DARK MATTER DATA AND QUARTIC SELF-COUPLINGS IN INERT DOUBLET MODEL*

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We analyse the thermal evolution of the Universe in the Inert Doublet Model for three viable regions of Dark Matter mass: low, medium and high DM mass. Those three regions exhibit different behaviour in the possible types of evolution. We argue that the quartic self-couplings in IDM are significant parameters for the astrophysical analysis.

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1. Thermal evolution of the Universe in IDM

The Inert Doublet Model (IDM) [1, 2] is a Z_2-symmetric 2HDM, which for a special set of parameters may provide the Dark Matter (DM) candidate. The model contains two scalar SU(2) doublets: a "standard" scalar (Higgs) doublet $\Phi_{\rm S}$ and a "dark" scalar doublet $\Phi_{\rm D}$. $\Phi_{\rm S}$ is responsible for the electroweak symmetry breaking and masses of fermions and gauge bosons as in the Standard Model (SM), while $\Phi_{\rm D}$ does not receive vacuum expectation value (v.e.v.) and does not couple to fermions. In the model the discrete D-symmetry of the Z_2 type is present

$$D: \quad \Phi_{\rm S} \xrightarrow{D} \Phi_{\rm S}\,, \qquad \Phi_{\rm D} \xrightarrow{D} -\Phi_{\rm D}\,, \qquad \text{SM fields} \xrightarrow{D} \text{SM fields}\,. \tag{1}$$

All the degrees of freedom of the dark doublet $\Phi_{\rm D}$ are realized as the massive D-scalars: two charged $D^{\pm}$ and two neutral D_H and D_A. They possess a conserved multiplicative quantum number, *the odd D-parity*, and therefore the lightest particle among them can be considered as a candidate for the DM particle.

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The D-symmetric potential V, which can describe IDM, is

$$V = -\frac{m_{11}^2}{2}\Phi_{\rm S}^\dagger\Phi_{\rm S} - \frac{m_{22}^2}{2}\Phi_{\rm D}^\dagger\Phi_{\rm D} + \frac{\lambda_1}{2}\left(\Phi_{\rm S}^\dagger\Phi_{\rm S}\right)^2 + \frac{\lambda_2}{2}\left(\Phi_{\rm D}^\dagger\Phi_{\rm D}\right)^2$$
$$+\lambda_3\left(\Phi_{\rm S}^\dagger\Phi_{\rm S}\right)\left(\Phi_{\rm D}^\dagger\Phi_{\rm D}\right) + \lambda_4\left(\Phi_{\rm S}^\dagger\Phi_{\rm D}\right)\left(\Phi_{\rm D}^\dagger\Phi_{\rm S}\right) + \frac{\lambda_5}{2}\left[\left(\Phi_{\rm S}^\dagger\Phi_{\rm D}\right)^2 + \text{h.c.}\right] \quad (2)$$

with all parameters real and $\lambda_5 < 0$ [3]. *Positivity conditions* imposed on the potential guarantee that the extremum with the lowest energy will be the global minimum of the potential (vacuum). Relevant conditions are: $\lambda_{1,2} > 0$, $R + 1 > 0$; $R = \lambda_{345}/\sqrt{\lambda_1\lambda_2}$, $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$.

The Yukawa interaction of SM fermions ψ_f with only one scalar doublet $\Phi_{\rm S}$ have the same form as in the SM with the change $\Phi \to \Phi_{\rm S}$ (Model I for a general 2HDM).

We consider *thermal evolution of the Lagrangian*, following the approach presented in [3, 4, 5]. In the first approximation the Yukawa couplings and the quartic coefficients of V are constant, while the quadratic parameters m_{ii}^2 $(i = 1, 2)$ vary with temperature T as follows [3, 6]

$$\begin{aligned} m_{ii}^2(T) &= m_{ii}^2 - c_i T^2\,, \\ c_1 &= \frac{3\lambda_1 + 2\lambda_3 + \lambda_4}{6} + \frac{3g^2 + g'^2}{8} + \frac{g_t^2 + g_b^2}{2}\,, \\ c_2 &= \frac{3\lambda_2 + 2\lambda_3 + \lambda_4}{6} + \frac{3g^2 + g'^2}{8}\,. \end{aligned} \quad (3)$$

In this work we limit ourselves to positive c_1, c_2 as we consider only the restoration of EW symmetry for high T (the negative values of $m_{11}^2(T)$ and $m_{22}^2(T)$ for high enough T).

As the Universe is cooling down the potential V (2), with temperature dependent quadratic coefficients (3), may have different ground states [3]. The general form of the neutral extremum is

$$\langle\Phi_{\rm S}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_{\rm S} \end{pmatrix}, \qquad \langle\Phi_{\rm D}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_{\rm D} \end{pmatrix}, \qquad \left(v^2 = v_{\rm S}^2 + v_{\rm D}^2\right). \quad (4)$$

EW symmetric extremum (EWs) is realized if $v_{\rm D} = v_{\rm S} = 0$. Here all fermions and bosons are massless and EW symmetry is conserved.

Inert extremum $\rm I_1$ can be realized if $v_{\rm D} = 0$, $v_{\rm S}^2 = v^2 = m_{11}^2/\lambda_1$. If $\rm I_1$ is a vacuum then in the scalar sector there are four dark scalar particles D_H, D_A, $D^\pm$ and the SM-like Higgs particle $h_{\rm S}$. The lightest dark particle is stable and so it is a good DM candidate. Assuming that DM particles are neutral, we consider such variant of IDM in which D_H is a DM candidate, meaning $M_{D^\pm}$, $M_{D_A} > M_{D_H}$. Various theoretical and experimental

constraints apply for the IDM (see *e.g.* [7, 8, 9, 10, 11, 12, 13]). EWPT and collider data (LEP II, Tevatron, LHC) constrain the allowed regions of the masses of $h_{\rm S}$ and dark scalars. The relic density measurements and the direct detection experiments can be used to constrain the DM mass and the DM-Higgs self-coupling λ_{345}. However, they do not constrain the DM quartic self-coupling λ_2.

Inertlike extremum $\rm I_2$ is mirror-symmetric to $\rm I_1$ as $v_{\rm S} = 0$, $v_{\rm D}^2 = v^2 = m_{22}^2/\lambda_2$. Fermions are massless at the tree-level (Model I), while gauge bosons are massive. There are four scalars $S_H, S_A, S^{\pm}$ (no DM candidate as D-symmetry is spontaneously violated) and the Higgs particle $h_{\rm D}$ with no interaction with fermions.

Mixed extremum M is a standard 2HDM extremum with $v_{\rm S}, v_{\rm D} \neq 0$. Fermions and bosons are massive and there are 5 Higgs particles: CP-even h and H, CP-odd A and charged $H^{\pm}$, none of them can be a DM candidate.

We assume that today inert phase $\rm I_1$ with the DM candidate D_H is realized. However, the sequences of transitions between different vacua (called here *rays*) were possible in the past [3].

There are three types of sequences that start in the EWs symmetric phase and end in the inert phase. First one is a single phase transition EWs $\rightarrow \rm I_1$. It is realized by rays I a, b, c (for $R > 1$, $0 < R < 1$ and $-1 < R < 1$, respectively), rays II a, b ($R > 1$ and $0 < R < 1$) and ray III (only for $R > 1$). The difference between those rays is the status of the $\rm I_2$ extremum: for ray I it is not an extremum; for ray II it is an extremum, but not a minimum; for ray III it is a local minimum, but not the global one.

Second type of sequence, EWs $\rightarrow \rm I_2 \rightarrow I_1$, can be realized only if $R > 1$ and is represented by the rays IV and V. In this case the EWSB is a phase transition of a second-order, while the last transition $\rm I_2 \rightarrow I_1$ is of the first-order.

Only for $R > 1$ there is an unique opportunity of coexistence of minima (vacuum $\rm I_1$ and local metastable minimum $\rm I_2$) for rays III, IV and V. For ray IV the coexistence is temporary and the local minimum $\rm I_2$ disappears for low temperatures, while for rays III and V it still exists for $T = 0$.

In the $0 < R < 1$ case there is a possibility of having three phase transitions in the sequence EWs $\rightarrow \rm I_2 \rightarrow M \rightarrow I_1$ (ray VI). All transitions here are of the second-order.

2. Phenomenological analysis

For phenomenological studies it is useful to choose the physical masses M_{h_S}, M_{D_H}, M_{D_A}, $M_{D^{\pm}}$ and the scalar self-couplings as the input parameters. In this analysis we chose two self-couplings, λ_{345} and λ_2, which have different properties and play different role in the analysis [14].

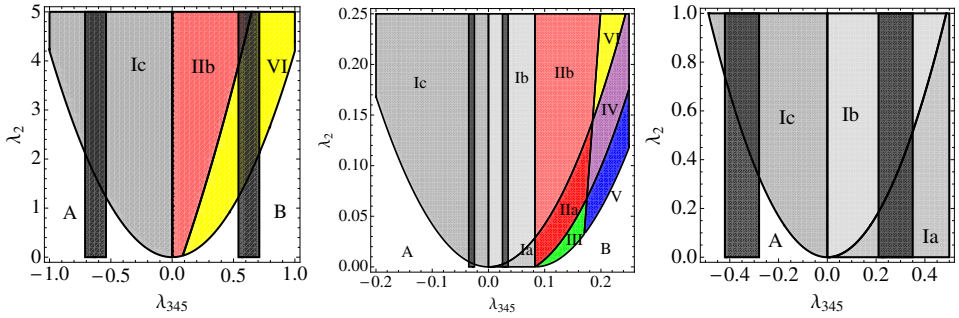
λ_{345} is a triple and quartic coupling of the DM particle and SM-like Higgs, *i.e.* $D_H D_H h_S$ or $D_H D_H h_S h_S$. In wide range of DM mass this parameter governs the main annihilation channel into pair of fermions via Higgs exchange: $D_H D_H \to h_S \to f\bar{f}$ with the cross-section $\sigma \propto \lambda_{345}^2/(4M_{D_H}^2 - M_{h_S}^2)^2$. For this reason this parameter, along with the DM mass, influences strongly the value of the DM relic density $\Omega_{\rm DM}h^2$. It also plays an important role in the direct detections experiments, as DM–nucleon elastic scattering cross-section is given by $\sigma_{{\rm DM},N} \propto \lambda_{345}^2/(M_{D_H} + M_N)^2$ [2].

The remaining self-coupling, λ_2, is a quartic coupling of DM particle. For this reason the exact value of λ_2 does not influence $\Omega_{\rm DM}h^2$ directly. However, this parameter limits λ_{345} through the positivity constraints and is important for the type of evolution.

Below we present the analysis done in the $(\lambda_{345}, \lambda_2)$ phase space for the three regions of DM mass for chosen benchmark points:

1. $M_{D_H} = 5$ GeV, $M_{D_A} = 105$ GeV, $M_{D^\pm} = 110$ GeV, $M_{h_S} = 120$ GeV,
2. $M_{D_H} = 50$ GeV, $M_{D_A} = 120$ GeV, $M_{D^\pm} = 120$ GeV, $M_{h_S} = 120$ GeV,
3. $M_{D_H} = 800$ GeV, $M_{D_A} = 801$ GeV, $M_{D^\pm} = 801$ GeV, $M_{h_S} = 120$ GeV.

Figures 1 (a), (b), (c) show the possible rays for each region, as well as the 3σ WMAP-allowed regions $0.085 < \Omega_{\rm DM}h^2 < 0.139$ [15]. Note that the region A is excluded by the positivity constraints and in the region B I_1 is only a *local minimum* and not the vacuum. Each ray is realized in the separate region of $(\lambda_{345}, \lambda_2)$ phase space. Furthermore, different types of evolution are possible in the cases of low, medium and high DM mass.



(a) low DM mass (b) medium DM mass (c) high DM mass

Fig. 1. Possible rays for different regions of DM mass. Vertical bounds denote regions allowed by WMAP measurements; region A is excluded by the positivity constraints, in region B, I_1 is only a *local* minimum.

2.1. Low DM mass

The low mass region, which resembles the singlet scalar DM, has been shown to fit into the CoGeNT, DAMA/Libra and CRESST-II signal [16, 17, 18, 19, 20], however it appears to be excluded by the XENON100 results [21]. In this region the DM particle is much lighter than all other scalar particles with $M_{D_H} \approx (4\text{–}8)$ GeV and $M_{D_A} \approx M_{D^\pm} \approx 100$ GeV [8]. Large mass splittings between the D_H and other scalar particles do not allow for the coannihilation. To have the correct WIMP cross-section and proper relic density rather large λ_{345} is needed [14].

In this region the possible types of evolution are limited to three rays only (ray I c, II b, VI, Fig. 1 (a)) and there is no coexistence of minima.

Notice, that to fit into the WMAP data we need not only large λ_{345}, but also large $\lambda_2 \approx 1$. The smaller values of λ_2 are excluded by positivity constraints (region A) or I_2 vacuum (region B). However, larger λ_{345} corresponds to the lower temperature of the final phase transition. In this example for the sequence EWs $\to \mathrm{I}_2 \to \mathrm{M} \to \mathrm{I}_1$ it occurs at $T_{\mathrm{M}\to\mathrm{I}_1} = 6$ GeV, so $M_{D_H} \approx T_{\mathrm{M}\to\mathrm{I}_1}$. The recent analysis [6] shows that in this case the lowest order of the thermal corrections to V is not sufficient.

2.2. Medium DM mass

In this region the DM mass is of the order of $M_{D_H} \approx (45\text{–}160)$ GeV. Mass splitting between D_H and $D^\pm$ should be large: $M_{D^\pm}$–$M_{D_H} \approx (50\text{–}90)$ GeV. Constraints for M_{D_A}–M_{D_H} have been derived in [11]. If this value is large with $M_{D_A} \approx M_{D^\pm}$ then there is no coannihilation. For M_{D_A}–$M_{D_H} < 8$ GeV this effect influences strongly the value of DM relic density [8, 9].

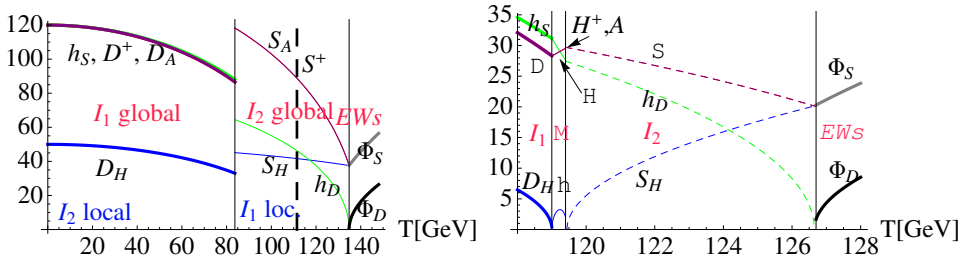
For medium DM mass, regardless of the exact values of the mass splittings, all rays are possible (Fig. 1 (b)). This is the only case when one can have the first-order phase transition for rays IV and V — those rays are not possible for low or high DM mass.

In this region $\Omega_{\mathrm{DM}}h^2$ is very sensitive to the exact value of M_{D_H} and mass splittings. Therefore, we cannot make a general statement that a certain ray will always give a proper relic density [14]. However, some properties of this mass region are independent of M_{D_H}. For example, complex sequences (rays IV–VI) require rather large λ_{345}. This, however, leads to the similar problem as in the low DM mass case: the temperature of the final phase transition is lower and especially for ray V further thermal corrections to the potential are needed.

2.3. High DM mass

As shown in [9] in the high mass region the mass of the dark matter particle should be over 500 GeV, with the small mass splittings between the dark particles. Also the *perturbative unitarity* may give the relevant

constraints [22]. To have a proper relic density we need rather large absolute values of $|\lambda_{345}|$ and λ_2 [14]. Only three rays are possible; they correspond to the sequence with a single phase transition EWs $\rightarrow$ I_1 and differ only by the value of R (rays I a, I b, I c, Fig. 1 (c)). Other types of evolution require $\lambda \approx O(10\text{–}20)$ [14].



(a) ray V, $\lambda_2 = 0.0684$ (b) ray VI, $\lambda_2 = 0.1672$

Fig. 2. Temperature evolution of masses of scalar particles in different sequences. $M_{D_H} = 50$ GeV, $M_{D_A} = 120$ GeV, $M_{D^\pm} = 120$ GeV, $M_{h_S} = 120$ GeV, $\lambda_{345} = 0.1945$. Notation: $D = D^\pm$, D_A, $S = S^\pm$, S_A.

2.4. Role of λ_2 self-coupling

λ_2 self-coupling is a significant parameter in IDM — as shown in the previous sections. It not only limits λ_{345} through the positivity constraints, but also is important for the type of evolution. In this section, we fix the scalar masses and λ_{345} self-coupling and let λ_2 vary. Depending on the value of λ_2, different types of evolution are realized.

Figures 2 (a), (b) show the thermal evolution of the mass parameters during evolution of the Universe [23]. In the case of ray V (Fig. 2 (a)) there are two phase transitions. EWSB into I_2 phase happens for $T = 134.8$ GeV. Dashed line shows the appearance of the local minimum I_1 during the inertlike phase of the evolution. I_1 becomes a global minimum after first-order phase transition which takes place for $T = 83.7$ GeV. From this point I_2 is a local minimum that still exists for $T = 0$.

Figure 2 (b) shows the evolution according to ray VI. Here, after EWSB at $T = 126.7$ GeV Universe enters the inertlike phase with massless fermions and massive gauge bosons. This minimum becomes a saddle point for $T = 119.4$ GeV and the second-order transition to the M phase takes place. This phase is short-lived and soon, at $T = 119.0$ GeV, there is another second-order transition into I_1 phase. There is no coexistence of minima at any point in time during evolution.

Notice, that although the temperature of EWSB is similar in both cases, the final phase transition happens at different temperatures and it is much lower for ray V. As discussed, this ray is the most likely to require further thermal corrections to the potential [6].

3. Conclusions

In this paper we studied the temperature evolution of the Universe in IDM. We also discuss the significance of the quartic self-coupling λ_2. This parameter does not influence DM relic density directly and it cannot be accessible in the colliders. However, it is related to the value of λ_{345} coupling through the positivity constraints. It also plays an important role in the evolution, as its different values lead to the different types of the evolution of the Universe.

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