Vol. 42 (2011) ACTA PHYSICA POLONICA B No 11

# STRUCTURE OF A WARM HYPERON STAR*

Ilona Bednarek, Monika Pieńkos

Institute of Physics, University of Silesia
Uniwersytecka 4, 40-007 Katowice, Poland

(Received October 20, 2011)

The analysis based on the relativistic mean field (RMF) theory with the isoscalar–isovector interactions has been performed in order to study properties of a warm, asymmetric nuclear matter in $\beta$-equilibrium. The influence of this additional meson interactions on the composition and structure of a neutron star has been studied. The obtained results refer to the case with only nucleon degree of freedom and to a neutron star matter which includes hyperons.

DOI:10.5506/APhysPolB.42.2221
PACS numbers: 97.60.Jd, 26.60.Dd, 26.60.Kp, 26.60.–c

## 1. Introduction

The well established scenario of a neutron star early evolution predicts the existence of a trapped-neutrino phase for which the entropy per baryon $s = 1$ and the total number of leptons per baryon $Y_L \simeq 0.4$ [1, 2]. In recent years, there has been extensive progress in theoretical and experimental description of a warm, neutron rich nuclear matter [3, 4] which resembles the matter in proto-neutron star interiors. Thus a special attention should be focused on the proper description of asymmetric nuclear matter at finite temperature. This can be done extending the isovector meson sector by the inclusion of mixed isoscalar–isovector meson interactions which modify the density dependence of the symmetry energy and through this neutron star properties.

Since the expected density in neutron star interiors exceeds several times the nuclear saturation density, at sufficiently high densities hyperons are possible to appear. The onset of hyperon formation depends on the hyperon–nucleon interaction [5, 6]. The higher the density the more hyperons are expected to populate. They can be formed both in leptonic and baryonic processes.

* Presented by M. Pieńkos at the XXXV International Conference of Theoretical Physics "Matter to the Deepest", Ustroń, Poland, September 12–18, 2011.

The main goal of this paper is to provide a systematic analysis of the influence of the additional isovector–isoscalar meson couplings on neutron star's physical characteristics. This has been done for the matter containing only nucleons and for matter which includes also hyperons.

## 2. The model

The dynamics of the model considered is determined by a Lagrangian density which embodies contributions from baryons, mesons and leptons

$$\begin{aligned}\mathcal{L} = &\sum_B \overline{\psi}_B \left[\gamma^\mu i D_\mu - (M_B - g_{B\sigma}\sigma - g_{B\sigma^*}\sigma^*)\right]\psi_B + \tfrac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \tfrac{1}{2}m_\sigma^2\sigma^2 - \tfrac{1}{3}g_3\sigma^3 \\ &- \tfrac{1}{4}g_4\sigma^4 + \tfrac{1}{2}\partial^\mu\sigma^*\partial_\mu\sigma^* - \tfrac{1}{2}m_{\sigma^*}^2\sigma^{*2} + \tfrac{1}{2}m_\omega^2\left(\omega^\mu\omega_\mu\right) + \tfrac{1}{2}m_\rho^2\left(\rho^{\mu a}\rho_\mu^a\right) + \tfrac{1}{2}m_\phi^2\left(\phi^\mu\phi_\mu\right) \\ &- \tfrac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} - \tfrac{1}{4}R^{\mu\nu a}R_{\mu\nu}^a - \tfrac{1}{4}\Phi^{\mu\nu}\Phi_{\mu\nu} + \tfrac{1}{4}c_3\left(\omega^\mu\omega_\mu\right)^2 + \sum_{l=e,\mu}\overline{\psi}_l\left(i\gamma^\mu\partial_\mu - m_l\right)\psi_l \\ &+ \Lambda_{\mathrm{V}}\left(g_{N\omega}g_{N\rho}\right)^2\left(\omega^\mu\omega_\mu\right)\left(\rho^{\mu a}\rho_\mu^a\right) + \Lambda_4\left(g_{N\sigma}g_{N\rho}\right)^2\sigma^2\left(\rho^{\mu a}\rho_\mu^a\right) .\end{aligned} \tag{1}$$

Baryon fields $\Psi_B^T = (\psi_N, \psi_\Lambda, \psi_\Sigma, \psi_\Xi)$ are composed of the following isomultiplets:

$$\Psi_N = \begin{pmatrix}\psi_p \\ \psi_n\end{pmatrix}, \qquad \Psi_\Lambda = \psi_\Lambda ,$$
$$\Psi_\Sigma = \begin{pmatrix}\psi_{\Sigma^+} \\ \psi_{\Sigma^0} \\ \psi_{\Sigma^-}\end{pmatrix}, \qquad \Psi_\Xi = \begin{pmatrix}\psi_{\Xi^0} \\ \psi_{\Xi^-}\end{pmatrix}.$$

The meson sector includes isoscalar–scalar field $\sigma$, isoscalar–vector field $\omega$, isovector–vector field $\rho$. The strange mesons $\sigma^\star$ and $\phi$ have been included in order to describe the strongly attractive $\Lambda\Lambda$ interaction [7]. The meson part contains also additional couplings between isoscalar and isovector mesons. The strength of these interactions are characterized by the coupling constants $\Lambda_{\mathrm{V}}$ and $\Lambda_4$. The covariant derivative equals $D_\mu = \partial_\mu + ig_{B\omega}\omega_\mu + ig_{B\phi}\phi_\mu + ig_{B\rho}I_{3B}\tau^a\rho_\mu^a$, and the field tensors $\Omega_{\mu\nu}, \Phi_{\mu\nu}$ and $R_{\mu\nu}^a$ are defined as:

$$\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu , \qquad \Phi_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu , \qquad R_{\mu\nu}^a = \partial_\mu\rho_\nu^a - \partial_\nu\rho_\mu^a .$$

Neutron star matter is constrained by the charge neutrality and equilibrium under the weak processes $B_1 + l \leftrightarrow B_2 + \nu_l$, where $B_1$ and $B_2$ denote baryons and $l$ and $\nu_l$ lepton and neutrino of the same flavor. When neutrinos are trapped in the matter the $\beta$-equilibrium condition takes the form $\mu_B = b_B\mu_n - q_B(\mu_l - \mu_{\nu_l})$, where $b_B$ and $q_B$ denotes respectively the baryon number and charge of baryon $B$. This builds relations between chemical potentials of particular hyperons:

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n \,, \qquad \mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e \,, \qquad \mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e \,.$$

All calculations have been done within the RMF approximation, where the meson fields are considered as classical fields and the fields operators are replaced by their expectation values. In the case of homogenous neutron star matter the non-zero expectation values of meson fields are:

$$\langle \sigma \rangle = s_0 \,, \qquad \langle \sigma^\star \rangle = s_0^\star \,, \qquad \langle \omega_0 \rangle = w_0 \,, \qquad \langle \rho_0^3 \rangle = r_0 \,, \qquad \langle \phi_0 \rangle = f_0 \,.$$

Vector meson–hyperon coupling constants are taken from the additive quark model [7]. Whereas in the scalar sector the scalar couplings $g_{Y\sigma}$, $Y$ stands for $\Lambda$, $\Sigma$ and $\Xi$ hyperons, have to reproduce the estimated values of the potential felt by a single $\Lambda$, $\Sigma$ and $\Xi$ in the saturated nuclear matter. The general form of the potential which describes hyperon–nucleon and hyperon-hyperon interaction can be written in the form which indicates the existence of correlations between the scalar and vector coupling constants

$$\begin{aligned} U_Y^{(B)} &= g_{Y\sigma} s_0 - g_{Y\omega} w_0 + g_{Y\sigma^\star} s_0^\star - g_{Y\phi} f_0 \\ &= M_Y - M_{Y^\star}(s_0, s_0^*) - (g_{Y\omega} w_0 + g_{Y\phi} f_0) \,. \end{aligned} \tag{2}$$

$M_Y^\star(s_0, s_0^\star)$ denotes the effective hyperon mass, the couplings of the $\sigma^\star$ and $\phi$ mesons with nucleons are equal zero $(g_{N\sigma^\star} = g_{N\phi} = 0)$. The analysis based on data collected for double hypernucleus ${}^{6}_{\Lambda\Lambda}$He done by Takahashi *et al.* [8] indicates that the $\Lambda\Lambda$ interaction is rather weak. The potential well depth evaluated for this data equals $U_\Lambda^{(\Lambda)} \simeq 5$ MeV. Using the meson exchange potentials for hyperon–hyperon interactions from Nijmegen Model [7] lead to a rough estimate which can be applied to determine the coupling of hyperons to the strange meson $\sigma^*$

$$U_\Xi^{(\Xi)} \simeq U_\Lambda^{(\Xi)} \simeq 2U_\Xi^{(\Lambda)} \simeq 2U_\Lambda^{(\Lambda)} \,. \tag{3}$$

The strange sector parameters of the model are collected in Table I. The extension of the isovector sector requires the adjustment of the $g_{N\rho}$–$\Lambda_{\rm V}$ and $g_{N\rho}$–$\Lambda_4$ pairs of coupling constants to keep the value of the symmetry energy $E_{\rm sym} \approx 26$ MeV at the density corresponding to $k_{\rm NF} = 1.15$ fm$^{-3}$ [9, 10].

In order to study the influence of nonlinear isovector–isoscalar $\rho$–$\omega$ and $\rho$–$\sigma$ couplings, the $\Lambda_{\rm V}$ and $\Lambda_4$ parameters have been fixed at the value of 0.0165. This provides the possibility to modify the density dependence of the symmetry energy in a range that gives the slope and curvature of the symmetry energy in agreement with that extracted from experiments.

TABLE I

The parameters of the strange sector. The hyperon–scalar meson coupling constants result from the fit to hypernuclear potentials.

| $Y$ | $U_{\Sigma}^{(N)}$ | $g_{\sigma Y}$ | $g_{Y\omega}/g_{N\omega}$ | $g_{\sigma^* Y}$ |
|---|---|---|---|---|
| $\Lambda$ | $-28$ MeV | 6.16894 | 2/3 | 5.48186 |
| $\Sigma$ | 30 MeV | 4.47586 | 2/3 | 5.48186 |
| $\Xi$ | $-18$ MeV | 3.20124 | 1/3 | 11.5156 |

The generalization of the theory to the finite temperature case requires the definition of a thermodynamic potential $\Omega$ [11] which for the model specified by the Lagrangian function (1) allows one to calculate the energy density $\varepsilon$ and pressure $P$ of the system:

$$\begin{aligned}\varepsilon \;=\; & \tfrac{1}{2}m_\sigma s_0^2+\tfrac{1}{3}g_3 s_0^3+\tfrac{1}{4}g_4 s_0^4+\tfrac{1}{2}m_{\sigma^*}s_0^{*2}+\tfrac{1}{2}m_\omega^2 w_0^2+\tfrac{3}{4}c_3 w_0^4\\ &+\tfrac{1}{2}m_\rho^2 r_0^2+\Lambda_4(g_\sigma g_\rho)^2 s_0^2 r_0^2+3\Lambda_V(g_\omega g_\rho)^2 w_0^2 r_0^2\\ &+\tfrac{1}{2}m_\phi^2 f_0^2+\sum_{i=B,l}\left\{\frac{2J_i+1}{(2\pi)^3}\int d^3k E_i^*(k)\left(f_i^+(k)+f_i^-(k)\right)\right\}, \end{aligned} \tag{4}$$

$$\begin{aligned}P \;=\; & -\tfrac{1}{2}m_\sigma s_0^2-\tfrac{1}{3}g_3 s_0^3-\tfrac{1}{4}g_4 s_0^4-\tfrac{1}{2}m_{\sigma^*}s_0^{*2}+\tfrac{1}{2}m_\omega^2 w_0^2+\tfrac{1}{4}c_3 w_0^4\\ &+\tfrac{1}{2}m_\rho^2 r_0^2+\tfrac{1}{2}m_\phi^2 f_0^2+\Lambda_4(g_\sigma g_\rho)^2 s_0^2 r_0^2+\Lambda_V(g_\omega g_\rho)^2 w_0^2 r_0^2\\ &+\sum_{i=B,l}\left\{\frac{2J_i+1}{3(2\pi)^3}\int d^3k\frac{k^2}{E_i^*}(k)\left(f_i^+(k)+f_i^-(k)\right)\right\}, \end{aligned} \tag{5}$$

where for the $i$-th species $J_i$ denotes the spin and $f_i^{\pm}$ the distribution functions for particle (+) and antiparticles (−)

$$f_i^{\pm}=\frac{1}{\exp\left[\,\left(E_i^\star(k)\mp\nu_i\right)/T\right]+1}\,. \tag{6}$$

The chemical potential and effective energy are defined as: $\nu_i=\mu_i-g_{i\omega}w_0-g_{i\rho}r_0^3 I_{3i}-g_{i\phi}f_0$ and $E_i^\star(k)=\sqrt{k^2+M_i^{\star 2}}$ . In the case of baryons the effective mass takes the form $M_B^\star=M_B-g_{B\sigma}s_0-g_{B\sigma^\star}s_0^\star$, for leptons $M_l^\star=m_l$, $l=e,\mu$. The proto-neutron star matter with trapped neutrinos is characterized by a fixed lepton fraction $Y_{L_l}=Y_{L_{e,\mu}}+Y_{\mu_{e,\mu}}$. Since there are no muons in the phase of a hot proto-neutron star the lepton fraction reduces to $Y_L=Y_e+Y_{\mu_e}$ and has been estimated at the value of $\simeq 0.4$.

## 3. Results

The analysis of proto-neutron and neutron star models has been done on the basis of TM1 parameterization (Table II). The essential results of the performed analysis are the EOSs of dense matter obtained for two compositions: for the case of nucleons only — the left panel of Fig. 1 and for the case which additionally includes hyperons — right panel of Fig. 1. In the case when only nucleons are considered, neutrino trapping leads to the softening of the EOS and the inclusion of the mixed meson interactions does not modify the EOS considerably.

TABLE II

The TM1 parameter set [12, 13].

| $M$ [MeV] | $m_\sigma$ [MeV] | $m_\omega$ [MeV] | $m_\rho$ [MeV] | $g_3$ [fm$^{-1}$] | $g_4$ | $c_3$ | $g_\sigma$ | $g_\omega$ |
|---|---|---|---|---|---|---|---|---|
| 938.919 | 511.2 | 783 | 770 | 7.232 | 0.618 | 71.3 | 10.029 | 12.614 |

The right panel of Fig. 1 depicts the much stronger influence of the additional meson couplings on the EOS of neutron star matter which includes hyperons. The stiffening of the EOS due to mixed meson interactions is even more evident in the case without neutrino trapping.

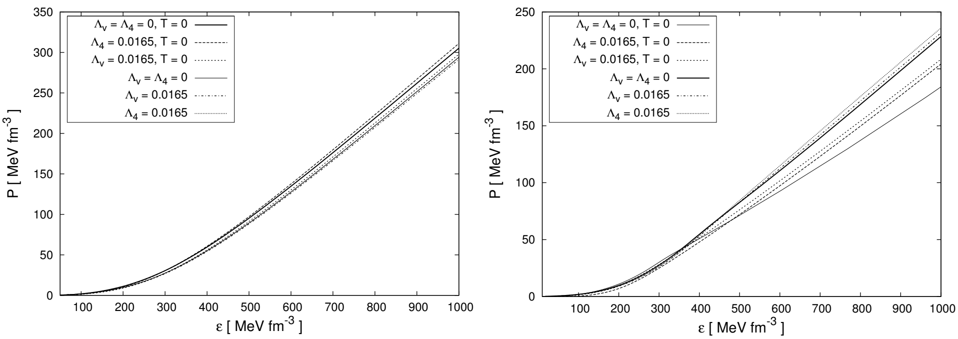


Fig. 1. Equations of state of the nonstrange (left panel) and strangeness-rich (right panel), warm neutrino-rich $Y_l = 0.4$ and cold neutrion-free matter. The obtained results include EOSs calculated for $\Lambda_V = \Lambda_4 = 0$ and for $\Lambda_V = \Lambda_4 = 0.0165$.

The alterations in the form of the EOS and especially in the high density limit have inevitable consequences for neutron star parameters. This manifests itself as a sensitivity of neutron star masses and radii to the stiffness of the EOS and allows one to study the influence of nonlinear vector meson interaction terms on neutron star parameters. The obtained mass–radius relations for nuclear matter with and without hyperons are shown respectively in the right and left panel of Fig. 2. The value of maximum masses and corresponding radii together with the radii calculated for the $M = 1.4M_\odot$ configuration are collected in Table III.

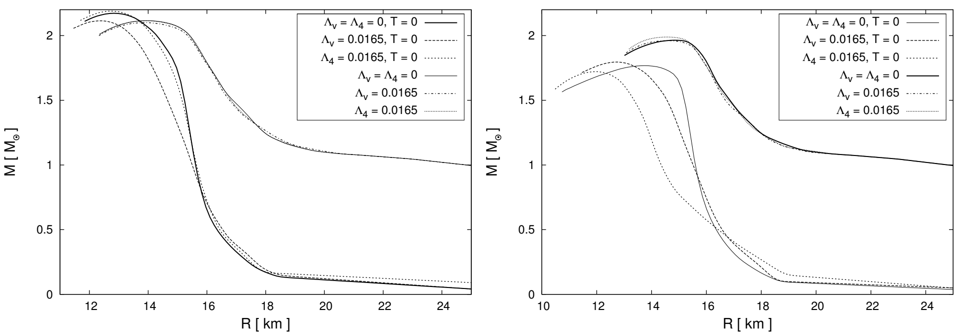


Fig. 2. The mass–radius relations for neutron star in the case of nucleons only (left panel) and in the case with hyperons (right panel), including solutions for neutrino-rich and neutrino-poor matter.

TABLE III

Neutron star parameters: the maximum mass $M_{\rm max}$, the radius for the maximum mass configuration $R(M_{\rm max})$, and the radius for the $M = 1.4M_\odot$ configuration $R(1.4M_\odot)$ calculated for different strength of the mixed meson interactions.

| Nucleons only | | | | Nucleons and hyperons | | |
|---|---|---|---|---|---|---|
| $M_{\rm max}$ $[M_\odot]$ | $R(M_{\rm max})$ [km] | $R(1.4M_\odot)$ [km] | | $M_{\rm max}$ $[M_\odot]$ | $R(M_{\rm max})$ [km] | $R(1.4M_\odot)$ [km] |
| $\Lambda_v = \Lambda_4 = 0,\ g_\rho = 9.2644$ | | | | | | |
| 2.17 | 12.80 | 15.32 | cold | 1.77 | 13.79 | 15.33 |
| 2.12 | 14.00 | 17.33 | warm | 1.96 | 14.82 | 17.22 |
| $\Lambda_v = 0.0165,\ g_\rho = 10.037$ | | | | | | |
| 2.12 | 12.34 | 14.85 | cold | 1.80 | 12.71 | 14.79 |
| 2.10 | 13.82 | 17.13 | warm | 1.96 | 14.65 | 17.18 |
| $\Lambda_4 = 0.0165,\ g_\rho = 10.8873$ | | | | | | |
| 2.19 | 12.70 | 15.28 | cold | 1.72 | 11.99 | 13.65 |
| 2.12 | 13.99 | 17.18 | warm | 1.99 | 14.61 | 17.19 |

Charge neutrality and $\beta$-equilibrium impose constraints on a neutron star composition. The presented equilibrium conditions are decisive in determining concentrations of all constituents of the neutron star matter. A concentration of a particular component $i = B, l$ can be defined as $Y_i = n_i/n_b$, where $n_i$ denotes the density of the component $i$ and $n_b$ is the total baryon density. The nonlinear mixed meson couplings reduce population of protons for neutrino-trapped and neutrino-free matter. This effect is stronger for nonstrange matter. When the EOS includes contributions from hyperons, proton concentrations is affected only for moderate densities. Results are depicted in the upper panels of Fig. 3. In general, trapped neutrinos shift the appearance of $\Lambda$ and $\Xi$ hyperons to higher densities. The $\Lambda_{\mathrm{V}}$ and $\Lambda_4$ couplings in the case of neutrino free matter enhance the concentrations of hyperons whereas for the proto-neutron star matter the mixed vector meson interactions reduce hyperon abundance. Concentrations of $\Lambda$ and $\Xi$ hyperons are presented in the lower panels of Fig. 3. The absence of $\Sigma$ hyperons is due to their strong repulsion in nuclear medium.

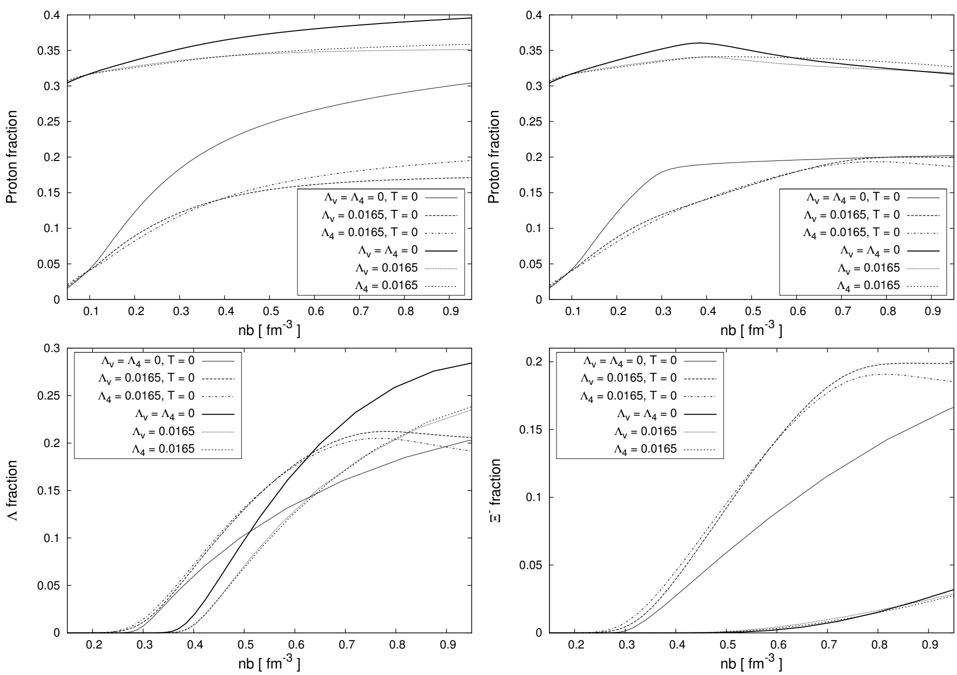


Fig. 3. Chemical composition of proto-neutron and neutron star matter for the case of nucleons only (upper left panel) and for the case including hyperons. Concentration of protons is shown in the upper panels, and fraction of $\Lambda$ and $\Xi^-$ particles — respectively lower left and lower right panel.

## 4. Conclusions

In this paper, a special class of the EOSs of asymmetric nuclear matter with zero and non-zero strangeness has been analyzed in a systematic approach within the RMF model. The basic characteristic of the considered model is the extended isovector meson sector which includes contributions from nonlinear isovector–isoscalar meson couplings. The results of the analysis performed for these nonlinear models have been compared with those obtained with the use of the standard TM1 parameter set. It has been found that neutron star parameters and compositions have been affected not only by neutrino trapping but by the mixed meson interactions as well. The results have been presented for non-strange and strangeness-rich matter.

The main results of performed calculations are the stiffening of the EOS and the increase of maximal mass value in the case of neutrino free and strangeness-rich matter as well as considerable change in the chemical composition of a neutron star matter in all analysed cases.

## REFERENCES

[1] M. Prakash *et al.*, *Lect. Notes Phys.* **578**, 364 (2001).

[2] I. Bednarek, M. Keska, R. Manka, *Int. J. Mod. Phys.* **D13**, 215 (2004).

[3] A. Gal, D.J. Millener, *Phys. Lett.* **B701**, 342 (2011) [`arXiv:1104.3429v2 [nucl-th]`].

[4] M.S. Ryu, B. Hong, T.I. Kang, *J. Korean Phys. Soc* **59**, 1605 (2011).

[5] I. Bednarek, *Acta Phys. Pol. B* **40**, 3071 (2009).

[6] I. Bednarek, R. Manka, *J. Phys. G* **36**, 095201 (2009).

[7] J. Schaffner-Bielich, A. Gal, *Phys. Rev.* **C62**, 034311 (2000).

[8] H. Takahashi *et al.*, *Phys. Rev. Lett.* **87**, 212502 (2001).

[9] C.J. Horowitz, J. Piekarewicz, *Phys. Rev. Lett.* **86**, 5647 (2001).

[10] C.J. Horowitz, J. Piekarewicz, *Phys. Rev.* **C66**, 055803 (2002).

[11] J.A. Pons, J.A. Miralles, M. Prakash, J.M. Lattimer, *Astrophys. J.* **553**, 382 (2001).

[12] Y. Sugahara, H. Toki, *Prog. Theor. Phys.* **92**, 803 (1994).

[13] I. Bednarek, R. Manka, *Int. J. Mod. Phys.* **D10**, 607 (2001).