electron colliders. For $v' \sim 10^{-10}$ GeV and $f = 1$ TeV an interesting and distinguishing feature of the littlest Higgs model is on stage. In this case, final decays of the neutral scalar and pseudoscalar are totally lepton-flavor violating with a collider signature of missing energy accompanied by a SM Z_L boson. For this value of v', the productions in the channel $e^+e^- \to Z_L\phi^0$ are not possible whereas in the channels $e^+e^- \to Z_L\phi^0\phi^0$ and $e^+e^- \to Z_L\phi^{\rm P}\phi^{\rm P}$ the productions of ϕ^0 and $\phi^{\rm P}$ are still observable. Although these channels contain high SM background, the productions and final lepton-flavor

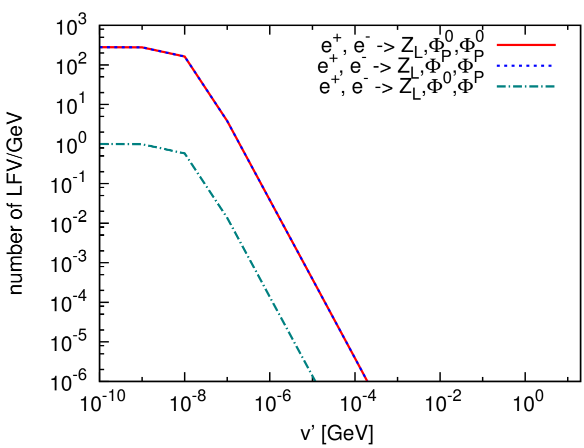

Fig. 11. Dependence of total number of lepton-flavour violating final states on the v.e.v. of the scalar triplet v' when $f = 1$ TeV and $s/s' = 0.80/0.60$ at $\sqrt{S} = 3$ TeV for Z_L associated pair production of neutral scalar and pseudoscalar.

violating decays of ϕ^0 and $\phi^{\rm P}$ can still be examined at e^+e^- colliders.

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REFERENCES

[1] N. Arkani-Hamed, A.G. Cohen, E. Katz, A.E. Nelson, *J. High Energy Phys.* **0207**, 034 (2002) [arXiv:hep-ph/0206021].

[2] N. Arkani-Hamed *et al.*, *J. High Energy Phys.* **0208**, 021 (2002) [arXiv:hep-ph/0206020].

[3] M. Schmaltz, *Nucl. Phys. Proc. Suppl.* **117**, 40 (2003) [arXiv:hep-ph/0210415].

[4] D.E. Kaplan, M. Schmaltz, *J. High Energy Phys.* **0310**, 039 (2003) [arXiv:hep-ph/0302049].

[5] J. Hubisz, P. Maeda, A. Noble, M. Perelstein, *J. High Energy Phys.* **01**, 135 (2006).

[6] J.L. Hewett, F.J. Petriello, T.G. Rizzo, *J. High Energy Phys.* **0310**, 062 (2003) [arXiv:hep-ph/0211218].

[7] M-C. Chen, S. Dawson, *Phys. Rev.* **D70**, 015003 (2004) [arXiv:hep-ph/0311032].

[8] W. Killian, J. Reuter, *Phys. Rev.* **D70**, 015004 (2004) arXiv:hep-ph/0311095.

[9] A.G. Dias, C.A. de S. Pires, P.S. Rodrigues da Silva, *Phys. Rev.* **D77**, 055001 (2008) [arXiv:0711.1154 [hep-ph]].

[10] C. Csaki *et al.*, *Phys. Rev.* **D68**, 035009 (2003) [arXiv:hep-ph/0303236].

[11] J.A. Conley, J. Hewett, M.P. Le, *Phys. Rev.* **D72**, 115014 (2005) [arXiv:hep-ph/0507198v2].

[12] M. Perelstein, *Prog. Part. Nucl. Phys.* **58**, 247 (2007) [arXiv:hep-ph/0512128].

[13] T. Han, H.E. Logan, B. McElrath, L.-T. Wang, *Phys. Rev.* **D67**, 095004 (2003) [arXiv:hep-ph/0301040].

[14] M. Schmaltz, D. Tucker-Smith, *Annu. Rev. Nucl. Part. Sci.* **55**, 229 (2005) [arXiv:hep-ph/0502182].

[15] G. Azuelos *et al.*, *Eur. Phys. J.* **C39**, 13 (2005).

[16] F. Ledroit, *AIP Conf. Proc.* **903**, 245 (2007) [arXiv:hep-ex/0610005].

[17] W. Kilian, D. Rainwater, J. Reuter, *Phys. Rev.* **D74**, 095003 (2006) [Erratum *Phys. Rev.* **D74**, 099905 (2006)] [arXiv:hep-ph/0609119].

[18] C.-X. Yue, W. Yang, F. Zhang, *Nucl. Phys.* **B716**, 199 (2005) [arXiv:hep-ph/0409066].

[19] A. Cagil, M.T. Zeyrek, *Phys. Rev.* **D80**, 055021 (2009) [arXiv:0908.3581 [hep-ph]].

[20] A. Cagil, *Nucl. Phys.* **B843**, 46 (2011) [arXiv:1010.0102 [hep-ph]].

[21] T. Han, H.E. Logan, B. Mukhopadhyaya, R. Srikanth, *Phys. Rev.* **D72**, 053007 (2005) [arXiv:hep-ph/0505260].

[22] S.R. Choudhury, N. Gaur, A. Goyal, *Phys. Rev.* **D72**, 097702 (2005) [arXiv:hep-ph/0508146].

[23] C.-X. Yue, S. Zhao, *Eur. Phys. J.* **C50**, 897 (2007) [arXiv:hep-ph/0701017].

[24] S.R. Choudhury *et al.*, *Phys. Rev.* **D75**, 055011 (2007) [arXiv:hep-ph/0612327].

[25] J. Brau *et al.*, arXiv:0712.1950 [physics.acc-ph].

[26] R.W. Assmann *et al.* [CLIC Study Team], A 3 TeV e^+e^- Linear Collider Based on CLIC Technology, CERN 2000-008, Geneva 2000; E. Accomando *et al.* [CLIC Physics Working Group], arXiv:hep-ph/0412251; H. Braun *et al.* [CLIC Study Team], CLIC 2008 Parameters, CERN-OPEN-2008-021, CLIC-NOTE-764.

[27] G.L. Fogli *et al.*, *Phys. Rev.* **D70**, 113003 (2004); M. Tegmark *et al.* [SDSS Collabration], *Phys. Rev.* **D69**, 103501 (2004).

[28] E.W. Otten, C. Weinheimer, *Rep. Prog. Phys.* **71**, 086201 (2008) [arXiv:0909.2104v1 [hep-ex]; Z.-z. Xing, *Phys. Lett.* **B679**, 255 (2009) [arXiv:0907.3014 [hep-ph]].

[29] K. Nakamura *et al.* [Particle Data Group], *J. Phys. G* **37**, 075021 (2010).

[30] A. Pukhov *et al.* [CalcHEP/CompHEP Collab.], hep-ph/9908288; hep-ph/0412191.