SUPERSIMPLICITY: A REMARKABLE HIGH ENERGY SUSY PROPERTY

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It is known that for any 2-to-2 process in MSSM, only the helicity conserving (HC) amplitudes survive asymptotically. Studying many such processes, at the 1-loop Electroweak (EW) order, it is found that their high energy HC amplitudes are determined by just three forms: a log-squared function of the ratio of two of the (s, t, u) variables, to which a π^2 is added; and two Sudakov-like ln- and $\ln^2$-terms accompanied by respective mass-dependent constants. Apart from a possible additional residual constant (which is also discussed), these HC amplitudes, may be expressed as linear combinations of the above three forms, with coefficients being rational functions of the (s, t, u) variables. This 1-loop property, called *supersimplicity*, is of course claimed for the 2-to-2 processes considered; but no violating examples are known at present. For $ug \to dW$, *supersimplicity* is found to be a very good approximation at LHC energies, provided the SUSY scale is not too high. SM processes are also discussed, and their differences are explored.

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1. Introduction

Supersymmetry is well-known for its remarkable properties controlling the hierarchy problem and improving the realization of Grand Unification [1]. More recently, two additional properties of Supersymmetry were noticed at the high energy behavior of the scattering amplitudes, where the soft supersymmetry (SUSY) breaking effects are minimized.

The first one concerns the differences in the coefficients of the 1-loop electroweak (EW) logarithmic behaviors contained in the so-called Sudakov terms, in SM and MSSM [2, 3, 4, 5]. The second one refers to the helicity conservation (HCns) property, which is specific to Supersymmetry.

This HCns property has been first proven to all orders in MSSM, at the approximation where all soft SUSY breaking effects, as well as the μ bilinear term of the scalar sector, are neglected [6, 7]. More explicitly it was showed that for any 2-to-2 processes

$$a_{\lambda_1} + b_{\lambda_2} \to c_{\lambda_3} + d_{\lambda_4} \,, \tag{1}$$

where λ_i denote the particle helicities, all amplitudes violating the helicity conservation rule

$$\lambda_1 + \lambda_2 = \lambda_3 + \lambda_4 \,, \tag{2}$$

must vanish at high energies and fixed angles in MSSM [6,7]; such amplitudes are called helicity violating (HV) amplitudes. Renormalizability is essential for the validity of HCns; all known anomalous couplings violate it [8].

So, only the helicity conserving (HC) amplitudes obeying (2), can survive asymptotically in MSSM. But in [6,7], nothing was said about the structure of the HC amplitudes at high energy, where mass effects may remain important, at least so far as they affect the scale of some logarithms. To study such mass effects in both, the HC and HV amplitudes, and investigate how HCns is realized in MSSM and violated in SM, many detail 1-loop EW calculations have been performed. The main results are summarized in the following paragraphs.

At the Born level, HCns is valid in both, the SM and the MSSM models. In such a case all HV amplitudes vanish asymptotically like inverse powers of the energy, while the HC ones tend to non-vanishing constants. Particularly for processes involving external gauge bosons, huge cancellations among the various diagrams contrive to establish HCns [9].

At the 1-loop EW level, with all mass terms retained, the high energy helicity amplitudes have been investigated, in both SM and MSSM, for gluon fusion producing a pair of gauge or Higgs bosons in [10,11], and for $ug \to dW$ in [12]. In all MSSM cases, it has then been studied how the high energy vanishing of all HV amplitudes is realized; usually like an inverse power of the energy, as the spartner contributions (sfermions and inos) cancel out the SM ones. In SM, on the contrary, it is only accidentally that the HV amplitudes may vanish asymptotically, and many cases have been identified where this does not happen [10, 11].

Concentrating on the HC asymptotic amplitudes in MSSM now, we distinguish two types of processes; those where there is no Born terms, and the ones in which Born terms are present. In each case, we define the 1-loop EW order property of *supersimplicity*, and explain how this definition is modified as we go from MSSM to SM.

In the first case, detail analytical studies at 1-loop EW order have recently been done for the gluon fusion to vector boson process $gg \to VV'$ [10], and the chargino and neutralino transitions $gg \to \tilde{\chi}_i \tilde{\chi}_j$ [13]. In these cases, there are no Born contributions and no Sudakov logarithms appear, implying no dependence on the SUSY breaking masses.

The HC asymptotic structure is then solely determined by forms like $\ln^2 + \pi^2$, where ratios of the (s,t,u) Mandelstam variables appear within the quadratic logarithms. The overall coefficients of these forms are solely determined by rational functions of (s,t,u), and there is no additional term. This is the *supersimplicity* structure in this case. All relevant formulae for this have already appeared in [10, 13], but they were not related to the concept of *supersimplicity*; this we do here.

The more important and new work in the present paper, still within MSSM, concerns the second type of the above processes for which Born terms are present. In this context we study the high energy 1-loop EW amplitudes for the 2-to-2 processes,

$$\begin{aligned} ug &\to dW\,, \qquad bg \to tW\,, \qquad bg \to tH^-\,, \qquad bg \to bZ\,, \\ bg &\to bH^0\,, \qquad gg \to t\bar{t}\,, \qquad gg \to \tilde{t}\bar{\tilde{t}}\,, \end{aligned} \tag{3}$$

and their SUSY transformed ones

$$\begin{aligned} \tilde{u}_L\tilde{g} &\to \tilde{d}_L\tilde{W}\,, \qquad \tilde{b}\tilde{g} \to \tilde{t}\tilde{W}\,, \qquad \tilde{b}\tilde{g} \to \tilde{t}\tilde{H}^-\,, \qquad \tilde{b}\tilde{g} \to \tilde{b}\tilde{Z}\,, \\ \tilde{b}\tilde{g} &\to \tilde{b}\tilde{H}^0\,, \qquad \tilde{g}\tilde{g} \to \tilde{t}\bar{\tilde{t}}\,, \qquad \tilde{g}\tilde{g} \to t\bar{t}\,. \end{aligned} \tag{4}$$

Note that the processes in (4) involve the gaugino and higgsino SUSY-counterparts of the charged and neutral gauge and Higgs boson processes in (3).

For the processes (3), (4), the content of *supersimplicity* is more involved. More explicitly, we find that the asymptotic HC amplitudes are now expressed as linear combinations of three possible forms, with coefficients being rational functions of the (s,t,u) variables. The first of these forms is the $\ln^2 + \pi^2$, one we have already seen for $gg \to VV', \tilde{\chi}_i\tilde{\chi}_j$. The other two forms consist of two Sudakov like terms, involving log and log-squared functions of a Mandelstam variable scaled by masses, to which respective "constants" are added, depending on ratios of masses.

The constants entering the definition of these three forms, greatly enhance the accuracy of the asymptotic expressions for the HC amplitudes, and allow to make valuable numerical predictions for physical observables. In addition to these forms, extra "residual constants" may also appear for the on-shell renormalized amplitudes of the MSSM processes (3), (4), at high energy.

Thus, *supersimplicity* completes the previously known rules for the purely logarithmic structure of Sudakov and angular depending terms, determining the high energy behavior of the 2-to-2 amplitudes [3, 4, 5].

While doing the analytical computations, we have also noticed an interesting recipe for obtaining the high energy MSSM results. This is based on the remark that it is often easier to first compute the relevant SUSY spartner process in (4), and then obtain the result for the actual process in (3), through a SUSY transformation. This is because the particles involved in the processes (4) have usually smaller spins than those in (3).

All together, the concept of *supersimplicity* in MSSM turns out to have three aspects: the simplicity of the high energy HC amplitudes; the recipe for computing these expressions by using the SUSY transformed processes; and the possibility of introducing a very simple renormalization scheme, the supersimplicity renormalization scheme (SRS), where only the above three forms appear asymptotically, without any additional constant. This SRS scheme may numerically be very close to the on-shell scheme. At least, this is what we have seen for $ug \to dW$, where the supersimplicity structure may be accurately (or approximately) valid at LHC, provided the SUSY scale is in the TeV range (or just above it).

The purpose of the present work is to describe this *supersimplicity* structure of the high energy HC amplitudes in MSSM, and to study its numerical accuracy for observable quantities. We repeat that this property is only defined at the 1-loop EW order.

Contents: Section 2 summarizes the MSSM *supersimplicity* structure of the processes $gg \to VV'$, involving no-Born term; based on the results of [10, 13]. In Section 3, the *supersimplicity* structure is described for processes (3), (4), which contain a Born-contribution. A detail study of $ug \to dW$ with numerical illustrations is also presented, while an analogous discussion of $bg \to bH_i^0$ appears in the Appendix. The results of Section 3 and the Appendix appear here for the first time. Finally, in Section 4, we present the Conclusions.

2. Supersimplicity for $gg \to VV'$

Here we summarize how *supersimplicity* appears in the 1-loop EW order results of [10, 11] for $gg \to VV'$, where V, V' are EW vector bosons. The results cover not only the MSSM case, but also the SM.

In MSSM, we of course have helicity conservation (HCns) at high energies. The asymptotic HV amplitudes thus vanish, while the HC ones are expressed through the form

$$r_{xy} \equiv \frac{-x-i\epsilon}{-y-i\epsilon} \quad \Rightarrow \quad \tilde{d}(r_{xy}) \equiv \ln^2 r_{xy} + \pi^2 \tag{5}$$

with x and y being any two of the (s,t,u) Mandelstam variables.

For transverse vector bosons, such high energy HC amplitudes are given by [14, 10]

$$F(gg \to ZZ)_{\mu\mu'\tau\tau'} = \alpha\alpha_s \frac{\left(9 - 18s_W^2 + 20s_W^4\right)}{24s_W^2 c_W^2} \delta_{\mu\mu'\tau\tau'}\,, \tag{6}$$

where (μ,μ') denote the initial gluon helicities, while (τ,τ') are the helicities of the final vector bosons, and

$$\begin{aligned} \delta_{+-+-} &= \delta_{-+-+} = -4\tilde{d}(r_{ts})\,, \\ \delta_{+--+} &= \delta_{-++-} = -4\tilde{d}(r_{us})\,, \\ \delta_{++++} &= \delta_{----} = -4\tilde{d}(r_{tu})\,, \end{aligned} \tag{7}$$

while all HV amplitudes satisfying $\mu + \mu' \neq \tau + \tau'$ vanish. A color factor δ^{ab}, with (a,b) describing the gluon SU(3) indices, is always removed from the amplitudes in (6). Similar expressions for $gg \to \gamma\gamma$, γZ, W^+W^- exist too [13].

Thus, the *supersimplicity* structure in this case MSSM means that all high energy transverse HC amplitudes are proportional[1] to the single form (5), without any additional constant.

Contrarily to the type of processes that we study in Sect. 3, where additional forms related to Sudakov logs appear; in the present case, no such Sudakov terms arise.

The derivation of the 1-loop asymptotic results (7), (6) from [10, 14] is quite laborious. A much simpler way to obtain them, is by looking at the SUSY-transformed processes

$$\tilde{g}\tilde{g} \to \tilde{B}\tilde{B}\,, \qquad \tilde{W}^{(3)}\tilde{W}^{(3)}\,, \qquad \tilde{W}^+\tilde{W}^-\,, \tag{8}$$

remembering that the signs of the gaugino–helicities are the same as those of the transverse gauge-bosons from which they were obtained, through the SUSY-transformation [6, 7]. In such a case, the box diagrams involve only 2 fermionic lines, each with only one γ^μ matrix. The calculation is then much simpler, leading, for transverse gauge bosons, to [13]

$$(-1)^{\tilde{\mu}-\tilde{\tau}'} F\left(\tilde{g}\tilde{g} \to \tilde{V}\tilde{V}'\right)_{\tilde{\mu}\tilde{\mu}'\tilde{\tau}\tilde{\tau}'} = F(gg \to VV')_{\mu\mu'\tau\tau'}\,, \tag{9}$$

[1] Real and Imaginary parts.

where $\tilde{\mu}$, $\tilde{\mu}'$, $\tilde{\tau}$, $\tilde{\tau}'$ are the gluino and gaugino helicities, which of course receive half integers values. The r.h.s. of (9) is of course determined by (6), and similar expressions for the other gauge bosons. As seen in (9), most of the gauge and gaugino asymptotic amplitudes, are identical. But for $\tilde{\mu} - \tilde{\tau}' = \pm 1$, sign differences appear, related to the way the fermionic states in the l.h.s. of (9) are defined.

An important role for the validity of (9), is played by the fact that the asymptotic amplitudes for (6), (8) are mass-independent; this allows us to consider un-mixed states. This is not true for the processes in Section 3, where mass complications always appear in the HC asymptotic 1-loop amplitudes.

Results analogous to (9), are also true for longitudinal vector bosons, which necessarily include higgsino amplitudes in the l.h.s. [13,10,14]. Thus, in order to study the $gg \to VV'$ asymptotic behavior, it is advantageous to consider the SUSY-transformed process $\tilde{g}\tilde{g} \to \tilde{\chi}_i \tilde{\chi}_j$, with the appropriate gaugino and higgsino $\tilde{\chi}_i \tilde{\chi}_j$ components. Such a procedure simplifies the calculation a lot[2].

The asymptotic structure in SM is mutilated by additional A^S contributions, inducing non-vanishing HV asymptotic amplitudes, and at the same time also creating HC contributions which include forms other than (5). Explicitly, the SM asymptotic amplitudes for transverse final vector bosons are [10]

$$F(gg \to ZZ)^{SM}_{\mu\mu'\tau\tau'} = \alpha\alpha_{\rm s} \frac{(9 - 18s_W^2 + 20 s_W^4)}{24 s_W^2 c_W^2} \left[\delta_{\mu\mu'\tau\tau'} - 2A^S_{\mu\mu'\tau\tau'}\right] , \quad (10)$$

where $\delta_{\mu\mu'\tau\tau'}$ only contributes to the HC amplitudes and is given by (7); while A^S contributes, both to the HC and HV transverse amplitudes as

$$\begin{aligned}
A^S_{++++} &= A^S_{----} = 4 - \frac{4ut}{s^2}\tilde{d}(r_{tu}) + \frac{4(t-u)}{s}\ln\left(\frac{t}{u}\right) , \\
A^S_{+-+-} &= A^S_{-+-+} = 4 - \frac{4st}{u^2}\tilde{d}(r_{st}) + \frac{4(s-t)}{u}\ln\left(\frac{-s-i\epsilon}{-t}\right) , \\
A^S_{+--+} &= A^S_{-++-} = 4 - \frac{4su}{t^2}\tilde{d}(r_{su}) + \frac{4(s-u)}{t}\ln\left(\frac{-s-i\epsilon}{-u}\right) , \quad (11)
\end{aligned}$$

and

$$\begin{aligned}
A^S_{+++-} &= A^S_{+-++} = A^S_{++--} = A^S_{++-+} = A^S_{+---} = A^S_{---+} = A^S_{-+--} \\
&= A^S_{--++} = A^S_{--+-} = A^S_{-+++} = -4 . \quad (12)
\end{aligned}$$

[2] This way, one obtains that the $gg \to VH$ processes are mass suppressed, at high energy, because of the left–right orthogonality of the gaugino–higgsino contributions.

In all these SM cases, receiving no Born contribution, the asymptotic HV amplitudes behave like constants. On the contrary, the high energy HC amplitudes are linear combinations of the form (5) and single logarithms of ratios of the (s, t, u) variables; to which additional constants, like those in (11) are added. Thus, the *supersimplicity* structure is somewhat reduced in SM.

Such linear logarithmic and additional constant terms are never seen in the MSSM case (6) [10, 11].

The asymptotic HC amplitudes involving longitudinal ZZ and W^+W^- final states, in both SM and MSSM, are solely determined by the single logarithmic form (5), with coefficients being rational functions of the Mandelstam variables; see Eqs. (25) of [10].

3. Processes with Born terms at high energies

We here consider the high energy behavior of the processes (3), (4), which receive non-vanishing Born contributions; the results thus obtained have not appeared in previous publications. According to these, the high energy behavior of the 1-loop HC amplitudes is determined by the form (5), as those of the previous section; but in addition to it, two new forms appear, containing the so-called Sudakov $\ln^2$ and ln terms [15], to which specific "constant" corrections are added.

The coefficients of $\ln^2$ are known to be identical in MSSM and SM, while those of the linear-ln terms are clearly different, even when disregarding the mass-scales inside logarithms [2, 3, 4].

The emphasis here though, is on the aforementioned "constant" corrections, which accompany the logarithms and depend on ratios of masses, in both, MSSM and SM. The "augmented Sudakov logarithms" thus introduced in Section 3.1, considerably enhance the accuracy of the high energy expressions.

In MSSM, the only asymptotically non-vanishing amplitudes for the processes (3), (4) at high energy, are the HC ones [6,7]. At the 1-loop EW order, a simple correspondence between the amplitudes of (3) and those of (4) has been found. This is not an exact equality, like in (9) but an equivalence of the forms which are of course mass dependent. Thus, the results for (3) may be simply obtained by renaming those of the corresponding process in (4). The external and internal masses of the process (4) have just to be replaced by the ones of[3] (3).

As the complexity of the calculation increases with the spin of the particles involved, the computation of the processes (4) is usually much simpler than those of the processes in (3). Thus, it is often advantageous to first

[3] This is facilitated when chargino, neutralino and squark mixings are neglected.

calculate the HC amplitudes in the interesting SUSY-transformed process of type (4), and then translate the result to the one for the original process in (3).

We next turn to the augmented Sudakov logarithms, mentioned above.

3.1. The augmented Sudakov forms and supersimplicity

For any 2-to-2 processes, at 1-loop EW order, in either MSSM or SM, there are two augmented Sudakov forms; the form $\ln^2$ and the form ln. The $\ln^2$-form is generated completely from each contributing diagram; *i.e.* they are not the result of combining contributions from different diagrams. This is also true for the form in (5). In contrast, for the linear ln-form, different diagrams, including self-energy contributions, conspire to generate it; this happens in the same way the divergent parts cancel.

In both cases, the Sudakov logs are accompanied by dimensionless "constants" depending on one of the external masses of the considered process, and two internal masses of the generating diagrams. These diagrams of course contain a vertex where the two internal lines join to produce the external one.

The augmented Sudakov $\ln^2$-form is generated by triangular or box-diagrams with gauge boson exchanges, and it involves the logarithm-squared of a Mandelstam (s,t,u) variable scaled by a gauge boson mass[4], in all examples we know [2,3,4]. Its general structure is

$$\overline{\ln^2 s_V} \equiv \ln^2\left(\frac{-s-i\epsilon}{m_V^2}\right) + 2L_{a_1Vc_1} + 2L_{a_2Vc_2}\,, \tag{13}$$

and similarly for the t,u variables[5]. Here[6] $m_V = m_W, m_Z, m_\gamma$. The constant term in the r.h.s. of (13) is given by [16,17,18]

$$L_{aVc} \equiv L(p_a,m_V,m_c) = \mathrm{Li}_2\left(\frac{2p_a^2+i\epsilon}{m_V^2-m_c^2+p_a^2+i\epsilon+\sqrt{\lambda(p_a^2+i\epsilon,m_V^2,m_c^2)}}\right) + \mathrm{Li}_2\left(\frac{2p_a^2+i\epsilon}{m_V^2-m_c^2+p_a^2+i\epsilon-\sqrt{\lambda(p_a^2+i\epsilon,m_V^2,m_c^2)}}\right), \tag{14}$$

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[4] It is conceivable that other masses of internally exchanged particles may also affect this scale; *e.g.* a Higgs mass.

[5] For $V=W$, the notation s_W in (13) should not be confused by the coincidence with the notation for the sine of the Weinberg-angle.

[6] To regularize infrared singularities we use $m_\gamma = m_Z$. The same choice was made in [12].

where Li_2 is a Spence function and

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc\,. \tag{15}$$

The complex quantities L_{aVc} of (14), are ubiquitous in the asymptotic expansion of the Passarino–Veltman (PV) functions [19, 18]. The first index in them refers to an external particle (a) of the considered processes, with its mass and momentum satisfying $p_a^2 = m_a^2$; while the other two indices describe the masses (m_V, m_c) of two internal particles (V, c) in the generating diagram, joining to the aVc-vertex. Since any V internal line has two ends, there are always two such terms generated by each contributing diagram, called $L_{a_1Vc_1}$ and $L_{a_2Vc_2}$, which lead to the two[7] last terms in (13).

We next turn to the augmented Sudakov ln forms, generated by bubble[8], triangular or box diagrams. These diagrams always involve two internal lines (i, j), joining to a vertex, where an external particle (a) is produced, through a non-vanishing (ija)-coupling. Its general form is

$$\overline{\ln s_{ij}} \equiv \ln \frac{-s - i\epsilon}{m_i m_j} + b_0^{ij}\left(m_a^2\right) - 2\,, \tag{16}$$

and similarly for the t, u variables. Here $b_0^{ij}(m_a^2)$ is a finite part of the standard B_0 bubble-function, defined as [19, 18]

$$\begin{aligned} b_0^{ij}(m_a^2) \;\equiv\; & b_0\left(m_a^2; m_i, m_j\right) = 2 + \frac{1}{m_a^2}\left[\left(m_j^2 - m_i^2\right)\ln\frac{m_i}{m_j}\right. \\ & \left. + \sqrt{\lambda\left(m_a^2 + i\epsilon, m_i^2, m_j^2\right)}\,\mathrm{arccosh}\left(\frac{m_i^2 + m_j^2 - m_a^2 - i\epsilon}{2m_i m_j}\right)\right]. \end{aligned} \tag{17}$$

In MSSM, the content of *supersimplicity* for the Born-containing processes (3), (4) is the following. First, a *supersimplicity* renormalization scheme (SRS) may be defined for these process, where the asymptotic HC amplitudes only contain linear combinations of the above three forms, with coefficients being rational functions of the (s, t, u) variables. Then, the high energy HC amplitudes, in the usual on-shell (OS) scheme [20], may be <u>completely</u> expressed as the addition of the aforementioned SRS amplitudes, to which a "residual constant" is added. This "residual constant" then acts as a counter term relating the SRS and on-shell schemes, and it may be very small; at least this is what we have found in the $ug \to dW$ case of Section 3.2.

[7] If c_1 or c_2, is actually a mixed state of several particles, then all of them will appear in (13), increasing the number of terms in it.

[8] Relevant for self-energy and counter term contributions.

What happens in SM? In this case, helicity conservation is not valid to all orders; but it holds at the Born level, for any 2-to-2 process. Because of this, for Born-involving processes, the high energy HV amplitudes are usually much smaller than the HC ones. This is also true for $\gamma\gamma \to \gamma\gamma$, γZ, ZZ [21, 22, 14]; but not for $gg \to VV'$ [10, 11].

Concentrating on the HC amplitudes, and restricting to the Born-involving processes (3), we find that the high energy structure in this SM case may again be described by the forms (5), (13), (16), (with different coefficients of course), but this time an additional form also appears involving linear logarithms of ratios of any two of the (s, t, u)-variables; *i.e.* there are four different forms in the SM case. In addition to them though, "SM residual constants" are needed to describe the on-shell amplitudes.

Again, a renormalization scheme, in analogy to SRS, may be defined for SM, where all asymptotic HC amplitudes are expressed as linear combinations of the aforementioned four forms, without any additional residual constant.

Below we call this scheme also SRS, in spite of the fact that we now refer to SM and not to MSSM. Again, the aforementioned "SM residual constants" act as counter terms relating SRS and to the on-shell scheme.

In the next section we give illustrations of the asymptotic HC amplitudes for $ug \to dW$, in both MSSM and SM. Corresponding results for $bg \to bH_i^0$, appear in the Appendix.

Before finishing this subsection we also add a remark on the no-Born processes $\gamma\gamma \to \gamma\gamma$, γZ, ZZ, for which, of course, no renormalization scheme dependence arises. In such a case, the high energy HC amplitudes in MSSM only contain the forms (5), (13) [21, 22, 14]. In SM though, the asymptotic HC amplitudes contain also linear logarithms involving rations of two of the (s, t, u)-variables, as well as additional constant terms. In both cases the contribution of the form (13) is induced by the W-loop.

3.2. *High energy $ug \to dW$ amplitudes at 1-loop EW order*

In order to appreciate the usefulness and accuracy of the *supersimplicity* description, we here present analytical expressions for the high energy HC amplitudes of the process $ug \to dW$, to the 1-loop EW order. Previous semi-analytical results for these have appeared in [23]; but there, the numerical components were blurring the picture and the *supersimplicity* structure was not visible.

We choose this process, because its external particles are rather light, so that the asymptotic region may be approached quickly, provided the SUSY scale is not too high. Moreover, since these external particles exist already in SM, the analysis may be done, both in MSSM and SM. This will be helpful in clarifying the SM–MSSM differences.

The complete EW 1-loop helicity amplitudes have already been computed, in the on-shell renormalization scheme [20], in both MSSM and SM [12]. Denoting the $ug \to dW$ helicity amplitudes as[9] $F_{\lambda\mu\tau\mu'}$, the two independent HC amplitudes are F_{-+-+} and F_{----}.

At high energies, the on-shell (OS) HC amplitudes may be written as

$$F^{\rm OS}_{-\pm-\pm} = F^{\rm Born}_{-\pm-\pm}\left[1+\frac{\alpha}{4\pi}(C_{-\pm-\pm}+\delta C_{\rm residual})\right] , \tag{18}$$

where

$$F^{\rm Born}_{-+-+} = \frac{eg_s}{\sqrt{2}s_W}\left(2\cos\frac{\theta}{2}\right) , \qquad F^{\rm Born}_{----} = \frac{eg_s}{\sqrt{2}s_W}\left(\frac{2}{\cos\frac{\theta}{2}}\right) , \tag{19}$$

describe their asymptotic Born expressions. A color matrix factor $\lambda^a/2$, acting between the initial u and final d quarks, is always removed from (18), (19). The *supersimplicity* structure is contained in $C_{-\pm-\pm}$, while $\delta C_{\rm residual}$ denotes the residual constant correction needed in the on-shell scheme.

In MSSM, the results for $C_{-\pm-\pm}$, may be computed in 2 different manners. Either through a lengthy direct computation of the $ug \to dW$ diagrams; or in a much simpler way, by looking at the SUSY transformed process $\tilde{u}_L\tilde{g} \to \tilde{d}_L\tilde{W}$. In both cases of course, the asymptotic limit of the PV functions given in [18] is used, which suffices for determining the high energy 2-to-2 physical amplitudes, up to energy suppressed terms.

In the first manner based on the complete $ug \to dW$ results [12], the only needed augmented Sudakov $\ln^2$ forms of type (13), are

$$\begin{aligned}
\overline{\ln^2 t_Z} &\equiv \ln^2\frac{-t-i\epsilon}{m_Z^2}+2(L_{dZd}+L_{uZu}) ,\\
\overline{\ln^2 u_Z} &\equiv \ln^2\frac{-u-i\epsilon}{m_Z^2}+2(L_{WZW}+L_{uZu}) ,\\
\overline{\ln^2 u_W} &\equiv \ln^2\frac{-u-i\epsilon}{m_W^2}+2(L_{WWZ}+L_{uWd}) ,\\
\overline{\ln^2 s_Z} &\equiv \ln^2\frac{-s-i\epsilon}{m_Z^2}+2(L_{dZd}+L_{WZW}) ,\\
\overline{\ln^2 s_W} &\equiv \ln^2\frac{-s-i\epsilon}{m_W^2}+2(L_{dWu}+L_{WWZ})
\end{aligned} \tag{20}$$

while for the augmented Sudakov ln forms of type (16), the relevant internal particles ij are such that either $ij = qV$ with ($V = W,\ Z,\ \gamma$) and ($q = u, d$), or $ij = \tilde{q}_L\tilde{\chi}_j$ with $\tilde{\chi}_j$ being a chargino or neutralino and

[9] The indices describe respectively the u, g, d, and W helicities.

($\tilde{q}_L = \tilde{u}_L, \tilde{d}_L$) leading to respective quantities like $b_0^{uW}(m_d^2)$, $b_0^{uZ}(m_u^2)$ or $b_0^{\tilde{u}_L\tilde{\chi}_j^+}(m_d^2)$, $b_0^{\tilde{u}_L\tilde{\chi}_j^0}(m_u^2)$ *etc.*

Using then (18), the complete 1-loop EW results for $ug \to dW$ [12], lead to

$$\begin{aligned}
C^{\rm MSSM}_{-+-+} = & \frac{(1-10c_W^2)}{36c_W^2 s_W^2}\left[-\overline{\ln^2 t_Z} - \frac{t}{u}\left(\ln^2 r_{ts}+\pi^2\right) + \ln^2 r_{tu} + \pi^2\right] \\
& + \frac{1}{2s_W^2}\left[-\overline{\ln^2 u_Z} - \overline{\ln^2 u_W} - \overline{\ln^2 s_Z} - \overline{\ln^2 s_W} + 2\left(\ln^2 r_{us}+\pi^2\right)\right] \\
& + \frac{(1+8c_W^2)}{24c_W^2 s_W^2}\left[\overline{\ln s_{uZ}} + \overline{\ln s_{dZ}}\right] + \frac{3}{4s_W^2}\left[\overline{\ln s_{dW}} + \overline{\ln s_{uW}}\right] \\
& - \sum_i \left\{ \frac{\left|Z^N_{1i}s_W + 3Z^N_{2i}c_W\right|^2}{72c_W^2 s_W^2}\overline{\ln s_{\tilde{u}_L\tilde{\chi}_i^0}} + \frac{\left|Z^N_{1i}s_W - 3Z^N_{2i}c_W\right|^2}{72c_W^2 s_W^2}\overline{\ln s_{\tilde{d}_L\tilde{\chi}_i^0}} \right. \\
& \left. + \frac{|Z^-_{1i}|^2}{4s_W^2}\overline{\ln s_{\tilde{d}_L\tilde{\chi}_i^+}} + \frac{|Z^+_{1i}|^2}{4s_W^2}\overline{\ln s_{\tilde{u}_L\tilde{\chi}_i^+}} \right\}, \qquad (21)
\end{aligned}$$

$$\begin{aligned}
C^{\rm MSSM}_{----} = & \frac{(1-10c_W^2)}{36c_W^2 s_W^2}\left[-\overline{\ln^2 t_Z} - \frac{t}{s}\left(\ln^2 r_{tu}+\pi^2\right) + \ln^2 r_{ts} + \pi^2\right] \\
& + \frac{1}{2s_W^2}\left[-\overline{\ln^2 u_Z} - \overline{\ln^2 u_W} - \overline{\ln^2 s_Z} - \overline{\ln^2 s_W} + 2\left(\ln^2 r_{us}+\pi^2\right)\right] \\
& + \frac{(1+8c_W^2)}{24c_W^2 s_W^2}\left[\overline{\ln u_{uZ}} + \overline{\ln u_{dZ}}\right] + \frac{3}{4s_W^2}\left[\overline{\ln u_{dW}} + \overline{\ln u_{uW}}\right] \\
& - \sum_i \left\{ \frac{\left|Z^N_{1i}s_W + 3Z^N_{2i}c_W\right|^2}{72c_W^2 s_W^2}\overline{\ln u_{\tilde{u}_L\tilde{\chi}_i^0}} + \frac{\left|Z^N_{1i}s_W - 3Z^N_{2i}c_W\right|^2}{72c_W^2 s_W^2}\overline{\ln u_{\tilde{d}_L\tilde{\chi}_i^0}} \right. \\
& \left. + \frac{\left|Z^-_{1i}\right|^2}{4s_W^2}\overline{\ln u_{\tilde{d}_L\tilde{\chi}_i^+}} + \frac{\left|Z^+_{1i}\right|^2}{4s_W^2}\overline{\ln u_{\tilde{u}_L\tilde{\chi}_i^+}} \right\}, \qquad (22)
\end{aligned}$$

which indeed contain only the forms[10] (5), (13), (16). The coefficients Z^N and Z^+, Z^- in (21), (22) describe the neutralino and chargino mixing matrices respectively [24].

The high energy HC amplitudes in the SRS scheme simply become

$$F^{\rm SRS}_{-\pm-\pm} = F^{\rm Born}_{-\pm-\pm}\left[1 + \frac{\alpha}{4\pi}C_{-\pm-\pm}\right]. \qquad (23)$$

Substituting in it, the MSSM result (21), (22), we obtain the high energy MSSM HC amplitudes in the SRS scheme.

[10] Note that (C_{-+-+}, C_{----}) are related to each-other through an $s \leftrightarrow u$ interchange. The same is true for the SM results in (28), (29).

We next discuss the additional "residual" contribution needed for calculating the on-shell (OS) result; compare (18) and note that the on-shell scheme has also been used in the exact 1-loop calculation of [12]. The "residual" contribution in (18) arises from the u- and d-quark wave function renormalization constants [25]

$$\delta Z_L^q = \frac{\alpha}{4\pi}\left[-c_q^{ij}\left(\Delta - \ln\frac{m_i m_j}{\mu^2} + b_0^{ij}\right)\right] + \overline{\delta Z_L^q}\,, \tag{24}$$

with Δ being the usual divergent contribution and c_q^{ij} is the coupling coefficient for the b_0^{ij} bubble (17). The W field renormalization constants are [25]

$$\delta Z_1^W - \delta Z_2^W + \tfrac{1}{2}\delta\Psi_W \equiv \frac{\alpha}{4\pi}\left[-\frac{2\Delta}{s_W^2} + \frac{1}{s_W^2}\left(2\ln\frac{m_Z m_W}{\mu^2} - 2b_0^{ZW}\right)\right] + \overline{\delta_W}\,, \tag{25}$$

$$\delta\Psi_W = -\mathrm{Re}\hat{\Sigma}_{WW}^{T'}\left(m_W^2\right) = -\left\{\mathrm{Re}\Sigma_{WW}^{T'}\left(m_W^2\right) + \delta Z_W\right\}\,. \tag{26}$$

Ignoring the square bracket parts in (24), (25), that are already contained in the supersimplicity $C_{-\pm-\pm}$-results (21), (22), the actual residual correction in (18) may be written as

$$\begin{aligned}
\delta_{\mathrm{OS}} &\equiv \frac{\alpha}{4\pi}\delta C_{\mathrm{residual}} = \tfrac{1}{2}\left[\overline{\delta Z_L^u} + \overline{\delta Z_L^d}\right] + \overline{\delta_W} \\
&= \frac{\alpha}{2\pi s_W^2}\left[\ln\frac{m_W}{m_Z} + b_0^{ZW}\left(m_W^2\right)\right] \\
&\quad + \tfrac{1}{2}\left[\delta\Psi_W + \left(\delta Z_L^d + \delta Z_L^u\right) - \left(\delta Z_L^d + \delta Z_L^u\right)_{(B_1\to -B_0/2)}\right]\,,
\end{aligned} \tag{27}$$

where $(B_0,\ B_1)$ are the standard PV bubble functions [19].

In MSSM, the supersimplicity expressions (21), (22) may also be obtained in a much simpler way, by considering the process $\tilde{u}_L\tilde{g} \to \tilde{d}_L\tilde{W}$. The HC asymptotic amplitudes in this case are determined in terms of $(C_{++},\ C_{--})$ defined in analogy to (18), with their indices describing the $\tilde{g}$, $\tilde{W}$ helicities. In this case, the first and third indices in the Sudakov $\ln^2$ forms L_{aVc} of (20) are changed to $L_{\tilde{a}V\tilde{c}}$; while the linear Sudakov ln forms defined in (16) acquire constant contributions like[11] $b_0^{W\tilde{u}_L}(m_{\tilde{d}}^2)$ or $b_0^{\tilde{W}u}(m_{\tilde{d}}^2)$ *etc.* Transforming $(C_{++},\ C_{--})$ back to the $ug \to dW$ case, we recover exactly (21), (22).

[11] Note that the b_0^{ij} functions in the $\tilde{u}_L\tilde{g} \to \tilde{d}_L\tilde{W}$ are calculated at squark-masses, as opposed to the $ug \to dW$ case, where they are calculated at the much smaller values of the u- and d-quark masses.

In SM, no SUSY transformation trick is applicable. In order to get the SM high energy amplitudes, we have to work with the complete 1-loop results of [12], suppressing the SUSY exchange diagrams. Ignoring also the small high energy HV amplitudes, and using for the HC ones the same definitions (18), (23), we get

$$
\begin{aligned}
C^{\rm SM}_{-+-+} = & \frac{(1-10c_W^2)}{36c_W^2 s_W^2}\left[-\overline{\ln^2 t_Z}+\frac{t^2}{u^2}\left(\ln^2 r_{ts}+\pi^2\right)+\ln^2 r_{tu}+\pi^2-\frac{2s}{u}\ln r_{ts}\right] \\
& +\frac{1}{2s_W^2}\left[-\overline{\ln^2 u_Z}-\overline{\ln^2 u_W}-\overline{\ln^2 s_Z}-\overline{\ln^2 s_W}+2\left(\ln^2 r_{us}+\pi^2\right)\right] \\
& +\frac{(1+8c_W^2)}{24c_W^2 s_W^2}\left[\overline{\ln s_{uZ}}+\overline{\ln s_{dZ}}\right]+\frac{3}{4s_W^2}\left[\overline{\ln s_{dW}}+\overline{\ln s_{uW}}\right] , \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
C^{\rm SM}_{----} = & \frac{(1-10c_W^2)}{36c_W^2 s_W^2}\left[-\overline{\ln^2 t_Z}+\frac{t^2}{s^2}\left(\ln^2 r_{tu}+\pi^2\right)+\ln^2 r_{ts}+\pi^2-\frac{2u}{s}\ln r_{tu}\right] \\
& +\frac{1}{2s_W^2}\left[-\overline{\ln^2 u_Z}-\overline{\ln^2 u_W}-\overline{\ln^2 s_Z}-\overline{\ln^2 s_W}+2\left(\ln^2 r_{us}+\pi^2\right)\right] \\
& +\frac{(1+8c_W^2)}{24c_W^2 s_W^2}\left[\overline{\ln u_{uZ}}+\overline{\ln u_{dZ}}\right]+\frac{3}{4s_W^2}\left[\overline{\ln u_{dW}}+\overline{\ln u_{uW}}\right] , \qquad (29)
\end{aligned}
$$

expressed completely in terms of the forms (5), (13), (16) and linear logarithms $(\ln r_{ts}, \ln r_{tu})$.

Thus, using (28), (29) in (23), we obtain the SM asymptotic HC amplitudes, in the SRS scheme. Correspondingly, the residual correction needed in the on-shell result (18) is again given by (27), where the SUSY contributions are now of course suppressed. Using (18), together with (28), (29), the on-shell asymptotic SM amplitudes are obtained.

Starting from (23), (18), the high energy MSSM or SM amplitudes in the SRS and OS schemes are related by

$$
F^{\rm OS}_{-\pm-\pm} = F^{\rm SRS}_{-\pm-\pm}\left[1+\frac{\alpha}{4\pi}\delta C_{\rm residual}\right] , \qquad (30)
$$

leading to the definition of their percentage difference as

$$
\delta_{\rm OS} \equiv \frac{\alpha}{4\pi}\delta C_{\rm residual} = \frac{F^{\rm OS}_{-\pm-\pm}-F^{\rm SRS}_{-\pm-\pm}}{F^{\rm SRS}_{-\pm-\pm}} . \qquad (31)
$$

Note that (30), (31) clearly indicate that the real quantity $\delta_{\rm OS}$ acts like a residual counter term relating the SRS and OS schemes.

We repeat that (18), (23), (30), (31) are valid in both, SM and MSSM, provided of course, that the appropriate $C_{-\mp-\mp}$ and $\delta C_{\rm residual}$ are used.

To compare the high energy MSSM and SM predictions for the[12] HC amplitudes in the SRS scheme, we simply need to identify the differences between (21), (22) and (28), (29). Such differences appear in the coefficients of the forms of type (5) and (16); and most strikingly, in the SM linear logarithms of ratios of the (s, t, u)-variables, that never appear in MSSM.

Constant "residual" contributions to the high energy HC amplitudes, in either MSSM or SM, (beyond those entering the aforementioned log-involving forms) can never appear in the SRS scheme. They can appear in the on-shell scheme given by (18) though, due to the residual counter term (31), determined by (27).

Coming now to the magnitude of the counter term $\delta_{\rm OS}$, relating the OS and SRS schemes, we find from (27) the numerical value

$$\delta_{\rm OS} = \frac{\alpha}{4\pi}\delta C_{\rm residual} \simeq 0.0289 \tag{32}$$

in the SM case; while the results for a wide class of MSSM benchmarks, are shown in the last column of Table I. The lower part of this table covers all ATLAS benchmarks of [30], while the upper part covers also possibilities of very heavy squarks and sleptons [27, 28, 29]. The counter terms $\delta_{\rm OS}$ appear rather insensitive to the model, the differences being at the unobservable permil level.

TABLE I

Input parameters at the grand scale, for some cMSSM models with $\mu > 0$, and the $\delta_{\rm OS}$ results. All dimensional parameters in GeV.

	$m_{1/2}$	m_0	A_0	$\tan\beta$	$\delta_{\rm OS}$
SPS1a' [26]	250	70	−300	10	0.0286
mSP4 [27]	137	1674	1985	18.6	0.0292
BBSSW [28]	900	4716	0	30	0.0299
BKPU [29]	2900	8700	0	50	0.0298
ATLAS SU1 [30]	350	70	0	10	0.0289
ATLAS SU2	300	3550	0	10	0.0297
ATLAS SU3	300	100	−300	6	0.0288
ATLAS SU4	160	200	−400	10	0.0283
ATLAS SU6	375	320	0	50	0.0290
ATLAS SU8.1	360	210	0	40	0.0290
ATLAS SU9	425	300	20	20	0.0291

[12] For $ug \to dW$ above 0.5 TeV, the HV amplitudes are much smaller than the HC ones, in all MSSM benchmarks of Table I, and in SM [12].

Consequently, the *supersimplicity SRS* amplitudes and cross-sections from (23), very closely approximate the on-shell ones from (18). Because of this, only the on-shell asymptotic results are plotted in the figures, where we compare them to the complete one loop results in the same scheme [12].

Thus, in Figs. 1, 2, 3, we show the HC amplitudes and the sum over amplitudes-squared

$$\sum_{\lambda\mu\tau\mu'} |F_{\lambda\mu\tau\mu'}|^2$$

for the MSSM benchmarks in the first three lines of Table I, while in Fig. 4, the analogous results for SM are given.

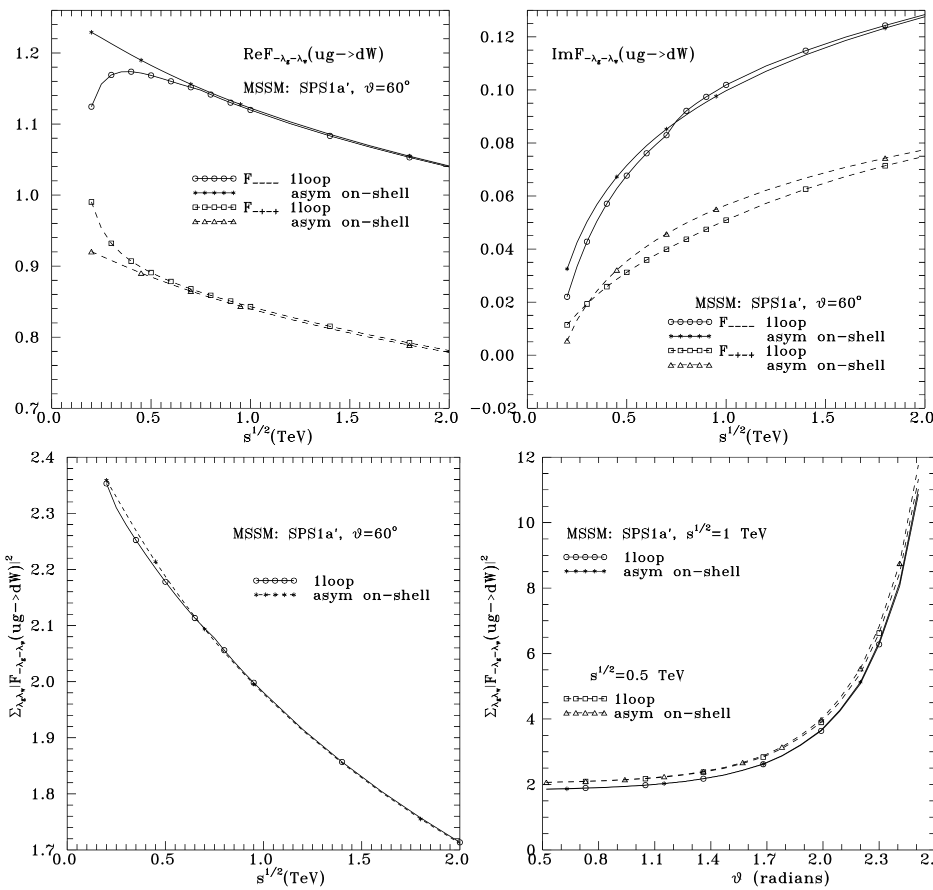

Fig. 1. The complete 1-loop results for $ug \to dW$ in SPS1a' at the on-shell scheme [12], are compared to their high energy "supersimplicity" approximation. Upper panels: Energy dependence of Real (left) and Im (right) parts of the HC amplitudes F_{----} and F_{-+-+} at $\theta = 60°$. Lower panels: Sum over all amplitudes squared; energy dependence at $\theta = 60°$ (left); angular dependence (right).

As seen in Figs. 1, the high energy *supersimplicity* structure is rather quickly established for SPS1a' [26].

In contrast, Figs. 2, 3 indicate a much slower supersimplicity approach, for the mSP4 [27] and BBSSW [28] benchmarks, induced by a considerably bigger SUSY scale; compare Table I. This seems stronger for the imaginary parts of the amplitudes, which are more sensitive to virtual thresholds. In any case the effect lies at the 1% percent level, which could be observable.

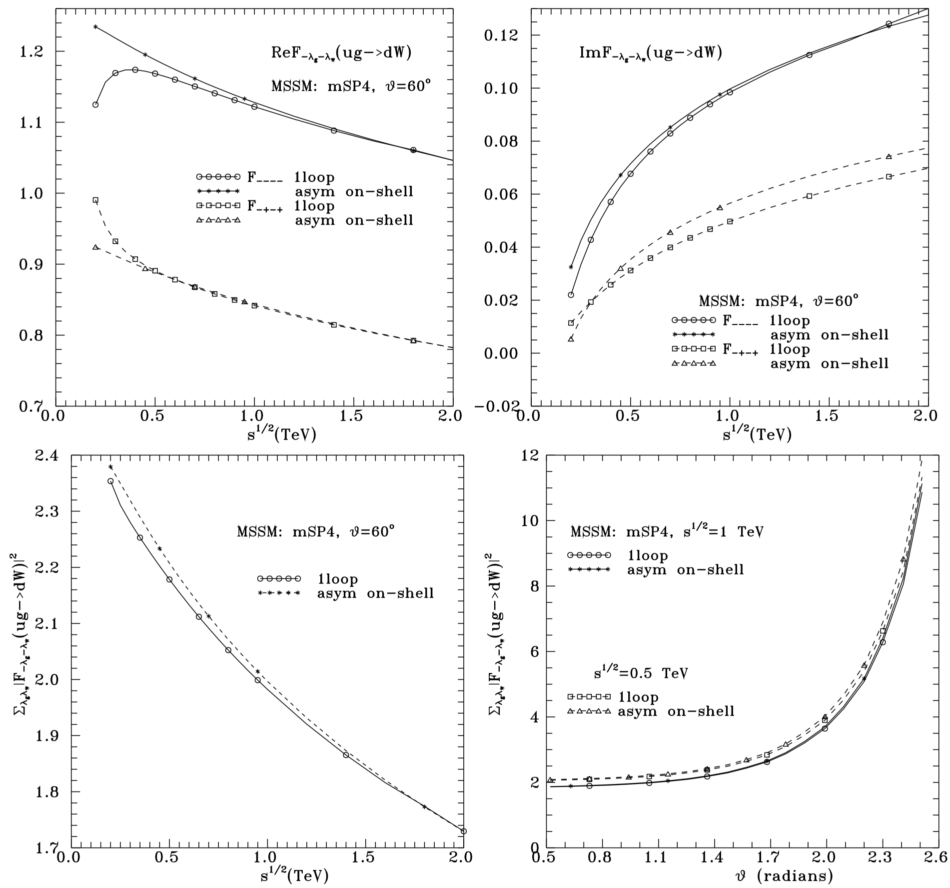

Fig. 2. mSP4-results as in Fig. 1.

Corresponding results for SM are shown in Fig. 4.

In the lower right parts of all these figures, the angular distributions of the exact 1-loop and the asymptotic expressions (18), are compared. As seen there, they roughly agree, already at 0.5 TeV and a wide range of angles, not only in SM, but also for the MSSM benchmarks [26,27,28], even though the SUSY scale reaches quite high values.

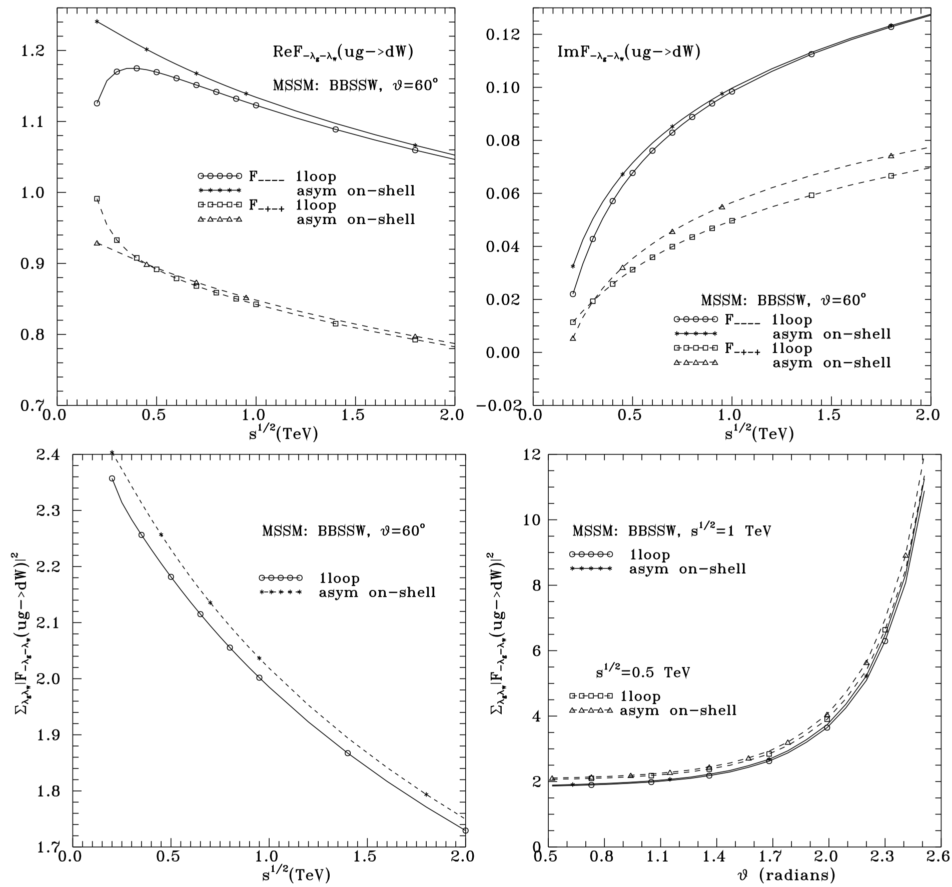

Fig. 3. BBSSW-results as in Fig. 1.

These remarks suggests a possibly simpler way to compare theory with future experimental data. This could be done by using the supersimplicity SRS expressions of (23), combined with an arbitrary real constant describing the residual counter term needed for describing the on-shell amplitudes. Only one experimental input, at an arbitrary energy and angle, should then be sufficient to fix the theoretical result. We can then get a feeling of the energy domain in which the supersimplicity expressions constitute a good approximation.

On the basis of the preceding discussion, we conclude, that the high energy supersimplicity expressions (21), (22) for MSSM, and (28), (29) for SM, may adequately describe $ug \to dW$ at LHC energies. The great virtue of these expressions, is that they are analytical and very simple. Provided therefore the SUSY scale is not too high, they constitute an efficient instrument for identifying the physics responsible for the various effects. Particularly in the MSSM case, they help identifying what are the SUSY-mass-combinations that mostly influence the various LHC observables. If needed, the accuracy of these predictions may be further increased by including the residual counter term corrections $\delta_{\rm OS}$, given in Table I and (32).

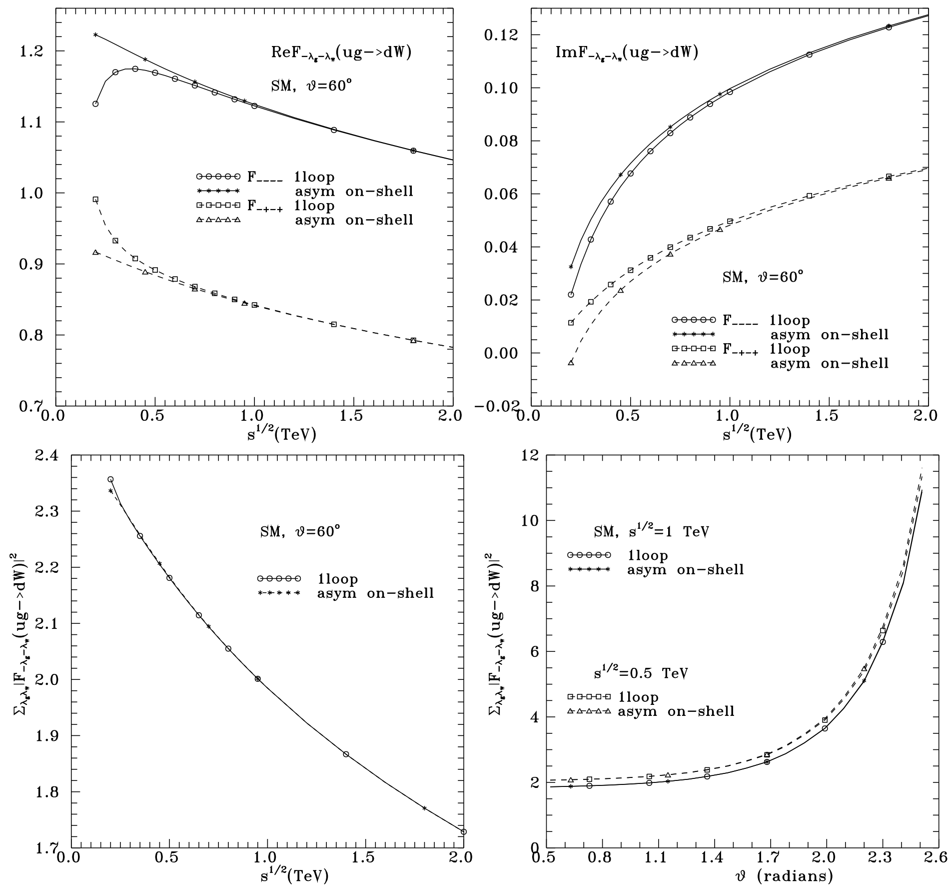

Fig. 4. SM-results as in Fig. 1.

4. Summary and prospectives for further studies

By studying a large number of 2-to-2 MSSM process, at the 1-loop EW order, we have found that a remarkably simple structure arises for the HC amplitudes, which are the only surviving ones at high energy.

At such energies and apart from a "residual constant", these amplitudes involve at most three different forms; namely (5), (13), (16), containing the well known logarithmic terms [2,3,4], to which definite constants are added. The identification of these constants, which greatly increase the accuracy of the high energy predictions, is the main contribution of this work.

The MSSM high energy physical amplitudes are then expressed as linear combinations of these forms, with coefficients being rational functions of the s, t, u variables; and occasionally an additional residual constant. We have called this very simple structure of the high energy MSSM amplitudes, *supersimplicity*.

Analogous results are also true for the SM case though, where four log-involving forms are needed and additional constants are inevitable, for describing the HC amplitudes. We should remember though that in SM, the HV amplitudes may occasionally be important.

If the Born-contribution is non-vanishing, a special renormalization scheme, called SRS, can be consistently defined in either MSSM or SM, where the validity of supersimplicity becomes asymptotically exact, at the 1-loop EW level. By this we mean that the asymptotic SRS HC amplitudes are expressed as a linear combinations of three (four) forms for MSSM (SM) respectively, without any residual constants. These SRS amplitudes are related to the usual on-shell scheme, by adding to it a Born-like contribution multiplied by a real residual counter term, relating the two schemes.

For $ug \to dW$, this residual counter term has been found very small, for a wide range of MSSM benchmarks in Table I and for SM. Thus, for this process at least, the *supersimplicity* structure is very accurate. For achieving this, a very important role is played by the constants added to the logarithms in the forms (5), (13), (16), which greatly enhance the accuracy of the previously known logarithmic results [2, 3, 4]. This can be seen in Figs. 1–4, where the exact 1-loop results [12] are compared to the on-shell asymptotic ones, for three MSSM benchmarks with a wide range of input parameters and SM. These results show that the supersimplicity expressions provide a good description even at rather low energies, when the SUSY scale is in the one TeV range. Even if the SUSY scale is higher, these expressions constitute a good approximation at the percent level.

Concentrating on MSSM, we emphasize that the *supersimplicity* description of the asymptotic HC amplitudes in terms of the three forms (5), (13), (16), is not only a property of the Born-term involving processes (3), (4); but it has also been seen in $gg \to VV'$, where just the form (5) suffices.

This is also valid for the much more complicated processes $\gamma\gamma \to \gamma\gamma$, $\gamma Z, ZZ$, whose asymptotic HC amplitudes may be fully expressed in terms of the forms (5), (13), again without any additional constant [21, 22, 14].

We are planning to further explore this in other processes, trying to see if there are any exceptions. At present, we have partial results for the process $e^+e^- \to f\bar{f}$ and its SUSY transformed $\tilde{e}^+\tilde{e}^- \to \tilde{f}\tilde{\bar{f}}$, which are consistent with those presented here.

We repeat that *supersimplicity* is an 1-loop MSSM property, realized at the high energy region, where the SUSY breaking effects are either minimized (as in the processes of Sect. 3), or vanish completely (as for the no

Born processes of Sect. 2). Diagrammatically, its realization involves two steps. First, the establishment of Helicity Conservation, which is due to SUSY cancellations between fermionic and bosonic diagrams; and second, the actual derivation of *supersimplicity*, for the helicity conserving amplitudes, which are the only ones that survive asymptotically.

At the technical level, the easiest way to establish *supersimplicity* for processes involving external gauge bosons[13], is to use their SUSY-transformed process, find the asymptotic HC amplitudes there, and then transform back to the original process, appropriately changing the internal and external masses in the forms (16), (13).

In SM, there is no helicity conservation theorem in general. Nevertheless, restricting to HC amplitudes, a corresponding analysis may be made. The main result now is that an additional fourth form appears, involving linear logarithms of ratios of Mandelstam variables; and additional constants may be occasionally needed.

In conclusion, we emphasize that *supersimplicity*, describing the leading HC amplitudes through formulae of a few lines in MSSM, is appealing from two aspects. The first one is theoretical; the simplicity of these formulae allows one to immediately read what are the main high-energy features of the electroweak contributions to the process considered and what is the role of supersymmetry.

The second one concerns the future comparison with experiments. In this paper we have concentrated on the 1-loop electroweak effects. A complete analysis will of course require the additional treatment of the QED and QCD corrections (in particular soft photon and gluon radiation) for which there exist specific codes. For what concerns QCD in particular, we just note that the SUSY QCD high energy contribution should behave similarly to the EW gauge part effects studied here. Thus, the Sudakov logarithms in *e.g.* [31] should be replaced by augmented forms similar to those of Section 3 (replacing charginos by gluinos). In addition, a study of the relevant background processes will also be necessary for each case. Nevertheless, our modest work may be useful in this respect also, since it could allow someone to get a general feeling of the high energy effects, by using simple formulas like (21), (22), instead of the enormous codes containing the exact 1-loop EW virtual corrections.

We are grateful to Jacques Layssac for help in using the $ug \to dW$ code. G.J.G. is partially supported by the European Union contracts MRTN-CT-2006-035505 HEPTOOLS and ITN programme "UNILHC" PITN-GA-2009-237920.

[13] These are the processes where HCns is most intriguing [9].

Appendix

High energy structure of $bg \to bH_i^0$, at 1-loop EW

In analogy to the $ug \to dW$ analysis in Subsect. 3.2, we here present the high energy HC amplitudes for the process $bg \to bH_i^0$, which is sensitive to the Higgs (Yukawa) sector, in both MSSM and SM. Here $H_i^0 = h^0$, H^0, A^0, G^0, describes any of the neutral Higgs or Goldstone bosons in MSSM while in SM, $H_i^0 = H_{\rm SM}$, G^0 .

The helicity amplitudes for $bg \to bH_i^0$ are denoted by $F_{\lambda\mu\tau}$, with (λ,τ) describing the helicities of the initial and final b-quark, while μ denotes the helicity of the gluon. The asymptotic Born contributions to these processes are

$$F^{\rm Born}_{-++} = -\sqrt{2} c^{\rm L}_{H_i^0} g_s \frac{t}{u} \cos\frac{\theta}{2} \,, \qquad F^{\rm Born}_{+--} = -\sqrt{2} c^{\rm R}_{H_i^0} g_s \frac{t}{u} \cos\frac{\theta}{2} \,, \tag{A.1}$$

with the MSSM couplings being

$$\begin{aligned} c^{\rm L}_{H^0} &= c^{\rm R}_{H^0} = -\frac{e m_b \cos\alpha}{2 s_W m_W \cos\beta} \,, \qquad & c^{\rm L}_{h^0} = c^{\rm R}_{h^0} = \frac{e m_b \sin\alpha}{2 s_W m_W \cos\beta} \,, \\ c^{\rm L}_{A^0} &= -c^{\rm R}_{A^0} = -i \frac{e m_b \tan\beta}{2 s_W m_W} \,, \qquad & c^{\rm L}_{G^0} = -c^{\rm R}_{G^0} = i \frac{e m_b}{2 s_W m_W} \,. \end{aligned} \tag{A.2}$$

Using the SUSY transformed process $\tilde{b}\tilde{g} \to \tilde{b}\tilde{\chi}_i^0$ for simplifying the calculations, and selecting the higgsino components, one gets in the SRS scheme in MSSM

$$F^{\rm SRS}_{\mp\pm\pm} = F^{\rm Born}_{\mp\pm\pm} \left[1 + \frac{\alpha}{4\pi} C_{\mp\pm\pm}(s,t,u) \right] \,, \tag{A.3}$$

where

$$\begin{aligned} C^{\rm MSSM}_{+--}(s,t,u) &= C^{\rm MSSM}_{-++}(u,t,s) \\ &= -\frac{1}{18 c_W^2} \left[-\overline{\ln^2 t_Z} + \left(\ln^2 r_{ts} + \pi^2\right) + \left(\ln^2 r_{tu} + \pi^2\right) \right] \\ &\quad -\frac{\left(1 + 2c_W^2\right)}{12 s_W^2 c_W^2} \overline{\ln^2 s_Z} - \frac{\left(1 + 8c_W^2\right)}{12 s_W^2 c_W^2} \frac{s}{t} \left(\ln^2 r_{us} + \pi^2\right) \\ &\quad + \frac{1}{6 c_W^2} \left[-\overline{\ln^2 u_Z} - \frac{u}{t} \left(\ln^2 r_{us} + \pi^2\right) \right] . \end{aligned} \tag{A.4}$$

Thus, in MSSM, only the forms (5) and (13) appear, while no Sudakov linear log forms, like those defined in (16), arise. Note that (A.4) holds the same for all H_i^0, since no couplings like those in (A.2) appear in it.

The needed Sudakov $\ln^2$ forms in (A.4), are

$$\begin{aligned}
\overline{\ln^2 t_Z} &= \ln^2 \frac{-t-i\epsilon}{m_Z^2} + 4L_{bZb}\,, \\
\overline{\ln^2 s_Z} &= \ln^2 \frac{-s-i\epsilon}{m_Z^2} + 2\left(L_{H_i^0 Z\varphi^0} + L_{bZb}\right)\,, \\
\overline{\ln^2 s_W} &= \ln^2 \frac{-s-i\epsilon}{m_W^2} + 2\left(L_{bWt} + L_{H_i^0 W\varphi^-}\right)\,, \\
\overline{\ln^2 u_Z} &= \ln^2 \frac{-u-i\epsilon}{m_Z^2} + 2\left(L_{H_i^0 Z\varphi^0} + L_{bZb}\right)\,,
\end{aligned} \tag{A.5}$$

where φ^0, φ^- respectively describe mixtures of the Higgs or Goldstone internal lines in the contributing diagram. Together with the corresponding V-internal lines, these generate the terms $L_{H_i^0 Z\varphi^0}$, $L_{H_i^0 W\varphi^-}$, contributing to the H_i^0 production. The explicit meanings of these terms are

$$\begin{aligned}
L_{H^0 Z\varphi^0} &= \frac{\sin\beta\sin(\beta-\alpha)}{\cos\alpha} L_{H^0ZA^0} + \frac{\cos\beta\cos(\beta-\alpha)}{\cos\alpha} L_{H^0ZG^0}\,, \\
L_{H^0 W\varphi^-} &= \frac{\sin\beta\sin(\beta-\alpha)}{\cos\alpha} L_{H^0WH^-} + \frac{\cos\beta\cos(\beta-\alpha)}{\cos\alpha} L_{H^0WG^-}\,, \\
L_{h^0 Z\varphi^0} &= \frac{\sin\beta\cos(\beta-\alpha)}{\sin\alpha} L_{h^0ZA^0} - \frac{\cos\beta\sin(\beta-\alpha)}{\sin\alpha} L_{h^0ZG^0}\,, \\
L_{h^0 W\varphi^-} &= \frac{\sin\beta\cos(\beta-\alpha)}{\sin\alpha} L_{h^0WH^-} - \frac{\cos\beta\sin(\beta-\alpha)}{\sin\alpha} L_{h^0WG^-}\,, \\
L_{A^0 Z\varphi^0} &= \frac{\cos\alpha\sin(\beta-\alpha)}{\sin\beta} L_{A^0ZH^0} + \frac{\sin\alpha\cos(\beta-\alpha)}{\sin\beta} L_{A^0Zh^0}\,, \\
L_{A^0 W\varphi^-} &= L_{A^0WH^-}\,, \\
L_{G^0 Z\varphi^0} &= \frac{\cos\alpha\cos(\beta-\alpha)}{\cos\beta} L_{G^0ZH^0} - \frac{\sin\alpha\sin(\beta-\alpha)}{\cos\beta} L_{G^0Zh^0}\,, \\
L_{G^0 W\varphi^-} &= L_{G^0WG^-}\,,
\end{aligned} \tag{A.6}$$

where (14) should be used. Notice that in the r.h.s. of all equations (A.6), the sum of the coefficients of the L_{abc} forms equals to 1, as it should.

As we have already said, the results (A.3), (A.4) were derived by working with the process $\tilde{b}\tilde{g} \to \tilde{b}\tilde{\chi}_i^0$, and their logarithmic behavior in (A.3), (A.4) should agree with the old Sudakov structure established directly for $bg \to bH_i^0$ [4]. As has amply been pointed out above, the absence of linear logs in (A.4), is an MSSM feature.

To check what happens in the SM cases, a direct diagrammatic computation must be made.

For $H_i^0 = H_{\rm SM}$, we would then use $L_{H_{\rm SM}ZG^0}$, $L_{H_{\rm SM}WG^-}$ in (A.5) and the couplings

$$c^{\rm L}_{H^0_{\rm SM}} = c^{\rm R}_{H^0_{\rm SM}} = -\frac{em_b}{2s_W m_W}\,. \tag{A.7}$$

Compared to the MSSM expressions (A.3), (A.4), the 1-loop SM correction contains typical linear terms $\ln r_{us}$, together with contributions of the forms (5), (16). We find for $H_i^0 = H_{\rm SM}$,

$$\begin{aligned} C^{\rm SM}_{\mp\pm\pm} - C^{\rm MSSM}_{\mp\pm\pm} &= \frac{1+2c_W^2}{2s_W^2c_W^2}\left[-\frac{su}{2t^2}\left(\overline{\ln^2 r_{us}+\pi^2}\right)+\frac{u}{t}\ln r_{us}\right] \\ &\quad +\frac{\overline{\ln u_{ZG^0}}}{2s_W^2c_W^2}+\frac{\overline{\ln u_{WG}}}{s_W^2}-\frac{m_t^2}{2s_W^2m_W^2}\left[\overline{\ln u_{tG}}+\frac{u}{t}\ln r_{us}\right]\,, \end{aligned} \tag{A.8}$$

where $G \equiv G^{\pm}$ denotes a charged Goldstone boson.

For the case $H_i^0 = G^0$ in SM, one should use $L_{G^0ZH_{\rm SM}}$, $L_{G^0WG^-}$, that leads to

$$\begin{aligned} C^{\rm SM}_{\mp\pm\pm} - C^{\rm MSSM}_{\mp\pm\pm} &= \frac{1+2c_W^2}{2s_W^2c_W^2}\left[-\frac{su}{2t^2}\left(\overline{\ln^2 r_{us}+\pi^2}\right)+\frac{u}{t}\ln r_{us}\right] \\ &\quad +\frac{\overline{\ln u_{ZH_{SM}}}}{2s_W^2c_W^2}+\frac{\overline{\ln u_{WG}}}{s_W^2}-\frac{m_t^2}{2s_W^2m_W^2}\left[\overline{\ln u_{tG}}+\frac{u}{t}\ln r_{us}\right]\,. \end{aligned} \tag{A.9}$$

In (A.8), (A.9) as well as in (A.4), only contributions of the *supersimplicity* structure arise, containing the forms (5), (13), (16) and linear logarithms of ratios of the s,t,u variables appear. Therefore, these are the HC amplitudes in the SRS scheme. To find the on-shell amplitudes, the counter term contributions, analogous to (27), must be calculated. This has not been done here.

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