

ON STRANGE SU(3) PARTNERS OF Θ^+ KLAUS GOEKE^a, MAXIM V. POLYAKOV^{a,b}, MICHAL PRASZALOWICZ^c^aInstitut für Theoretische Physik II, Ruhr-Universität
Bochum, 44780, Germany^bPetersburg Nuclear Physics Institute, Gatchina
188 300, St. Petersburg, Russia^cThe M. Smoluchowski Institute of Physics, Jagiellonian University
Reymonta 4, 30-059 Kraków, Poland*(Received November 9, 2010)*

We propose a scenario in which Roper octet can mix with a putative antidecuplet of exotic baryons and predict the properties of its strange members. We show that $1795 \text{ MeV} < M_{\Sigma_{10}^-} < 1830 \text{ MeV}$ and $1900 \text{ MeV} < M_{\Xi_{10}^-} < 1970 \text{ MeV}$. We also estimate total widths: $10 \text{ MeV} < \Gamma_{\Sigma_{10}^-} < 30 \text{ MeV}$ and $\Gamma_{\Xi_{10}^-} \sim 10 \text{ MeV}$ and branching ratios for different decay modes.

DOI:10.5506/APhysPolB.42.61

PACS numbers: 14.20.Pt

1. Introduction

At the end of 2002 Japanese collaboration LEPS at Spring-8 [1] and the bubble chamber experiment DIANA at ITEP, Moscow [2], announced evidence of a strange baryon, called Θ^+ , whose quantum numbers cannot be constructed from 3 quarks. In the quark model, this truly exotic state: $(uudd\bar{s})$, is believed to be rather heavy (1800–1900 MeV) and wide. The excitement created by the findings of LEPS and DIANA was due to the low mass of the putative Θ^+ , of the order of 1530 MeV, and a very small width. Such properties, however, are natural in chiral models, where the antistrange quark is excited in form of a chiral field rather than as a constituent quark. Early predictions of the Θ^+ mass in different versions of the chiral models were very close to the experimental numbers of LEPS and DIANA [3, 4], and moreover the width, as estimated by Diakonov, Petrov and Polyakov in 1997, was very small [5].

The discovery by LEPS and DIANA triggered both experimental and theoretical activity in a somewhat extinct field of hadron spectroscopy. Present experimental situation is, however, rather confusing. Many collaborations announced the existence of Θ^+ in different experimental setups,

but the comparable number of searches ended with null result. Furthermore, dedicated high statistics runs with CLAS detector at CEBAF did not show any signal of Θ^+ contradicting previous findings of the same collaboration [6]. On the other hand, LEPS has confirmed its original result [7, 8]. Also high statistics analysis by DIANA turned out to be positive [9]. We refer the reader to recent reviews of the experimental situation [10–12]. For the purpose of the present study we assume that Θ^+ exists with a mass equal 1540 MeV and total width $\Gamma < 1$ MeV.

One of the immediate consequences of a possible existence of Θ^+ is the existence of the whole $SU(3)$ multiplet: $\overline{10}$ (see Fig. 1). Indeed, NA49 experiment at CERN [13] announced discovery of another exotic state, namely $\Xi_{\overline{10}}^{--}(1860)$. Unfortunately, the searches of $\Xi_{\overline{10}}$ by other groups have not confirmed the results of NA49.

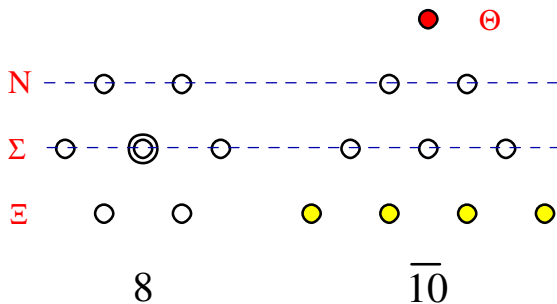


Fig. 1. $SU(3)$ weight diagrams for octet and antidecuplet. States that can mix lie on dashed lines.

Apart from truly exotic states that cannot be constructed from three quarks, antidecuplet contains cryptoexotic states that are primarily built from 5 quarks, however, their quantum numbers can be constructed from three quarks as well. These are nucleon-like states ($N_{\overline{10}}$) and Σ -like states ($\Sigma_{\overline{10}}$) that are the subject of the present paper. The interpretation of these states is not well understood: one may try to associate them with some known resonances, as it was done in the case of $N_{\overline{10}}$ in the original paper of Diakonov, Petrov and Polyakov for example [5], or one may postulate the existence of new, yet undiscovered resonances with nucleon or Σ quantum numbers. In this paper, we follow the latter approach trying to predict the range of masses and widths for cryptoexotic $\Sigma_{\overline{10}}$ and also $\Xi_{\overline{10}}$ states.

In the Chiral Quark-Soliton model (χ QSM) the spin-parity quantum numbers of the antidecuplet members are unambiguously predicted to be $J^P = \frac{1}{2}^+$ [5], so that $N_{\overline{10}}$ and $\Sigma_{\overline{10}}$ are predicted to be P_{11} resonances. One of the striking properties of $N_{\overline{10}}$ is that it can be excited by an electromagnetic probe from the neutron target much stronger than from the proton one [14].

The photoexcitation of charged isocomponent of $N_{\overline{10}}$ is possible only due to $SU_{\mathfrak{f}}(3)$ violation; therefore its suppression by a factor $\sim 1/10$ in the amplitude is expected.

The existence of a new nucleon resonance with the mass near ~ 1700 MeV was suggested in Refs. [15,16]. The authors of Ref. [15] used the Gell-Mann–Okubo mass relations in the presence of mixing, in order to predict the mass of this new nucleon resonance. As an input for the Gell-Mann–Okubo mass formula the authors of Ref. [15] used the mass of the Ξ_{10}^{--} baryon reported by the NA49 Collaboration [13]. In Ref. [16], in order to constrain the mass of this possible new narrow N^* , the modified PWA of πN scattering data was employed. It was found that the easiest way to accommodate a narrow N^* is to set its mass around 1680 MeV and quantum numbers to P_{11} ($J^P = \frac{1}{2}^+$). In the same paper, the width of the possible N^* was analyzed in the framework of χ QSM. It was found that the width of new N^* is in the range of tens of MeV with very small πN partial width of $\Gamma_{\pi N} \leq 0.5$ MeV [16]. One should stress that the decay to πN is not suppressed in the $SU_{\mathfrak{f}}(3)$ limit and it can be made small only if the symmetry violation is taken into account. It follows that the preferred decay channels are ηN , $\pi \Delta$ and $K \Lambda$ [5, 16, 17, 18, 19, 20].

The search of the new nucleon resonance has been performed in the η photoproduction on the neutron at GRAAL [21, 22]. The narrow peak in the quasi-free neutron cross-section and in the ηn invariant mass spectrum has been observed. The original observation of Refs. [21, 22] has been recently confirmed by two other groups: CBELSA/TAPS [24] and LNS-Sendai [25]. All three experiments found an enhancement in the quasi-free cross-section on the neutron. In addition, the GRAAL and CBELSA/TAPS groups have observed a narrow peak in the ηn invariant mass spectrum at 1680–1685 MeV. The corresponding data can be explained by existence of a new narrow resonance [26, 27, 28]. One should, however, stress that the part of above mentioned experimental results may have a different interpretation that does not require to postulate a new narrow nucleon resonance [29, 30, 31].

Further evidence for a new narrow nucleon resonance came from the analysis of the Σ beam asymmetry in η photoproduction on the proton [23, 32, 33, 34]. In these papers, the narrow structure in the Σ beam asymmetry around invariant mass ~ 1685 MeV has been observed. That structure can be interpreted as the contribution of a narrow nucleon resonance with the mass 1685 MeV, total width ≤ 25 MeV and the photocoupling to the proton much smaller than to the neutron; the properties that are expected for the nonstrange partner of Θ^+ [5, 14, 16, 17, 18, 19, 20]. The properties of possible new narrow nucleon resonance that crystallized out recent experiments on η photoproduction are summarized in Ref. [34]. Throughout this paper we shall assume that the new $N(1685)$ nucleon resonance exists with total width below 25 MeV.

One of the striking and to some extent counterintuitive properties of Θ^+ is its small width. In particular, DIANA [9] that has doubled the statistics of their formation experiment $K^+n(\text{Xe}) \rightarrow K^0p$ as compared to the original result reported in Ref. [2], claims $M_\Theta = 1537 \pm 2$ MeV and the width $\Gamma_\Theta = 0.39 \pm 0.10$ MeV (with possible systematic uncertainties). The only other available formation experiment with the secondary kaon beam at BELLE sets an upper limit $\Gamma_\Theta < 0.64$ MeV (at a 90% confidence level) [35] which is beyond the above value. Also the reanalysis of the old KN scattering data [36] shows that there is room for the exotic resonance with a width below 1 MeV.

The small width implies that the coupling $g_{\Theta NK}$ is at least an order of magnitude smaller than $g_{\pi NN} \approx 13$. The small value of $g_{\Theta NK}$ appears naturally in a relativistic field-theoretic approach to baryons, allowing for a consistent account for multi-quark components in baryons; in particular in Refs. [37, 38] an upper bound $\Gamma_\Theta \approx 2$ MeV has been obtained without any parameter fixing. Recent calculation of the Θ^+ width in χ QSM [20] also gave small width of 0.71 MeV. The width below 1 MeV also comes out from the parameter-free QCD sum rules analysis [39].

In any case the small value of $g_{\Theta NK}$ is related to the small value of $G_{\overline{10}}$, *i.e.* of the reduced matrix element that is responsible for the direct decay $\overline{10} \rightarrow 8$. Indeed

$$g_{\Theta NK} = G_{\overline{10}} + \sin \alpha H_{\overline{10}}, \quad (1)$$

where α is the mixing angle between the nucleon-like states in octet and antidecuplet, and $H_{\overline{10}}$ denotes the transition reduced matrix element $\overline{10} \rightarrow \overline{10}$ (see Table I for definitions). For ordinary baryons we expect terms like $\sin \alpha H_{\overline{10}}$ ($\sin \alpha$ being of the order of m_s) to be small in comparison with the leading term. For antidecuplet both terms $G_{\overline{10}}$ and $\sin \alpha H_{\overline{10}}$ are small and comparable in magnitude. They may add or cancel depending on the decay channel (for Θ^+ we have only two equal decay channels $\Theta^+ \rightarrow NK$) violating completely the $\text{SU}_F(3)$ relations between the decay couplings. In this way one is able to explain the suppression of πN decay channel in $N_{\overline{10}}$ decays mentioned above, and also the existence of $\overline{10} \rightarrow 10$ transition that is forbidden in the $\text{SU}_F(3)$ limit. We see therefore, that in the case of antidecuplet mixing is an important ingredient primarily to understand the decays, but also the masses [15, 16, 18, 19, 40].

Unfortunately, at least at the first sight, there is a large arbitrariness as far as mixing angles and transition matrix elements are concerned. Here we shall try to constrain them from the existing data. If the data are not available we shall make estimates based on χ QSM. We shall consider mixing of the ground state octet (8_1), the octet of the $N(1440)$ Roper resonance (8_2) and antidecuplet. In the $\text{SU}_F(3)$ limit all states in the ground state octet and in antidecuplet are separately degenerate in mass. When m_s

corrections are switched on the masses split and take values given by Gell-Mann–Okubo (GMO) mass formulae. At the same time the wave functions become mixtures of the original representation (8 or $\bar{10}$) and all other allowed $SU_{\text{fl}}(3)$ representations that appear in the tensor product $8 \otimes 8$ or $8 \otimes \bar{10}$. This introduces 8 – $\bar{10}$ mixing for nucleon-like and Σ -like states characterized by the angle α (note that due to the accidental equality of the pertinent $SU(3)$ Clebsch–Gordan coefficients mixing angles between N - and Σ -like states are equal in the leading order in m_s). We assume that ground state octet GMO states correspond to the physical states, which is true with 0.5% accuracy [15]. We assume next that antidecuplet GMO states undergo further mixing with the Roper octet. For Roper octet GMO mass formulae work with much worse accuracy of approximately 3% [15], so there is a need for additional mixing. Again both for N - and Σ -like states the mixing angle ϕ is the same. This is depicted in Fig. 2. Throughout this paper we take into account only these two mixings which we generally believe to be small.

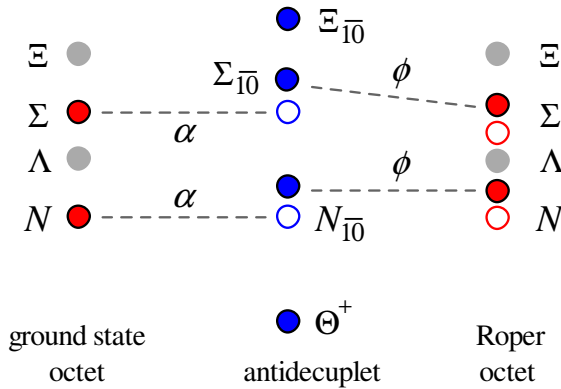


Fig. 2. Definition of mixing angles for nucleon-like states and Σ -like states. Full circles denote physical states, open circles in the case of Roper octet and $\bar{10}$ correspond to the GMO states that undergo further mixing with angle ϕ . Grey circles correspond to particles not considered in the present paper.

Our analysis is based on the following assumptions concerning antidecuplet. We assume $M_{\Theta^+} = 1540$ MeV and $\Gamma_{\Theta^+} < 1$ MeV. We follow analysis of Refs. [23, 34] assuming that $N_{\bar{10}}$ is a new resonance with mass 1685 MeV and total width $\Gamma_{N_{\bar{10}}} < 25$ MeV. We also assume hierarchy of the branching ratios that is described in more detail in Section 4. With these assumptions we are able to provide limits on the mixing angles both for the nucleon-like and Σ -like states. We find a small region in the space of mixing angles, where the required properties of $N_{\bar{10}}$ are reproduced. For these allowed angles we calculate masses of $\Sigma_{\bar{10}}$ and $\Xi_{\bar{10}}$ and their decay patterns.

The paper is organized as follows. In the next section we discuss general formulae for quantum mixing and define mixing angles and wave functions used throughout this paper. In Section 3 we calculate the decay constants in the presence of mixing. We define reduced matrix elements and discuss their hermiticity properties. Section 4 contains numerical results of our analysis. Finally, in Section 5 we briefly summarize our results and present conclusions.

2. Masses in the presence of mixing

2.1. Two state mixing

Before we discuss three state mixing, let us recall the formulae for two state mixing, that can be found for example in Ref. [15]. Consider perturbation Hamiltonian ($M_2 > M_1$, $V > 0$), where $V \sim m_s$

$$H' = \begin{bmatrix} M_1 & -V \\ -V & M_2 \end{bmatrix}. \quad (2)$$

Here we have chosen “ $-$ ” sign in front of $V > 0$ in order to be in agreement with the sign dictated by the χ QSM. Let us consider for the moment $\overline{10}$ –8 mixing. In that case Hamiltonian (2) represents mixing between nucleon states or Σ states (with different entries for each case). Exotic states Θ^+ and $\Xi_{\overline{10}}$ remain unmixed. It is important to note here that the SU(3) Clebsch–Gordan coefficients are identical for N and Σ states and, therefore, V is the same.

Introducing

$$\delta M = M_2 - M_1, \quad \Delta = \sqrt{\delta M^2 + 4V^2} \quad (3)$$

we get the following mass eigenvalues

$$M_{1,2}^{\text{phys}} = \frac{1}{2} (M_1 + M_2 \pm \Delta) \quad (4)$$

and eigenvectors

$$\begin{aligned} |1^{\text{phys}}\rangle &= |1\rangle \frac{2V}{\sqrt{2\Delta(\Delta - \delta M)}} + |2\rangle \sqrt{\frac{\Delta - \delta M}{2\Delta}}, \\ |2^{\text{phys}}\rangle &= |1\rangle \frac{-2V}{\sqrt{2\Delta(\Delta + \delta M)}} + |2\rangle \sqrt{\frac{\Delta + \delta M}{2\Delta}}. \end{aligned} \quad (5)$$

The mixing angle α

$$\begin{bmatrix} |1^{\text{phys}}\rangle \\ |2^{\text{phys}}\rangle \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} |1\rangle \\ |2\rangle \end{bmatrix} \quad (6)$$

is conveniently defined through the second equation (5)

$$\tan \alpha = \frac{2V}{\Delta + \delta M} \simeq \frac{V}{\delta M} \simeq \frac{V}{\overline{M}_2 - \overline{M}_1}. \quad (7)$$

The last approximation consists in approximating δM by the difference of the mean multiplet masses in the spirit of the first order perturbation theory in m_s . Then Eq. (5) implies

$$\begin{aligned} M_1^{\text{phys}} &= \frac{1}{\cos 2\alpha} (M_1 \cos^2 \alpha - M_2 \sin^2 \alpha), \\ M_2^{\text{phys}} &= \frac{1}{\cos 2\alpha} (M_2 \cos^2 \alpha - M_1 \sin^2 \alpha). \end{aligned} \quad (8)$$

2.2. Three state mixing

Let us consider three states belonging to ordinary octet 8_1 , ‘‘Roper’’ octet 8_2 and antidecuplet $\overline{10}$, with unperturbed masses satisfying

$$M_{8_1} < M_{8_2} < M_{\overline{10}}. \quad (9)$$

The vector of nucleon-like (or Σ -like) states

$$\begin{bmatrix} |8_1\rangle \\ |8_2\rangle \\ |\overline{10}\rangle \end{bmatrix} \quad (10)$$

is a subject of mixing by the orthogonal matrix

$$\mathcal{O} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}. \quad (11)$$

Throughout this paper we neglect Roper-ground octet mixing and set $\theta = 0$. Angle α , as in Section 2.1 describes $\overline{10}$ – 8_1 mixing, angle ϕ is responsible for the mixing of Roper states with already mixed $\overline{10}$, as depicted schematically in Fig. 2.

Since we anticipate that the mixing will be not large, the physical states can be labeled by the $SU(3)$ representations so that

$$\begin{aligned} |8_1^{\text{phys}}\rangle &= \cos \alpha |8_1\rangle + \sin \alpha |\overline{10}\rangle, \\ |8_2^{\text{phys}}\rangle &= -\sin \phi \sin \alpha |8_1\rangle + \cos \phi |8_2\rangle + \sin \phi \cos \alpha |\overline{10}\rangle, \\ |\overline{10}^{\text{phys}}\rangle &= -\cos \phi \sin \alpha |8_1\rangle - \sin \phi |8_2\rangle + \cos \phi \cos \alpha |\overline{10}\rangle. \end{aligned} \quad (12)$$

2.3. Phenomenology of mixing

The sequential mixing (11) allows to calculate physical masses by iterative application of (8). After the first mixing of antidecuplet with the ground state octet, antidecuplet states satisfy Gell-Man–Okubo mass formulae (from now on particle names stand for their masses)

$$N_{\overline{10}} = \Theta^+ + \delta, \quad \Sigma_{\overline{10}} = \Theta^+ + 2\delta, \quad \Xi_{\overline{10}} = \Theta^+ + 3\delta. \quad (13)$$

States $N_{\overline{10}}$ and $\Sigma_{\overline{10}}$ then mix further with the Roper octet

$$\begin{aligned} M_{8_2} &= M_{\overline{10}}^{\text{phys}} \sin^2 \phi + M_{8_2}^{\text{phys}} \cos^2 \phi, \\ M_{\overline{10}} &= M_{\overline{10}}^{\text{phys}} \cos^2 \phi + M_{8_2}^{\text{phys}} \sin^2 \phi. \end{aligned} \quad (14)$$

These formulae are valid both for nucleon- and sigma-like states. Since physical masses of the nucleon-like states are known we can calculate bare $N_{\overline{10}}$ from (14)

$$N_{\overline{10}} = N_{\overline{10}}^{\text{phys}} \cos^2 \phi + N_{8_2}^{\text{phys}} \sin^2 \phi. \quad (15)$$

Since we know Θ^+ (which is not mixed) and $N_{\overline{10}}$ (from (15)) we can calculate δ and then $\Xi_{\overline{10}}^{\text{phys}} = \Xi_{\overline{10}}$ (because $\Xi_{\overline{10}}$ is not mixed as well) and finally $\Sigma_{\overline{10}}^{\text{phys}}$ from the last equation in (14) since both $\Sigma_{8_2}^{\text{phys}}$ and $\Sigma_{\overline{10}}$ (from (13)) are known

$$\Sigma_{\overline{10}}^{\text{phys}} = \frac{1}{\cos^2 \phi} \left(2N_{\overline{10}}^{\text{phys}} \cos^2 \phi + \left(2N_{8_2}^{\text{phys}} - \Sigma_{8_2}^{\text{phys}} \right) \sin^2 \phi - \Theta^+ \right), \quad (16)$$

$$\Xi_{\overline{10}}^{\text{phys}} = 3 \left(N_{\overline{10}}^{\text{phys}} \cos^2 \phi + N_{8_2}^{\text{phys}} \sin^2 \phi \right) - 2\Theta^+. \quad (17)$$

3. Decays in the presence of mixing

To calculate the decay width of baryon $B_1 \rightarrow B_2 + \varphi$ (where φ stands for the pseudoscalar meson) we shall use — following [5, 20] — the generalized Goldberger–Treiman relation employed first by Witten, Adkins and Nappi in Ref. [41]

$$\Gamma_{B_1 \rightarrow B_2 \varphi} = \frac{g_{B_1 B_2 \varphi}^2}{2\pi(M_1 + M_2)^2} p_\varphi^3. \quad (18)$$

Here, M_1 is the mass of the decaying baryon, M_2 the mass of the decay product, p_φ meson momentum given by

$$p_\varphi = \frac{\sqrt{(M_1^2 - (M_2 + m_\varphi)^2)(M_1^2 - (M_2 - m_\varphi)^2)}}{2M_1}. \quad (19)$$

The decay constant $g_{B_1 B_2 \varphi}$ stands for the matrix element of the tensor decay operator $O_\varphi^{(8)}$

$$g_{B_1 B_2 \varphi} = \left\langle B_2^{\text{phys}} \left| O_\varphi^{(8)} \right| B_1^{\text{phys}} \right\rangle. \quad (20)$$

Explicit form of the decay operator is known for example in χ QSM [5]. Here we shall simply assume, that $O_\varphi^{(8)}$ transforms as a φ component of the octet and as spin 1. This, together with the assumption that $O_\varphi^{(8)}$ satisfies hermiticity condition will allow us to express the relevant matrix elements between the unmixed states with the help of $SU_{\text{fl}}(3)$ isoscalar factors and the reduced matrix elements. Below we list all matrix elements needed in the present analysis. Decays to octet (ground state or Roper) are given by

$$\begin{aligned} \langle 8, B_2 | O_\varphi^{(8)} | \bar{10}, B_1 \rangle &= - \left[\begin{array}{cc|c} 8 & 8 & \bar{10} \\ \varphi & B_1 & B_2 \end{array} \right] G_{\bar{10}} \quad \left(\text{or } G_{\bar{10}}^{\text{R}} \right), \\ \langle 8, B_2 | O_\varphi^{(8)} | 10, B_1 \rangle &= \sqrt{2} \left[\begin{array}{cc|c} 8 & 8 & 10 \\ \varphi & B_1 & B_2 \end{array} \right] G_{10} \quad \left(\text{or } G_{10}^{\text{R}} \right). \end{aligned} \quad (21)$$

Diagonal matrix elements are defined as follows

$$\begin{aligned} \langle \bar{10}, B_2 | O_\varphi^{(8)} | \bar{10}, B_1 \rangle &= \sqrt{2} \left[\begin{array}{cc|c} 8 & \bar{10} & \bar{10} \\ \varphi & B_1 & B_2 \end{array} \right] H_{\bar{10}}, \\ \langle 8, B_2 | O_\varphi^{(8)} | 8, B_1 \rangle &= 2 \left[\begin{array}{cc|c} 8 & 8 & 8 \\ \varphi & B_1 & B_2 \end{array} \right] A + \sqrt{20} \left[\begin{array}{cc|c} 8 & 8 & 8' \\ \varphi & B_1 & B_2 \end{array} \right] B, \end{aligned} \quad (22)$$

with $A, B \rightarrow A^{\text{R}}, B^{\text{R}}$ when one of the octets is 8_2 . Note that transitions $8 \rightarrow \bar{10}$ are equal to $\bar{10} \rightarrow 8$ of (21) by the hermiticity requirement. For transitions $8 \rightarrow 10$ and $10 \rightarrow 8$ more care is needed since decuplet has spin $3/2$; we shall comment upon this later. Matrix elements used in this paper are displayed in Table I.

In the case of pion–nucleon coupling and Roper decays we may safely neglect small mixing corrections proportional to the sinuses of the mixing angles. With notation of Table I we have

$$g_{\pi NN} = \frac{\cos^2 \alpha}{\sqrt{3}} (A - 3B), \quad \varepsilon = \frac{F}{D} = -\frac{A}{3B}. \quad (23)$$

Furthermore Roper decay constants read

$$\begin{aligned} g_{RN\pi} &= \cos \phi \cos \alpha (A^{\text{R}} - 3B^{\text{R}}), \\ g_{RN\eta} &= -\cos \phi \cos \alpha (A^{\text{R}} + B^{\text{R}}). \end{aligned} \quad (24)$$

TABLE I

Matrix elements of the decay operator between $SU_{\mathbb{H}}(3)$ symmetry states.

$B_1 \rightarrow B_2 \varphi$	$\langle 8, B_2 O_\varphi^{(8)} \overline{10}, B_1 \rangle$	$\langle \overline{10}, B_2 O_\varphi^{(8)} \overline{10}, B_1 \rangle$	$\langle 8, B_2 O_\varphi^{(8)} 8, B_1 \rangle$
$\Theta^+ \rightarrow NK$	$G_{\overline{10}}$	$H_{\overline{10}}$	—
$N \rightarrow N\pi$	$\frac{1}{2}G_{\overline{10}}$	$\frac{1}{2}H_{\overline{10}}$	$A - 3B$
$N\eta$	$\frac{1}{2}G_{\overline{10}}$	$-\frac{1}{2}H_{\overline{10}}$	$-A - B$
$K\Lambda$	$-\frac{1}{2}G_{\overline{10}}$	—	$A - B$
$K\Sigma$	$\frac{1}{2}G_{\overline{10}}$	$H_{\overline{10}}$	$A + 3B$
$\Sigma \rightarrow N\overline{K}$	$\frac{1}{\sqrt{6}}G_{\overline{10}}$	$-\frac{2}{\sqrt{6}}H_{\overline{10}}$	$-\frac{2}{\sqrt{6}}(A + 3B)$
$\Sigma\eta$	$\frac{1}{2}G_{\overline{10}}$	—	$2B$
$\Lambda\pi$	$-\frac{1}{2}G_{\overline{10}}$	—	$2B$
$\Sigma\pi$	$\frac{1}{\sqrt{6}}G_{\overline{10}}$	$\frac{2}{\sqrt{6}}H_{\overline{10}}$	$\frac{4}{\sqrt{6}}A$
ΞK	$-\frac{1}{\sqrt{6}}G_{\overline{10}}$	—	$\frac{2}{\sqrt{6}}(A - 3B)$
$\Xi \rightarrow Xi\pi$	$-\frac{1}{\sqrt{2}}G_{\overline{10}}$	—	—
ΣK	$\frac{1}{\sqrt{2}}G_{\overline{10}}$	$-\frac{1}{\sqrt{2}}H_{\overline{10}}$	—

Since for the Roper octet $\varepsilon^R \approx 0.37$ [19], which makes $g_{RN\eta}$ very small (note that the tiny decay width of the Roper to $N\eta$ [42] results both from the smallness of the coupling constant and of the phase space), we will assume in the following that

$$B^R = -A^R \quad (25)$$

which corresponds to

$$\varepsilon^R = \frac{1}{3}. \quad (26)$$

For antidecuplet decays we keep mixing terms, since they are comparable in magnitude to the primary decay constants. For Θ^+ we get

$$g_{\Theta NK} = \cos \alpha G_{\overline{10}} + \sin \alpha H_{\overline{10}}. \quad (27)$$

Since $g_{\Theta NK}$ can be directly read off from Θ^+ decay width, it is convenient to express the remaining decay constants through $g_{\Theta NK}$ and $g_{\pi NN}$, ε and $g_{RN\pi}$. This leads to the following decay constants for $N_{\overline{10}}$

$$\begin{aligned}
g_{N_{\overline{10}}N\pi} &= \frac{1}{2} \cos \phi \cos \alpha g_{\theta NK} - \cos \phi \tan \alpha \sqrt{3} g_{\pi NN} - \tan \phi g_{RN\pi} \\
&\simeq \frac{1}{2} g_{\theta NK} - \sin \alpha \sqrt{3} g_{\pi NN} - \sin \phi g_{RN\pi}, \\
g_{N_{\overline{10}}N\eta} &= \frac{1}{2} \cos \phi \cos \alpha g_{\theta NK} - \frac{1}{2} \cos \phi \sin 2\alpha H_{\overline{10}} + \cos \phi \tan \alpha \frac{3\varepsilon - 1}{1 + \varepsilon} \frac{g_{\pi NN}}{\sqrt{3}} \\
&\simeq \frac{1}{2} g_{\theta NK} - \sin \alpha H_{\overline{10}} + \sin \alpha \frac{3\varepsilon - 1}{1 + \varepsilon} \frac{g_{\pi NN}}{\sqrt{3}}. \tag{28}
\end{aligned}$$

Approximate equalities in Eq. (28) correspond to the small mixing angle limit ($\cos(\text{angle}) = 1$).

Apart from physical decay constants mentioned above, $g_{N_{\overline{10}}N\eta}$ depends additionally on $H_{\overline{10}}$ that canceled in the expression for $g_{N_{\overline{10}}N\pi}$. This is *a priori*, apart from the mixing angles, new free parameter that cannot be constrained from the data. In what follows we shall estimate $H_{\overline{10}}$ using model calculations within the framework of χ QSM.

Decay constant to AK reads

$$\begin{aligned}
g_{N_{\overline{10}}AK} &= -\frac{1}{2} \cos \phi g_{\theta NK} + \frac{1}{2} \cos \phi \sin \alpha H_{\overline{10}} \\
&\quad - \frac{1}{2} \frac{\tan \phi}{\cos \alpha} g_{RN\pi} - \frac{\cos \phi \tan \alpha}{\cos \alpha} \frac{3\varepsilon + 1}{1 + \varepsilon} \frac{g_{\pi NN}}{\sqrt{3}} \\
&\simeq -\frac{1}{2} \left(g_{\theta NK} - \sin \alpha H_{\overline{10}} + \sin \phi g_{RN\pi} + \sin \alpha \frac{3\varepsilon + 1}{1 + \varepsilon} \frac{2g_{\pi NN}}{\sqrt{3}} \right). \tag{29}
\end{aligned}$$

In the $SU_{\text{fl}}(3)$ limit antidecuplet states cannot decay to decuplet. However, in the presence of mixing such decays are possible

$$\begin{aligned}
g_{N_{\overline{10}}\Delta\pi} &= -2 \cos \phi \tan \alpha g_{\Delta N\pi} - \tan \phi g_{R\Delta\pi} \\
&\simeq -2 \sin \alpha g_{\Delta N\pi} - \sin \phi g_{R\Delta\pi}, \tag{30}
\end{aligned}$$

where we have introduced new decay constant $g_{R\Delta\pi}$ describing Roper decay to $\Delta\pi$. Here a remark concerning factor 2 in front of $g_{\Delta N\pi}$ in Eq. (30) is in order. Factor 2 implies

$$g_{N\Delta\pi} = 2g_{\Delta N\pi}. \tag{31}$$

This relation follows from the fact, that the decay operator transforms as an octet in $SU_{\text{fl}}(3)$ but has also spin 1, since the decays considered here occur in P -wave. While calculating the width we average over initial spin and isospin and sum over the final state. In the case of Δ decay initial spin and

isospin are $3/2$ and averaging of amplitude $A_{\Delta \rightarrow N\pi}$ gives

$$\langle A_{\Delta \rightarrow N\pi}^2 \rangle = \frac{1}{2S^\Delta + 1} \frac{1}{2T^\Delta + 1} \sum_{S_z^\Delta, S_z^N} \sum_{T_z, T_z^\Delta, T_z^N} A^2 = \frac{1}{16} \sum_{S_z^\Delta, S_z^N} \sum_{T_z, T_z^\Delta, T_z^N} A^2, \quad (32)$$

where A is the reduced amplitude. When calculating transition $N \rightarrow \Delta\pi$ which appears due to the mixing $N_{\overline{10}} - N_{8_1}$ we have, due to hermiticity, the same amplitude A , however averaging is different

$$\langle A_{N \rightarrow \Delta\pi}^2 \rangle = \frac{1}{2S^N + 1} \frac{1}{2T^N + 1} \sum_{S_z^\Delta, S_z^N} \sum_{T_z, T_z^\Delta, T_z^N} A^2 = \frac{1}{4} \sum_{S_z^\Delta, S_z^N} \sum_{T_z, T_z^\Delta, T_z^N} A^2. \quad (33)$$

Hence

$$\sqrt{\langle A_{N \rightarrow \Delta\pi}^2 \rangle} = 2\sqrt{\langle A_{\Delta \rightarrow N\pi}^2 \rangle} \quad (34)$$

which is effectively (up to an overall phase) the same as (31).

Decay constant for $\Sigma_{\overline{10}}$ to nucleon reads as follows

$$\begin{aligned} g_{\Sigma_{\overline{10}} N \overline{K}} &= \frac{1}{\sqrt{6}} \left(\cos \phi \cos \alpha g_{\theta NK} - \frac{3}{2} \cos \phi \sin 2\alpha H_{\overline{10}} \right. \\ &\quad \left. - \tan \phi g_{RN\pi} - \cos \phi \tan \alpha \frac{1-\varepsilon}{1+\varepsilon} 2\sqrt{3} g_{\pi NN} \right) \\ &\simeq \frac{1}{\sqrt{6}} \left(g_{\theta NK} - 3 \sin \alpha H_{\overline{10}} - \sin \phi g_{RN\pi} - \sin \alpha \frac{1-\varepsilon}{1+\varepsilon} 2\sqrt{3} g_{\pi NN} \right). \end{aligned} \quad (35)$$

Decays to Σ take the following form

$$\begin{aligned} g_{\Sigma_{\overline{10}} \Sigma \pi} &= \frac{1}{\sqrt{6}} \left(\cos \phi \cos \alpha g_{\theta NK} + \frac{1}{2} \cos \vartheta \cos \phi \sin 2\alpha H_{\overline{10}} \right. \\ &\quad \left. - \tan \phi g_{RN\pi} - \cos \phi \tan \alpha \frac{4\varepsilon}{1+\varepsilon} \sqrt{3} g_{\pi NN} \right) \\ &\simeq \frac{1}{\sqrt{6}} \left(g_{\theta NK} + \sin \alpha H_{\overline{10}} - \sin \phi g_{RN\pi} - \sin \alpha \frac{4\varepsilon}{1+\varepsilon} \sqrt{3} g_{\pi NN} \right), \end{aligned} \quad (36)$$

$$\begin{aligned} g_{\Sigma_{\overline{10}} \Sigma \eta} &= \frac{1}{2} \left(\cos \phi \cos \alpha g_{\theta NK} - \frac{1}{2} \cos \phi \sin 2\alpha H_{\overline{10}} \right. \\ &\quad \left. + \tan \phi g_{RN\pi} + \cos \phi \tan \alpha \frac{4}{1+\varepsilon} \frac{g_{\pi NN}}{\sqrt{3}} \right) \\ &\simeq \frac{1}{2} \left(g_{\theta NK} - \sin \alpha H_{\overline{10}} + \sin \phi g_{RN\pi} + \sin \alpha \frac{4}{1+\varepsilon} \frac{g_{\pi NN}}{\sqrt{3}} \right). \end{aligned} \quad (37)$$

Decay to Λ is given by

$$\begin{aligned}
 g_{\Sigma_{\overline{10}}\Lambda\pi} &= \frac{1}{2} \left(-\cos\phi g_{\theta NK} + \cos\phi \sin\alpha H_{\overline{10}} \right. \\
 &\quad \left. + \frac{\tan\phi}{\cos\alpha} g_{RN\pi} + \frac{\cos\phi \tan\alpha}{\cos\alpha} \frac{4\varepsilon}{1+\varepsilon} \sqrt{3} g_{\pi NN} \right) \\
 &\simeq -\frac{1}{2} \left(g_{\theta NK} - \sin\alpha H_{\overline{10}} - \sin\phi g_{RN\pi} - \sin\alpha \frac{4\varepsilon}{1+\varepsilon} \sqrt{3} g_{\pi NN} \right). \quad (38)
 \end{aligned}$$

Finally, for the decays to decuplet we have

$$\begin{aligned}
 g_{\Sigma_{\overline{10}}\Sigma^*\pi} &= -\frac{1}{\sqrt{6}} g_{N_{\overline{10}}\Delta\pi}, \\
 g_{\Sigma_{\overline{10}}\Delta\pi} &= \sqrt{\frac{2}{3}} g_{N_{\overline{10}}\Delta\pi}. \quad (39)
 \end{aligned}$$

4. Where are and what are properties of strange particles in antidecuplet

In order to get some numerical insight into Eqs. (28), let us recall the results of χ QSM

$$G_{\overline{10}} = \sqrt{\frac{3}{5}} G_{\overline{10}}^{\chi\text{QSM}}, \quad H_{\overline{10}} = \frac{\sqrt{3}}{4} H_{\overline{10}}^{\chi\text{QSM}} \quad (40)$$

in notation of Ref. [17]. Taking fits to decays without mixing we get

$$G_{\overline{10}} \sim 1.3, \quad H_{\overline{10}} \sim -6.9. \quad (41)$$

In the present work we completely eliminate $G_{\overline{10}}$ through Eq. (27), however $H_{\overline{10}}$ reappears in other decay constants. Therefore, in the following we shall choose $H_{\overline{10}} = -7$, however, we also checked other values, namely -5 and -9 . Note, that even if we know some decay constant — such as $g_{\pi NN}$ — from experiment, we still have freedom in choosing the relative phase with which it enters Eqs. (27)–(30). Some of these phases can be absorbed to the sign of the mixing angles, the other ones have to be chosen arbitrarily. For example taking $g_{\pi NN} > 0$ we can absorb the relative phase into the sign of α (see Eq. (27)). Therefore, the sign of $H_{\overline{10}}$ cannot be absorbed into redefinition of α . Choosing negative $H_{\overline{10}}$ we have followed the sign dictated by the χ QSM, therefore, we have to check sensitivity of our predictions for $H_{\overline{10}} > 0$. We find that in this case the results are not compatible with experimentally acceptable pattern of decay widths. Similarly the phase of $g_{RN\pi}$ can be absorbed into the sign of angle ϕ , therefore, the relative phase

between $g_{\Delta N\pi}$ and $g_{R\Delta\pi}$ is not fixed (see Eq. (30)). In what follows we assume that both $g_{\Delta N\pi}$ and $g_{R\Delta\pi}$ are positive and show that for negative phase we again get results that are not compatible with experiment.

Throughout this paper we assume the following values for the parameters entering decay constants

$$g_{\pi NN} = 13.21, \quad \varepsilon = 0.56, \quad \varepsilon^R = \frac{1}{3} \quad (42)$$

and the following decay widths for Δ and Roper [42]

$$\Gamma_{\Delta \rightarrow N\pi} = 120 \text{ MeV}, \quad \Gamma_{R \rightarrow N\pi} = 152.1 \text{ MeV}, \quad \Gamma_{R \rightarrow \Delta\pi} = 58.5 \text{ MeV}. \quad (43)$$

In Figs. 3 and 4 we plot partial decay widths and branching ratios for $N_{\overline{10}}$ decays as functions of mixing angles α and ϕ . Total width has been calculated by adding all partial widths plus 10% for unaccounted three body decays. First of all let us observe that without mixing the dominant decay mode πN has branching ratio of 66%, next is ηN with branching ratio 20%

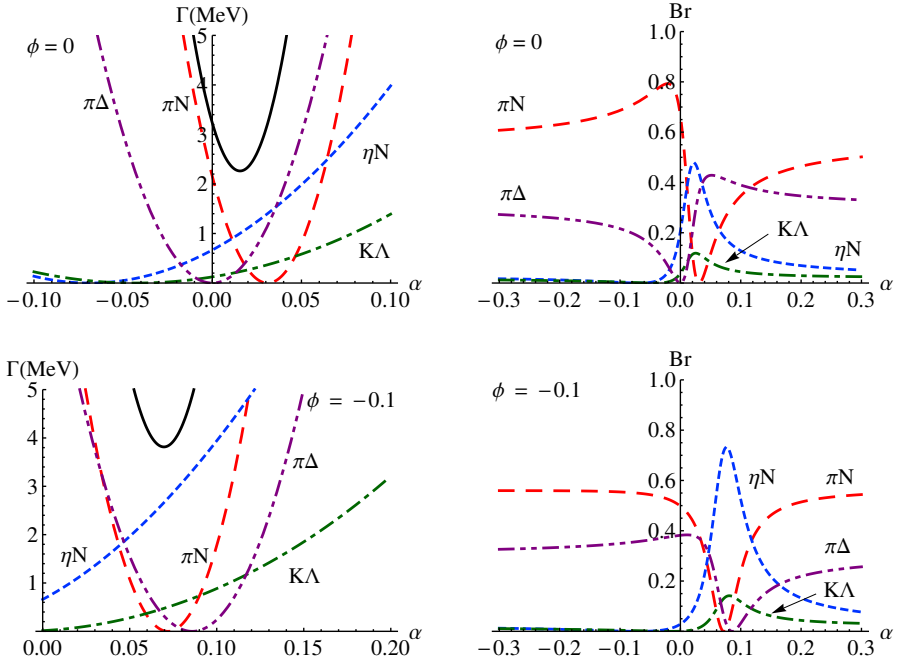


Fig. 3. Partial widths and branching ratio for $N_{\overline{10}}$ decays into: $N\pi$ — long dashed (red), $N\eta$ — short dashed (blue), ΔK — dash-dotted (dark green), $\Delta\pi$ — dash-dot-dotted (purple) and total width — solid (black) as functions of angle α for fixed ϕ : $\phi = 0$ — upper plots, $\phi = -0.1$ — lower plots. For all plots $H_{\overline{10}} = -7$.

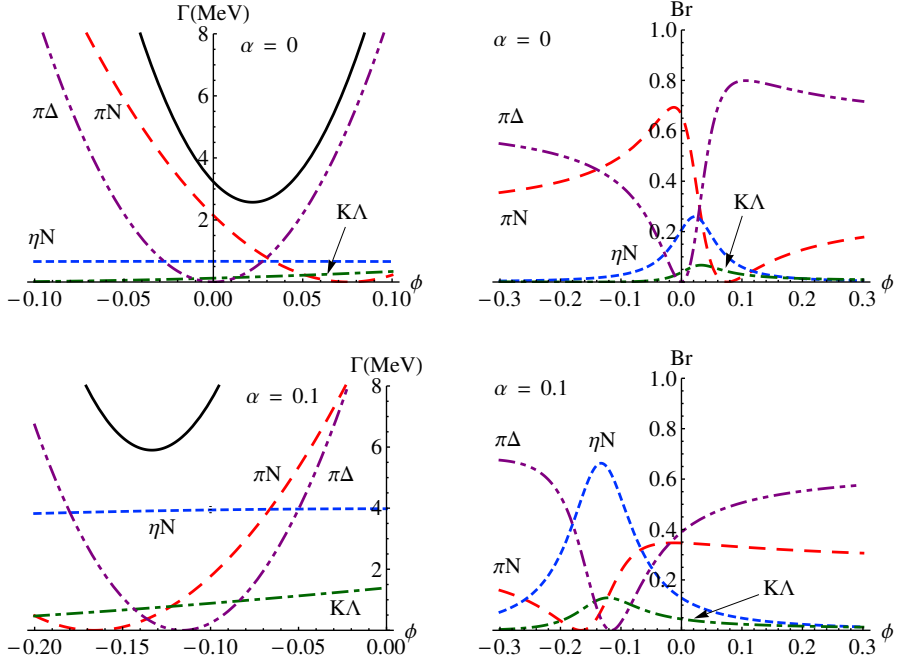


Fig. 4. Same as Fig. 3 but as functions of angle ϕ for fixed α : $\alpha = 0$ — upper plots, $\alpha = +0.1$ — lower plots.

and finally $K\Lambda$ — 4%. Decays to decuplet are forbidden. This decay pattern contradicts experiment (provided we want to interpret $N(1685)$ which decays predominantly to ηN , as a cryptoexotic member of antidecuplet). However, the branching ratios and decay widths change rather rapidly when mixing is included. We can see from Figs. 3 and 4 that for positive α and negative ϕ there exist regions, where the decay to ηN is dominant and the total width is relatively large. In order to find the preferable region in (α, ϕ) plane we impose the following requirements [16, 23, 34]

$$\begin{aligned} \Gamma(N_{\overline{10}} \rightarrow \pi N) &< 0.5 \text{ MeV}, \\ \text{Br}(N_{\overline{10}} \rightarrow \eta N) &> 0.2, \\ 5 \text{ MeV} &< \Gamma_{\text{tot}}(N_{\overline{10}}) < 25 \text{ MeV} \end{aligned} \quad (44)$$

and plot pertinent contours in Fig. 5. The allowed region defined in Eq. (44) should lie between the two continuous (black) ellipses corresponding to the allowed range of the total width, inside the outer dashed blue ellipse corresponding to $\text{Br}(N_{\overline{10}} \rightarrow \eta N) = 0.2$, and between two solid (red) lines, where $\Gamma(N_{\overline{10}} \rightarrow \pi N) < 5 \text{ MeV}$. We see that there is a rather narrow strip of allowed

angles concentrated in the vicinity of the point $\alpha \approx 0.11$ and $\phi \approx -0.2$. In Fig. 5 we also plot the dotted (orange) line inside the allowed region given by the equation

$$\phi(\alpha) = 0.0508 - 2.207\alpha, \quad 0.079 < \alpha < 0.159. \quad (45)$$

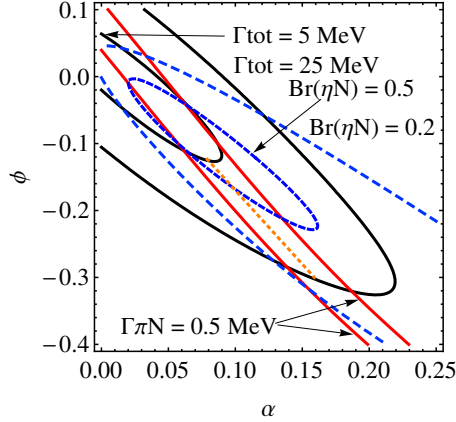


Fig. 5. Contour plot corresponding to the conditions (44). We also plot a dotted (orange) line inside the allowed region of mixing angles along which we later plot partial decays widths and branching ratios.

In the following figures we plot various quantities along the line (45), *i.e.* for mixing angles inside the allowed region, indicating the limits on angle α by vertical thin lines. First in Fig. 6 we show partial decay widths and branching ratios of $N_{\overline{10}}$. We see that indeed the decay to πN is strongly suppressed, however another channel, namely the decay to $\pi \Delta$, starts to dominate for larger mixing angles. In any case decays to decuplet are large (remember they are forbidden if there is no mixing) and this prediction provides a stringent test of our model.

Next, we come to the predictions for $\Sigma_{\overline{10}}$ and also for $\Xi_{\overline{10}}$. In Fig. 7 we plot masses of antidecuplet states as functions of ϕ . Solid lines correspond to the physical masses. Since we take Θ^+ and $N_{\overline{10}}$ as input they do not depend on mixing angle. Dashed lines correspond to GMO states before mixing. We see that

$$1795 \text{ MeV} < M_{\Sigma_{\overline{10}}} < 1830 \text{ MeV}, \quad (46)$$

$$1900 \text{ MeV} < M_{\Xi_{\overline{10}}} < 1970 \text{ MeV} \quad (47)$$

within the allowed limits of Fig. 5 (not only at the endpoints of the line (45)).

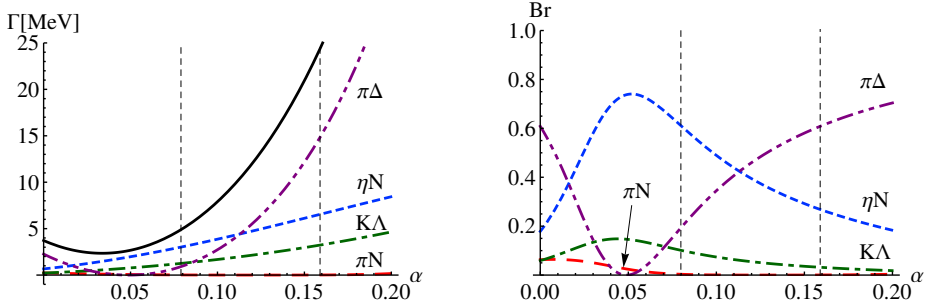


Fig. 6. Partial widths and branching ratio for N_{10} decays into: $N\pi$ — long dashed (red), $N\eta$ — short dashed (blue), ΛK — dash-dotted (dark green), $\Delta\pi$ — dash-dot-dot (purple) and total width — solid (black) plotted along the line (45). For all plots $H_{10} = -7$.

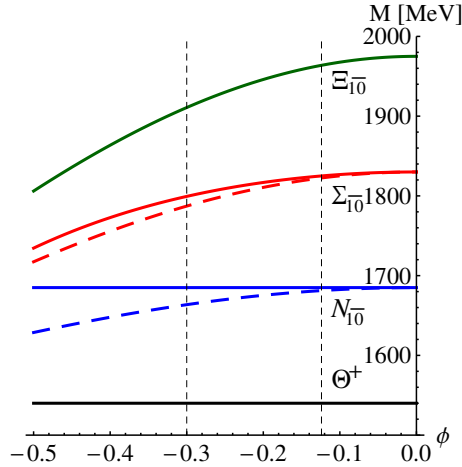


Fig. 7. Masses of antidecuplet states as functions of angle ϕ . Solid lines correspond to the physical masses, whereas dashed lines to the GMO masses before mixing with Roper octet. Note that for Θ^+ and Ξ_{10}^- GMO states and physical states coincide.

Finally, in Fig. 8 we plot partial widths for the decays of Σ_{10}^- . We see two dominating decay modes: $\Sigma_{10}^- \rightarrow KN$ and $\pi\Lambda$. In the right panel we magnify the scale to distinguish between the remaining decays. In Fig. 9 we plot the pertinent branching ratios. In Table II we give the range of widths within the allowed limits of Fig. 5 (not only along the line (45)).

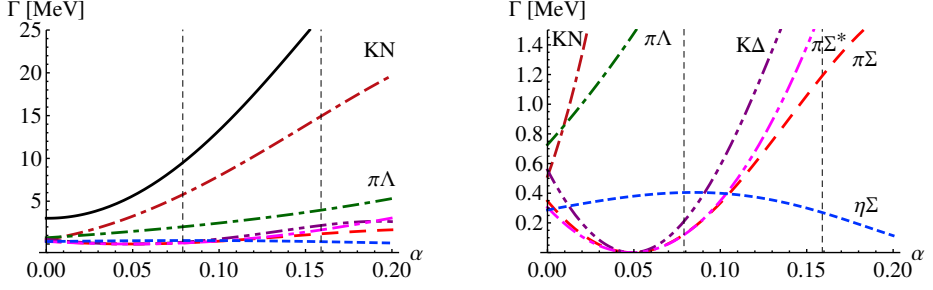


Fig. 8. Partial widths for Σ_{10} decays into: NK — double dash dotted (brown), $\Lambda\pi$ — dash dotted (dark green), $\Sigma\pi$ — long dashed (red), ΔK — double dash dotted (purple), $\Sigma\eta$ — short dashed (blue), $\Sigma^*\pi$ — dash-dot-dotted (pink) and total width — solid (black) plotted along the line (45). For all plots $H_{10} = -7$. On right panel we present the enlargement of the left plot in order to distinguish different decay modes that are below 1.2 MeV.

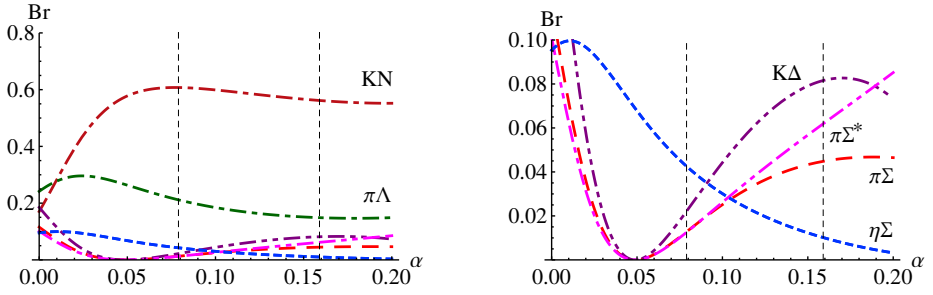


Fig. 9. Branching ratios for decays of Σ_{10} . All lines as in Fig. 8.

TABLE II

Range of decay widths of Σ_{10} in the region of allowed mixing angles.

Mode	Γ_{\min} [MeV]	Γ_{\max} [MeV]
KN	5.50	15.27
$\pi\Lambda$	1.70	4.40
$K\Delta$	0.09	2.57
$\pi\Sigma$	0.01	1.74
$\pi\Sigma^*$	0.05	1.95
$\eta\Sigma$	0.22	0.50
Total	9.7	26.9

Finally we also plot in Fig. 10 partial decay widths for the two allowed decays modes of $\Xi_{\overline{10}}$.

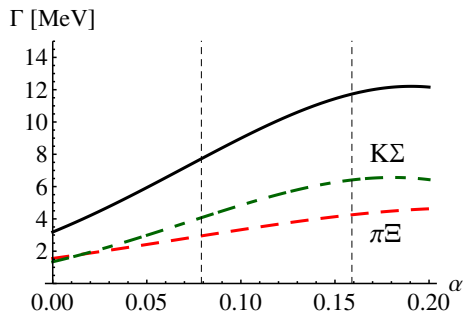


Fig. 10. Partial widths for $\Xi_{\overline{10}}$ decays into: $N\pi$ – long dashed (red), and total width — solid (black) plotted along the line (45). For all plots $H_{\overline{10}} = -7$.

5. Conclusions

In the present paper we have examined mixing scenario in which the crypto-exotic antidecuplet states mix with the respective states in the ground state octet and in the Roper octet. This scenario is motivated by recent experimental results on new narrow nucleon resonance $N(1685)$. Its small decay width $\Gamma < 25$ MeV and the fact that $n(1685)$ (neutron-like state) undergoes photoexcitation in η meson production and $p(1685)$ does not, is easily explained if these states are interpreted as members of antidecuplet. However, $N(1685)$ seems to have very small coupling to the πN channel and comparatively large one to ηN channel in contradiction to pure $SU_{\text{fl}}(3)$ predictions for these decays. Since Roper resonance has very small partial width to ηN one can adjust its mixing angle with $N(1685)$ to suppress πN coupling not affecting the ηN one. We have shown that there exists a small, but stable against variation of the unknown couplings, region in the space of mixing angles, where such mechanism is effectively working. Our main result concerning the strange members of antidecuplet is based on the observation that Σ -like states have the same mixing angles as the nucleonic states and on the fact that $\Xi_{\overline{10}}$ does not mix neither with Roper octet nor with the ground state octet. This allows us to constrain masses of these states and their decay patterns.

Before we discuss our results in more detail we want to stress that the mass limits given in Eqs. (46), (47) and the decay widths summarized in Table II should be considered as the qualitative ones. Variations of Roper decay constants within the experimental limits and Roper ε_{R} parameter would enlarge limits derived in this paper. Also our discussion concerning $H_{\overline{10}}$ parameter was only qualitative.

We predict that $\Sigma_{\overline{10}}$ has a mass around 1815 MeV within the limits of Eq. (46). There are no known Σ resonances in this energy range [42]. Total width of $\Sigma_{\overline{10}}$ does not exceed 30 MeV but is also constrained from below being larger than 10 MeV. Note that we have calculated total widths by adding all partial widths and 10% for unaccounted three body decays. Most prominent decay channels are KN and $\pi\Lambda$ with branching ratios approximately 60% and 20%, respectively. Due to the mixing $SU_{\text{fl}}(3)$ forbidden decays to decuplet are possible, but small, at the level of 5 to 9%.

The simplest way to detect $\Sigma_{\overline{10}}$ is its formation in K^-p scattering with the kaon beam of $p_{\text{lab}} \sim 1$ GeV. The corresponding resonance cross-section is about 5 millibarn. Note, however, that the non-resonant K^-p cross-section in this energy region is tens of millibarn. That requires high statistics experiment and detailed partial wave analysis. Additionally, the small width of predicted $\Sigma_{\overline{10}}$ demands high energy resolution of the kaon beam. $\Sigma_{\overline{10}}$ can be produced in the photoproduction experiments, *e.g.* in $\gamma + p \rightarrow \Sigma_{\overline{10}} + K_S$, note however, that according to simple estimate based on the U-spin the corresponding production cross-section is about three times smaller than the analogous $\gamma + p \rightarrow \Theta^+ + K_S$ cross-section, *i.e.* very small. One can reveal the small signal of the $\gamma + p \rightarrow \Sigma_{\overline{10}} + K_S \rightarrow p + K_L + K_S$ processes using its interference with much stronger amplitude of $\gamma + p \rightarrow \phi + p \rightarrow p + K_L + K_S$ [43]. Details of possible ways to see $\Sigma_{\overline{10}}$ in various processes we shall give elsewhere [44].

From (47) we see that $M_{\Xi_{\overline{10}}}$ is larger than 1900 MeV. The latter estimate is in disagreement with the result of NA49 [13]. There is one known three star resonance in the energy range (47), namely $\Xi(1950)$ of unknown spin and parity [42]. However, it has isospin $I = 1/2$ and therefore it cannot belong to $\overline{10}$. Indeed in Ref. [19] $\Xi(1950)$ has been attributed to the same octet as $N(1710)$.

Our results are based on, to some extent arbitrary, assignment of the relative signs of two reduced matrix elements defining decay constants: $H_{\overline{10}}$ and $g_{R\Delta\pi}$. We have fixed $H_{\overline{10}}$ to be negative ($H_{\overline{10}} \sim -7$) in agreement with χ QSM and $g_{R\Delta\pi}$ to be positive ($g_{R\Delta\pi} \sim 30$). In Fig. 11 we show contour plots corresponding to (44) with these phases inverted. We see that there is no common stable intersection region in the space of mixing angles for $H_{\overline{10}} > 0$ or/and $g_{R\Delta\pi} < 0$. In fact there is small allowed spot for $H_{\overline{10}} = -7$ and $g_{R\Delta\pi} = -30$, but it is unstable for small variations of these values or conditions (44). Finally, in the same figure we present contour plot for large width of Θ^+ , where also no allowed region of mixing angles exists.

To conclude: we have proposed a scenario in which Roper octet can mix with putative antidecuplet of exotic baryons and predicted the properties of its strange members $\Sigma_{\overline{10}}$ and $\Xi_{\overline{10}}$. We hope that these estimates will be helpful in eventual experimental searches.

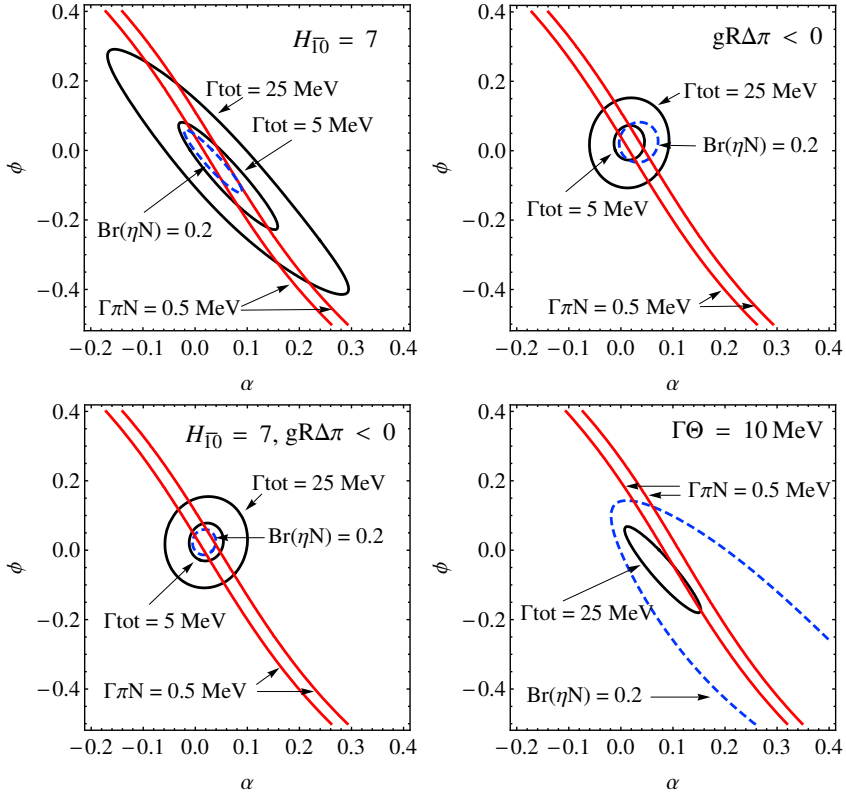


Fig. 11. Contour plots as in Fig. 5 for three unconstraint phases taking values different than in the main analysis, and for the standard phases but for large decay width of Θ^+ .

The authors are grateful to M. Amarian, D. Diakonov, V. Kuznetsov and V. Petrov for discussions. The paper was partially supported by the Polish–German cooperation agreement between the Polish Academy of Sciences (PAN) and Deutsche Forschungsgemeinschaft (DFG), and SFB TR16 (Bochum–Bonn–Giessen) of DFG.

REFERENCES

- [1] T. Nakano *et al.* [LEPS Collaboration], *Phys. Rev. Lett.* **91**, 012002 (2003) [[arXiv:hep-ex/0301020](#)].
- [2] V.V. Barmin *et al.* [DIANA Collaboration], *Phys. At. Nucl.* **66**, 1715 (2003) [*Yad. Fiz.* **66**, 1763 (2003)] [[arXiv:hep-ex/0304040](#)].

- [3] L.C. Biedenharn, Y. Dothan, *Monopolar Harmonics in $SU(3)_F$ as Eigenstates of the Skyrme–Witten Model for Baryons*, E. Gotsman, G. Tauber (eds.), *From $SU(3)$ to Gravity*, Cambridge University Press, 1985, pp. 15–34.
- [4] M. Praszalowicz, talk at the Workshop on Skyrmions and Anomalies, M. Jeżabek, M. Praszalowicz eds., World Scientific, 1987, p. 112; M. Praszalowicz, *Phys. Lett. B* **575**, 234 (2003) [arXiv:hep-ph/0308114].
- [5] D. Diakonov, V. Petrov, M.V. Polyakov, *Z. Phys. A* **359**, 305 (1997) [arXiv:hep-ph/9703373].
- [6] B. McKinnon *et al.* [CLAS Collaboration], *Phys. Rev. Lett.* **96**, 212001 (2006) [arXiv:hep-ex/0603028]; R. De Vita *et al.* [CLAS Collaboration], *Phys. Rev. D* **74**, 032001 (2006) [arXiv:hep-ex/0606062].
- [7] T. Hotta [LEPS Collaboration], *Acta Phys. Pol. B* **36**, 2173 (2005).
- [8] T. Nakano *et al.* [LEPS Collaboration], *Phys. Rev. C* **79**, 025210 (2009) [arXiv:0812.1035 [nucl-ex]].
- [9] V.V. Barmin *et al.* [DIANA Collaboration], *Phys. At. Nucl.* **70**, 35 (2007) [arXiv:hep-ex/0603017]; arXiv:0909.4183 [hep-ex].
- [10] T. Nakano, *Nucl. Phys. A* **755**, 3 (2005).
- [11] V.D. Burkert, *Int. J. Mod. Phys. A* **21**, 1764 (2006) [arXiv:hep-ph/0510309].
- [12] M.V. Danilov, R.V. Mizuk, *Phys. At. Nucl.* **71**, 605 (2008).
- [13] C. Alt *et al.* [NA49 Collaboration], *Phys. Rev. Lett.* **92**, 042003 (2004) [arXiv:hep-ex/0310014].
- [14] M.V. Polyakov, A. Rathke, *Eur. Phys. J. A* **18**, 691 (2003) [arXiv:hep-ph/0303138].
- [15] D. Diakonov, V. Petrov, *Phys. Rev. D* **69**, 094011 (2004) [arXiv:hep-ph/0310212].
- [16] R.A. Arndt *et al.*, *Phys. Rev. C* **69**, 035208 (2004) [arXiv:nucl-th/0312126].
- [17] J.R. Ellis, M. Karliner, M. Praszalowicz, *J. High Energy Phys.* **0405**, 002 (2004) [arXiv:hep-ph/0401127].
- [18] M. Praszalowicz, *Acta Phys. Pol. B* **35**, 1625 (2004) [arXiv:hep-ph/0402038].
- [19] V. Guzey, M.V. Polyakov, *AIP. Conf. Proc.* **775**, 3 (2005) [arXiv:hep-ph/0501010]; V. Guzey, M.V. Polyakov, *Ann. Phys.* **13**, 673 (2004); arXiv:hep-ph/0512355.
- [20] T. Ledwig, H.C. Kim, K. Goeke, *Phys. Rev. D* **78**, 054005 (2008) [arXiv:0805.4063 [hep-ph]].
- [21] V. Kuznetsov [GRAAL Collaboration], arXiv:hep-ex/0409032.
- [22] V. Kuznetsov *et al.*, *Phys. Lett. B* **647**, 23 (2007) [arXiv:hep-ex/0606065].
- [23] V. Kuznetsov *et al.*, *Acta Phys. Pol. B* **39**, 1949 (2008) [arXiv:0807.2316 [hep-ex]].
- [24] I. Jaegle *et al.* [CBELSA Collaboration and TAPS Collaboration], *Phys. Rev. Lett.* **100**, 252002 (2008) [arXiv:0804.4841 [nucl-ex]].

- [25] F. Miyahara *et al.*, *Prog. Theor. Phys. Suppl.* **168**, 90 (2007).
- [26] K.S. Choi, S.i. Nam, A. Hosaka, H.C. Kim, *Phys. Lett.* **B636**, 253 (2006) [arXiv:hep-ph/0512136]; S.i. Nam, K.S. Choi, A.Hosaka, H.C. Kim, *Prog. Theor. Phys. Suppl.* **168**, 97 (2007) [arXiv:0704.3101 [hep-ph]]; K.S. Choi, S.i. Nam, A. Hosaka, H.C. Kim, arXiv:0710.2185 [hep-ph]; K.S. Choi, S.i. Nam, A. Hosaka, H.C. Kim, *J. Phys. G* **36**, 015008 (2009) arXiv:0707.3854 [hep-ph].
- [27] Y.I. Azimov, V. Kuznetsov, M.V. Polyakov, I. Strakovsky, *Eur. Phys. J.* **A25**, 325 (2005) [arXiv:hep-ph/0506236].
- [28] A. Fix, L. Tiator, M.V. Polyakov, *Eur. Phys. J.* **A32**, 311 (2007) [arXiv:nucl-th/0702034].
- [29] V. Shklyar, H. Lenske, U. Mosel, *Phys. Lett.* **B650**, 172 (2007) [arXiv:nucl-th/0611036].
- [30] A.V. Anisovich *et al.*, *Eur. Phys. J.* **A41**, 13 (2009) [arXiv:0809.3340 [hep-ph]].
- [31] M. Doring, K. Nakayama, *Phys. Lett.* **B683**, 145 (2010) [arXiv:0909.3538 [nucl-th]].
- [32] V. Kuznetsov *et al.*, arXiv:0801.0778 [hep-ex].
- [33] V. Kuznetsov *et al.*, arXiv:hep-ex/0703003.
- [34] V. Kuznetsov, M.V. Polyakov, *JETP Lett.* **88**, 347 (2008) [arXiv:0807.3217 [hep-ph]].
- [35] K. Abe *et al.* [Belle Collaboration], *Phys. Lett.* **B632**, 173 (2006) [arXiv:hep-ex/0507014].
- [36] R.A. Arndt, I.I. Strakovsky, R.L. Workman, *Phys. Rev.* **C68**, 042201 (2003) *Erratum-ibid.* **C69**, 019901 (2004) [arXiv:nucl-th/0308012]; J. Haidenbauer, G. Krein, *Phys. Rev.* **C68**, 052201 (2003) [arXiv:hep-ph/0309243]; R.A. Arndt, I.I. Strakovsky, R.L. Workman, *Nucl. Phys.* **A754**, 261 (2005) [arXiv:nucl-th/0311030]; W.R. Gibbs, *Phys. Rev.* **C70**, 045208 (2004) [arXiv:nucl-th/0405024].
- [37] D. Diakonov, V. Petrov, *Phys. Rev.* **D72**, 074009 (2005) [arXiv:hep-ph/0505201].
- [38] C. Lorce, *Phys. Rev.* **D74**, 054019 (2006) [arXiv:hep-ph/0603231].
- [39] A.G. Oganesian, *Int. J. Mod. Phys.* **A22**, 2093 (2007) [arXiv:hep-ph/0608031].
- [40] S. Pakvasa, M. Suzuki, *Phys. Rev.* **D70**, 036002 (2004) [arXiv:hep-ph/0402079].
- [41] G.S. Adkins, C.R. Nappi, E. Witten, *Nucl. Phys.* **B228**, 552 (1983).
- [42] C. Amsler *et al.* [Particle Data Group], *Phys. Lett.* **B667**, 1 (2008).
- [43] M. Amarian, D. Diakonov, M.V. Polyakov, *Phys. Rev.* **D78**, 074003 (2008) [arXiv:hep-ph/0612150]; Y. Azimov, *J. Phys. G* **37**, 023001 (2010) [arXiv:0904.1376 [hep-ph]].
- [44] M. Amarian, M.V. Polyakov, M. Praszalowicz, in preparation.