SYMMETRY ENERGY AND NEUTRON STAR EQUATION OF STATE*

Ilona Bednarek, Monika Pieńkos

Institute of Physics, University of Silesia
Uniwersytecka 4, 40-007 Katowice, Poland

(Received October 27, 2011)

Information about many aspects of both nuclear physics and a neutron star structure are encoded in the density dependence of the symmetry energy. Despite the fact that considerable progress has been made in constraining the density dependence of the symmetry energy, it is still the most uncertain part of the asymmetric nuclear matter equation of state (EOS). The analysis of neutron star parameters which are most sensitive to the form of the symmetry energy in the case of nucleon and strangeness-rich matter has been done with the use of the relativistic mean field (RMF) approximation. The mixed vector meson interactions have been introduced to modify the density dependence of the symmetry energy.

DOI:10.5506/APhysPolB.42.2211
PACS numbers: 97.60.Jd, 26.60.Kp, 26.60.–c, 21.65.Cd

1. Introduction

Symmetry energy is the fundamental quantity that characterizes properties of asymmetric nuclear matter. The goal of this paper is to point out differences between properties of asymmetric nuclear matter calculated within the minimal model constructed on the basis of TM1 parametrization [1,2] and the model built for the significantly extended isovector sector. Properties and structure of a neutron star is determined by the EOS of asymmetric nuclear matter. Therefore, deeper understanding of the EOS of matter under extreme conditions of density and asymmetry is the basic issue at present. It has been shown that the energy per nucleon of asymmetric nuclear matter $\epsilon(n_b, f_a)$ expanded in the isospin asymmetry $f_a = (n_n - n_p)/n_b$, where n_p, n_n and n_b denote proton, neutron and baryon number densities, respectively, can be split into the part adequate for the symmetric nuclear

* Presented at the XXXV International Conference of Theoretical Physics "Matter to the Deepest", Ustroń, Poland, September 12–18, 2011.

matter, $\epsilon(n_b, 0)$ for $f_a = 0$, and the one characterizing the isospin dependence of the system

$$\epsilon(n_b, f_a) = \epsilon(n_b, 0) + E_{\mathrm{sym}}(n_b) f_a^2 + \mathcal{O}\left(f_a^4\right) + \ldots \,. \tag{1}$$

In the above, expression $E_{\mathrm{sym}}(n_b)$ denotes the bulk nuclear symmetry energy. Subsequently, the expansion of $\epsilon(n_b, 0)$ and $E_{\mathrm{sym}}(n_b)$ in n_b around the equilibrium density n_0 and the omission of the higher order terms in the isospin asymmetry leads to the well-known empirical parabolic approximation (PA)

$$\epsilon(n_b, f_a) = \epsilon(n_0) + \frac{1}{2!}\left(K_0 + K_{\mathrm{sym}} f_a^2\right)\chi^2 + \left(E_{\mathrm{sym}}(n_0) + L\chi\right) f_a^2 \tag{2}$$

expressed in terms of the dimensionless variable $\chi = (n_b - n_0)/3n_0$, which refers to the deviation of the density from the equilibrium density n_0. Equation (2) describes the density and asymmetry dependence of the nuclear matter EOS $\epsilon(n_b, f_a)$, and is characterized by a sequence of coefficients

$$\begin{aligned} E_{\mathrm{sym}}(n_0) \equiv J_0 &= \frac{1}{2}\frac{\partial^2 \epsilon(n_b, f_a)}{\partial f_a^2}\,, \qquad K_0 = 9n_0^2 \frac{\partial^2 \epsilon(n_b)}{\partial {n_b}^2}\,, \\ L &= 3n_0 \frac{\partial E_{\mathrm{sym}}(n_b)}{\partial n_b}\,, \qquad K_{\mathrm{sym}} = 9n_0^2 \frac{\partial^2 E_{\mathrm{sym}}(n_b)}{\partial n_b^2}\,. \end{aligned} \tag{3}$$

All derivatives are evaluated at the point n_0. One group of coefficients in the PA stands for the binding energy per nucleon of symmetric nuclear matter $\epsilon(n_0)$ and the incompressibility coefficient of symmetric nuclear matter K_0, which characterizes the curvature of $\epsilon(n_b)$ at n_0. The other group includes the value of the symmetry energy at equilibrium density $E_{\mathrm{sym}}(n_0) \equiv J_0$, the slope parameter L, and the curvature parameter K_{sym} which represents the isospin dependence of the incompressibility. These coefficients are related to the density dependence of the symmetry energy $E_{\mathrm{sym}}(n_b)$.

The main characteristics of neutron stars include the value of the maximum mass and the average neutron star radius. Whereas the maximum mass is governed by the incompressibility of symmetric nuclear matter the average radius mainly depends on the pressure of the isospin asymmetric matter. Neutron star radii are determined by the pressure at moderate densities. It has been shown that there is a relation between the pressure and the slope of the symmetry energy L in the vicinity of n_0. The most recent reported value of the symmetry energy coefficient $J_0 = 30.53$ MeV and the slope $L = 52.520$ MeV [3] are very important in constraining neutron star parameters and the form of the density dependence of the symmetry energy. Neutron star parameters most sensitive to the density dependence of the symmetry energy are: the mass-radius relation, the chemical composition, neutron star cooling rate, the core-crust transition density and moment of inertia.

2. The model

Neutron star structure is determined by the EOS of highly asymmetric nuclear matter in β-equilibrium. The most uncertain part of the EOS contains the symmetry energy term, whose density dependence is still poorly known.

The Walecka model [4] forms the basis for the construction of the EOS of neutron star matter. The study performs analysis of two separate cases. The one which refers to the non-strange matter comprises: nucleons, mesons (scalar σ, vector ω and vector–isovector ρ) and leptons. The extension of the meson sector to the case of non-zero strangeness requires the addition of two strange mesons–scalar $\sigma^\star$ and vector ϕ, as they are indispensable to describe the strongly attractive $\Lambda\Lambda$ interaction [5]. Both, the model with nucleons (N) only and the one which includes hyperons (Y) are supplemented by the non-linear isovector–isoscalar mixed vector meson interaction $\Lambda_{\rm V}(g_{N\omega}g_{N\rho})^2(\omega_\mu\omega^\mu)(\rho_{a\mu}\rho_a^\mu)$. This makes possible to modify the density dependence of the symmetry energy.

All calculations have been done within the RMF approximation, where the meson fields are considered as classical fields and the fields operators are replaced by their expectation values. In the case of homogeneous neutron star matter the non-zero expectation values of meson fields are

$$\begin{aligned}\langle\sigma\rangle &= s_0\,, \qquad \langle\sigma^\star\rangle = s_0^\star\,, \qquad \langle\omega_0\rangle = w_0\,,\\ \langle\rho_0^3\rangle &= r_0\,, \qquad \langle\phi_0\rangle = f_0\,.\end{aligned}$$

The energy density of the system is given by

$$\begin{aligned}E &= \tfrac{1}{2}m_\sigma^2 s_0^2 + \tfrac{1}{3}g_3 s_0^3 + \tfrac{1}{4}g_4 s_0^4 + \tfrac{1}{2}m_{\sigma^*}^2 s_0^{*2} + \tfrac{1}{2}m_\omega^2 w_0^2 + \tfrac{3}{4}c_3 w_0^4\\ &\quad +\tfrac{1}{2}m_\rho^2 r_0^2 + 3\Lambda_V(g_{N\omega}g_{N\rho})^2 w_0^2 r_0^2 + \tfrac{1}{2}m_\phi^2 f_0^2 + \sum_B E_B + E_l\,,\end{aligned}\tag{4}$$

where $E_i = \frac{2}{\pi^2}\int_0^{k_{F,i}} k^2 dk\sqrt{k^2+M_i^{\star 2}}$, $i = N, Y, l$ represents the nucleon, hyperon and lepton contributions to the energy density (4). In the case of baryons ($B = N+Y$) the effective mass takes the form $M_B^\star = M_B - g_{B\sigma}s_0 - g_{B\sigma^\star}s_0^\star$, for leptons $M_l^\star = m_l$, $l = e, \mu$.

In order to analyze the importance of the isovector sector, its further modification has been made which involve supplementary couplings between vector mesons ω, ρ and ϕ. Considering the mixed interactions between ω–ϕ and ρ–ϕ vector mesons an extended model constructed within the framework of the non-linear realization of chiral $\mathrm{SU(3)_L}\times\mathrm{SU(3)_R}$ symmetry has been applied. It has been shown that such a model describes simultaneously reasonably well finite nuclei and infinite nuclear matter properties and the value of hyperon potentials [6]. Calculations done for such an extended model lead to the following explicit formula for the energy density

$$
\begin{aligned}
E &= \tfrac{1}{2}m_\omega^2 w_0^2 + \tfrac{1}{2}m_\rho^2 r_0^2 + \tfrac{1}{2}m_\phi^2 f_0^2 + \tfrac{1}{2}m_\sigma^2 s_0^2 + \tfrac{1}{3}g_3 s_0^3 + \tfrac{1}{4}g_4 s_0^4 \\
&\quad + \tfrac{3}{4}c_3 w_0^4 + 3\Lambda_{\mathrm{V}}(g_{B\omega}g_{B\rho})^2 w_0^2 r_0^2 + \tfrac{3}{4}c_3 r_0^4 + \tfrac{3}{8}c_3 f_0^4 + \tfrac{1}{2}m_{\sigma^*}^2 s_0^{*2} \\
&\quad + \tfrac{3}{2}\left(\tfrac{3}{2}c_3 - \Lambda_{\mathrm{V}}(g_{B\omega}g_{B\rho})^2\right)\left(w_0^2 + r_0^2\right) f_0^2 + \tfrac{3}{4}\Lambda_{\mathrm{V}}(g_{B\omega}g_{B\rho})^2 f_0^4 \\
&\quad + \sum_B \tfrac{2}{\pi^2} \int\limits_0^{k_{F,B}} k^2 dk \sqrt{k^2 + (M_B - g_{B\sigma}s_0 - g_{B\sigma^*}s_0^*)^2} + E_l \,. \qquad (5)
\end{aligned}
$$

All calculations have been done on the basis of TM1 parameter set [1,2]. The extension of the isovector sector requires the adjustment of the $g_{N\rho}$–Λ_{V} pair of coupling constants to keep the value of the symmetry energy $E_{\mathrm{sym}} \approx 26$ MeV at the density corresponding to $k_{NF} = 1.15\,\mathrm{fm}^{-3}$. This gives the symmetry energy coefficient $J_0 = 33$ MeV [7,8].

Vector meson–hyperon coupling constants are taken from the additive quark model [5,6]. Whereas in the scalar sector the scalar couplings $g_{Y\sigma}$, Y stands for Λ, Σ and Ξ hyperons, have to reproduce the estimated values of the potential felt by a single Λ, Σ and Ξ in the saturated nuclear matter. The analysis based on data collected for double hypernucleus ${}^{6}_{\Lambda\Lambda}$He done by Takahashi *et al.* [9] indicates that the $\Lambda\Lambda$ interaction is rather weak. The potential well depth evaluated for this data equals $\mathrm{U}_{\Lambda}^{(\Lambda)} \simeq 5$ MeV. Usage of the meson exchange potentials for hyperon–hyperon interactions from Nijmegen Model [5] leads to a rough estimate which can be applied to determine the coupling of hyperons to the strange meson σ^*: $\mathrm{U}_{\Xi}^{(\Xi)} \simeq \mathrm{U}_{\Lambda}^{(\Xi)} \simeq 2\mathrm{U}_{\Xi}^{(\Lambda)} \simeq 2\mathrm{U}_{\Lambda}^{(\Lambda)}$. The general form of the potential which describes hyperon–nucleon and hyperon–hyperon interaction can be written in the form which indicate the existence of correlations between the scalar and vector coupling constants $\mathrm{U}_{Y}^{(B)} = g_{Y\sigma}s_0 - g_{Y\omega}w_0 + g_{Y\sigma^\star}s_0^\star - g_{Y\phi}f_0$, where $M_Y^\star(s_0, s_0^\star)$ denotes the effective hyperon mass, the couplings of the $\sigma^\star$ and ϕ mesons with nucleons are equal zero ($g_{N\sigma^\star} = g_{N\phi} = 0$). Parameters that enter the model are collected in Table I.

TABLE I

The parameters of the considered models calculated for different values of the parameter Λ_{V}. The first column includes the adjusted parameter $g_{N\rho}$, whereas the other columns parameters of the strange sector.

	Λ_{V}	$g_{N\rho}$	$g_{\sigma\Lambda}$	$g_{\sigma\Sigma}$	$g_{\sigma\Xi}$	$g_{\sigma^*\Lambda}$	$g_{\sigma^*\Sigma}$	$g_{\sigma^*\Xi}$
MIN	0	9.2644	6.169	4.476	3.0125	5.482	5.482	11.516
	0.0165	10.037	6.169	4.476	3.0125	5.482	5.482	11.516
EXT	0.015	9.9371	6.169	4.476	3.2012	5.482	5.482	11.375
	0.0165	10.037	6.169	4.476	3.201	5.482	5.482	11.372

3. Results and conclusion

The calculated EOSs for the non-strange and strangeness rich matter are presented in Fig. 1. In both cases the stiffness of the EOS is altered by the strength of the mixed vector meson interactions. When only nucleons are considered this additional vector meson coupling softens the EOS. In the case of strangeness-rich matter the extension of the isovector sector leads to the EOSs which are stiffer then the ones calculated for models without non-linear vector meson interactions. This effect is even more evident for the very extended model which connects the asymmetry and strangeness fraction of neutron star matter.

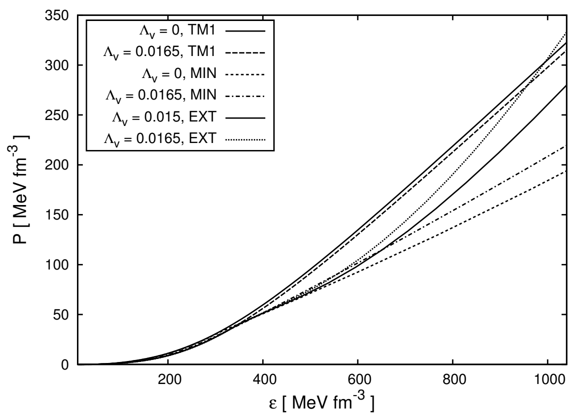

Fig. 1. EOS of the non-strange and strangeness-rich matter. The obtained results depict EOSs calculated for $\Lambda_V = 0$ and for $\Lambda_V = 0.0165$. In the case of hyperon-rich matter the abbreviations min. and ext. denote the minimal model with the ω–ρ coupling only and the model with different type of ω–ρ, ω–ϕ and ρ–ϕ couplings, respectively.

Global neutron star parameters such as the mass and radius and the internal structure of a neutron star can be determined by solving the Tolman–Oppenheimer–Volkoff (TOV) equation. The results obtained for the set of calculated EOSs lead to the mass-radius relations and allows one to determine the value of the maximum mass and radius which in a sense can be considered as a measure of the importance of particular non-linear couplings between vector mesons. In Fig. 2 the mass-radius relations are depicted. The varying value of the parameter Λ_V influences not only the value of the maximum mass but the value of radius as well. This is in accordance with the form of EOSs. The higher the value of the strength of the mixed vector meson coupling the more compact star has been obtained. The values of the mass and radius for the maximum mass configuration and the radius

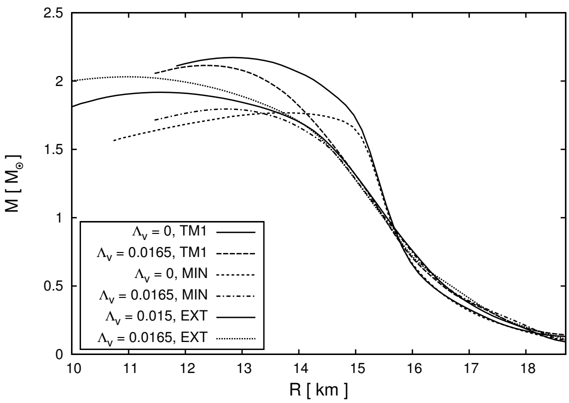

Fig. 2. Mass-radius relations calculated for the chosen values of parameter $\Lambda_{\rm V}$.

for $M = 1.4M_{\odot}$ for zero strangeness and for strangeness-rich matter are collected in Table II. It has been shown that in the very non-linear model the inclusion of hyperons does not soften the EOS; on the contrary the EOS become stiffer. The consequences for neutron star parameters are straightforward and appear as a considerable growth of neutron star masses.

TABLE II

Neutron star parameters: the maximum mass configuration $M_{\rm max}$, the radius of the maximum mass configuration $R(M_{\rm max})$ and the radius of the neutron star with the mass $M = 1.44M_{\odot}$. All parameters have been calculated for three separated cases: for the matter with nucleons only, for the model marked as minimal, which includes hyperons, however the vector meson sector besides the strange mesons contains only the mixed ω–ρ coupling. The extended model can be characterized by very rich spectrum of mixed vector meson interactions.

	$\Lambda_{\rm V}$	$M_{\rm max}$ $[M_{\odot}]$	$R(M_{\rm max})$ [km]	$R(1.4M_{\odot})$ [km]
Nucleons only	0	2.17	12.80	15.33
	0.0165	2.12	12.34	14.85
MIN	0	1.77	13.79	15.33
	0.0165	1.80	12.71	14.79
EXT	0.015	1.92	11.61	14.84
	0.0165	2.03	10.92	14.77

The essential part of the EOS is the symmetry energy. Its density dependence for the model with the extended isovector sector is given by the relation

$$E_{\rm sym}(n_b) = \frac{k_F^2}{6\sqrt{k_F^2 + M_N^{*2}}} + \frac{n_b}{8\left(m_\rho^2/g_\rho^2 + 2\Lambda_{\rm V}\omega_0^2\right)}\,.$$

For $\Lambda_{\rm V} = 0$ the symmetry energy varies linearly with the density. Fig. 3 allows one to compare the differences in density dependence of the symmetry energy which stem from different values of the parameter $\Lambda_{\rm V}$. The contribution to the symmetry energy coming from the non-linear vector meson mixed interactions makes the symmetry energy softer. This have been compared with the predictions of variational calculations obtained by Akmal *et al.* [10] (APR). Additionally, the analysis of data concerning isospin diffusion and neutron to proton double ratio made by Tsang *et al.* [11] which results in the following form of functional dependence of the symmetry energy $E_{\rm sym}(\rho) = 12.5(\rho/\rho_0)^{2/3}+17.6(\rho/\rho_0)^{\gamma}$, $\gamma = 0.4$–1.05, has been included.

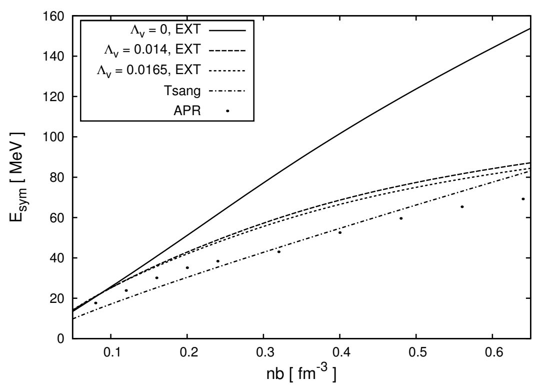

Fig. 3. Density dependence of the symmetry energy. The stiffest one has been obtained for the standard TM1 parameter set, without ω–ρ coupling. Two subsequent curves represent the much softer forms of the symmetry energy when the mixed vector meson interactions are included. Dots represent the result of Akmal *et al.* [10].

Charge neutrality and the condition of β equilibrium ($p+e^- \leftrightarrow n+\nu_e$) [6] impose constraints on neutron star composition. Neutrinos are not considered, since their mean free path is longer then the star radius. From the equilibrium conditions it is possible to determine all constituents of the neutron star matter. The concentrations of a particular component $i = B, l$ of the matter can be defined as $Y_i = n_i/n_b$, where n_i denotes the density of the component i and n_b is the total baryon number density. In Fig. 4 and Fig. 5 density fractions of protons and strange baryons as a function of the baryon number density for the fixed values of the parameter $\Lambda_{\rm V}$ are presented. Proton fraction which controls neutron star cooling is the most sensitive parameter to the density dependence of the symmetry energy. The non-linear vector meson interactions modify the proton fraction, and the most significant differences are between non-strange matter for the model in which the isovector part of interaction is described in terms of only ρ meson and the one which includes the additional mixed vector meson coupling. Considering the hyperon content of the considered models one can see that the first hyperon that appears is Λ and it is followed by Ξ^-. However, the initial rapid increase in population of Ξ^- hyperons has been suppressed leading to the reduced concentration of Ξ^- hyperons at sufficiently high density. For comparison the results obtained for the minimal model with hyperons have been included. It is evident that the additional non-linear couplings between vector mesons in the case of the extended vector meson sector modify the chemical composition of the neutron star shifting hyperon onset point to higher densities and reduces the strangeness content of the system. When the minimal hyperon model is considered the inclusion of ω–ρ coupling increases the population of hyperons in the system.

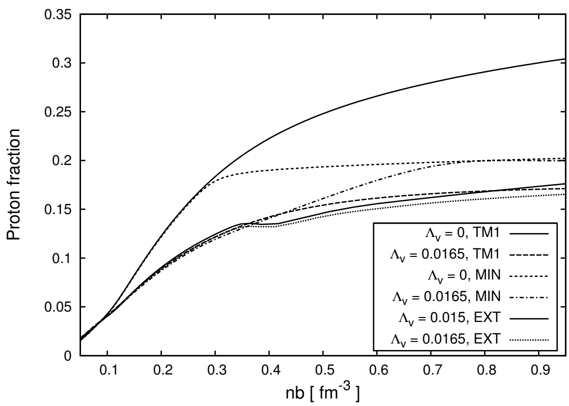

Fig. 4. Relative concentrations of protons for the presented models.

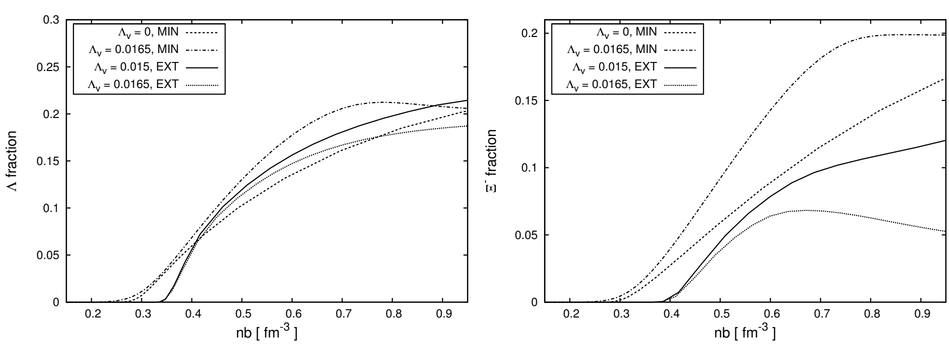

Fig. 5. Relative concentrations of strange baryons obtained for the minimal and extended models.

Current neutron star models indicate that their internal structure consists of several distinct parts with the innermost uniform liquid core being surrounded by a non-uniform region which constitutes the crust. For the inner part of the crust which spans from the neutron-drip density ($\rho_{\rm out} \sim 4 \times 10^{11} {\rm g cm}^{-3}$) to the transition density $\rho_{\rm t}$ the polytropic form of the EOS has been adopted $P_{\rm IC}(\epsilon) = AE + BE^{4/3}$, where E denotes the energy density. The two constants A and B have been determined from the condition that the pressure have to be continuous at the boundary between the inner crust and the core and at the boundary between the inner and the outer crust [12]. The transition density $\rho_{\rm t}$, pressure $P_{\rm t}$ and the symmetry energy slope L calculated in the presented model confirm the existence of $\rho_{\rm t}$–L and $P_{\rm t}$–L correlations. The obtained values of these parameters are in the experimentally accepted range: $\rho_{\rm t} = 0.08\,{\rm fm}^{-3}$, $P_{\rm t} = 0.64\,{\rm MeV\ fm}^{-3}$, and $L = 77.8$ MeV in the case of $\Lambda_{\rm V} = 0.015$ and $\rho_{\rm t} = 0.083\,{\rm fm}^{-3}$, $P_{\rm t} = 0.669\,{\rm MeV\ fm}^{-3}$, $L = 74$ MeV in the case of $\Lambda_{\rm V} = 0.0165$.

The results point out that neutron star properties and structure are significantly modified not only by the presence of hyperons and by the strength of hyperon–nucleon and hyperon–hyperon interactions but also by the non-linear mixed vector meson interactions. These additional vector meson couplings affect the isovector sector of the model and through this the density dependence of the symmetry energy which has profound influence on neutron star parameters.

REFERENCES

[1] Y. Sugahara, H. Toki, *Prog. Theor. Phys.* **92**, 803 (1994).

[2] I. Bednarek, R. Manka, *Int. J. Mod. Phys.* **D10**, 607 (2001).

[3] D.V. Shetty, S.J. Yennello, *Pramana* **75**, 259 (2010) [arXiv:1002.0313v4 [nucl-ex]].

[4] J.D. Walecka, *Theoretical Nuclear and Subnuclear Physics*, World Scientific, Singapore 2004.

[5] J. Schaffner-Bielich, A. Gal, *Phys. Rev.* **C62**, 034311 (2000).

[6] I. Bednarek, R. Manka, *J. Phys. G* **36**, 095201 (2009).

[7] C.J. Horowitz, J. Piekarewicz, *Phys. Rev. Lett.* **86**, 5647 (2001).

[8] C.J. Horowitz, J. Piekarewicz, *Phys. Rev.* **C66**, 055803 (2002).

[9] H. Takahashi *et al.*, *Phys. Rev. Lett.* **87**, 212502 (2001).

[10] A. Akmal, V.R. Pandharipande, D.G. Ravenhall, *Phys. Rev.* **C58**, 1804 (1998).

[11] M.B. Tsang *et al.*, *Phys. Rev. Lett.* **102**, 122701 (2009).

[12] M. Centelles *et al.*, *Phys. Rev. Lett.* **102**, 122502 (2009).