PRAGMATIC EXTENSIONS OF THE STANDARD MODEL*

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We discuss models that allow to ameliorate the Standard Model little hierarchy problem by adding extra scalar degrees of freedom. We argue that extra gauge-singlet real scalars can both soften the little hierarchy problem and provide a realistic source of Dark Matter. For that, a setup consistent with the present LHC bounds for the Higgs-boson mass is provided *e.g.* by $N=6$ scalars with masses $m\sim 1.5$–3 TeV and the UV cutoff $\Lambda\sim 4.5$–10 TeV. We explore the possibility that a second Higgs-boson doublet is added do the Standard Model in such a way that quadratic divergences in corrections to the scalar two-point Green functions are canceled. Although the cancellation allows for substantial amount of CP violation in the scalar potential, it is not consistent with Dark Matter being a component of one of the two Higgs doublets. Therefore, either a third (inert) doublet or a singlet must be added.

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1. Introduction

The goal of this study is to extend the Standard Model of electroweak interactions such that the little hierarchy problem is ameliorated, extra sources of CP violation emerge and candidates for Dark Matter are provided while all the successes of the SM are preserved. We will restrict ourselves to only those extensions that interact with the SM through renormalizable interactions. Since quadratic divergences in loop corrections to the Higgs boson mass are dominated by top-quark contributions, therefore we will introduce extra scalar degrees of freedom, so that they can balance the top contribution. The extensions we consider, although renormalizable, shall be treated as effective low-energy theories valid below a cutoff energy Λ. We are not going to discuss the UV completions of such models.

2. The little hierarchy problem

The quadratically divergent 1-loop correction to the Higgs boson (h) mass was first calculated by Veltman [1]

$$\delta^{(\mathrm{SM})} m_h^2 = \left[3m_t^2/2 - \left(6m_W^2 + 3m_Z^2\right)/8 - 3m_h^2/8\right]\Lambda^2/\left(\pi^2 v^2\right) , \qquad (1)$$

where Λ is a UV cutoff, that we adopt as a regulator, and $v \simeq 246$ GeV denotes the vacuum expectation value of the scalar doublet (small SM logarithmic corrections will be neglected).

The LHC data limit [2] the Higgs-boson mass to the range 115 GeV $< m_h <$ 145 GeV. The correction (1) can, therefore, exceed the mass itself even for small values of Λ, *e.g.* $\delta^{(\mathrm{SM})} m_h^2 \simeq m_h^2$ for $m_h = 130$ GeV already for $\Lambda \simeq 600$ GeV. This is considered as an indication of extensions of the SM with a typical scale of 1 TeV. Since no effects of such low energy new physics have been observed, that difficulty is known as the little hierarchy problem.

Here, our pragmatic task is to construct a simple modification of the SM within which δm_h^2 (the total correction to the SM Higgs boson mass squared) is suppressed only up to $\Lambda \lesssim$ 3–10 TeV. Since (1) is dominated by the fermionic (top quark) terms, the most economic way of achieving this is by introducing new scalars φ_i, 1-loop contributions of which, reduce those from the SM. In order to preserve SM predictions, we assume that φ_i are singlets under the SM gauge group. Then it is easy to observe that the theoretical expectations for all existing experimental tests remain the same if $\langle\varphi_i\rangle = 0$ (which we assume hereafter).

The most general scalar potential implied by $Z_2^{(i)}$ independent symmetries $\varphi_i \to -\varphi_i$ (imposed in order to prevent $\varphi_i \to hh$ decays) reads:

$$\begin{aligned} V(H,\varphi_i) &= -\mu_H^2|H|^2 + \lambda_H|H|^4 + \sum_{i=1}^{N}\left(\mu_\varphi^{(i)}\right)^2\varphi_i^2 \\ &\quad + \frac{1}{24}\sum_{i,j=1}^{N}\lambda_\varphi^{(ij)}\varphi_i^2\varphi_j^2 + |H|^2\sum_{i=1}^{N}\lambda_x^{(i)}\varphi_i^2 \,. \end{aligned} \tag{2}$$

In the following numerical computations we will assume that (2) has an $O(N)$ symmetry so that $\mu_\varphi^{(i)} = \mu_\varphi$, $\lambda_\varphi^{(ij)} = \lambda_\varphi$ and $\lambda_x^{(i)} = \lambda_x$ (small deviations from this assumption do not change our results qualitatively). The minimum of V is at $\langle H\rangle = v/\sqrt{2}$ and $\langle\varphi_i\rangle = 0$ when $\mu_H^2, \mu_\varphi^2 > 0$ and $\lambda_H, \lambda_\varphi > 0$ which we now assume. The masses for the SM Higgs boson and the new scalar singlets are $m_h^2 = 2\mu_H^2$ and $m^2 = 2\mu_\varphi^2 + \lambda_x v^2$ ($\lambda_H v^2 = \mu_H^2$), respectively.

Positivity of the potential at large field strengths requires that $\lambda_H, \lambda_\varphi$, $\lambda_x > 0$, or if $\lambda_x < 0$ then $\lambda_H\lambda_\varphi > 6\lambda_x^2$ must hold at the tree level. The high energy unitarity implies $\lambda_H \leq 4\pi/3$ (the SM requirement [3]) and $\lambda_\varphi \leq 8\pi$, $\lambda_x < 4\pi$ (known [4] for $N = 1$).

The existence of φ_i generates additional radiative corrections[1] to m_h^2. Then the extra contribution to m_h^2 reads

$$\delta^{(\varphi)}m_h^2 = -\left[N\lambda_x/\left(8\pi^2\right)\right]\left[\Lambda^2 - m^2\ln\left(1+\Lambda^2/m^2\right)\right]. \tag{3}$$

Adopting the parameterization $|\delta m_h^2| = |\delta^{(\mathrm{SM})}m_h^2 + \delta^{(\varphi)}m_h^2| = D_t m_h^2$, we can find the value of λ_x necessary to suppress δm_h^2 to a desired level (D_t) as a function of m, for any choice of m_h and Λ; examples are plotted in Fig. 1 for $N = 6$.

It should be noted that (in contrast to SUSY) the logarithmic terms in (3) can be relevant in canceling large contributions to δm_h^2. It is important to note that the required value of λ_x decreases as the number of singlets N grows. When $m \ll \Lambda$, the λ_x needed for the amelioration of the hierarchy problem is insensitive to m, D_t or Λ as illustrated in Fig. 1; analytically we find up to terms $\mathcal{O}\left(m^4/\Lambda^4\right)$

$$\lambda_x \simeq N^{-1}\left\{4.8-3\left(m_h/v\right)^2+2D_t\left[2\pi/(\Lambda/\ \mathrm{TeV})\right]^2\right\}\left[1-m^2/\Lambda^2\ln\left(m^2/\Lambda^2\right)\right]. \tag{4}$$

[1] The Λ^2 corrections to m^2 can also be tamed within the full model with additional fine tuning, but we will not consider them here, see [5].

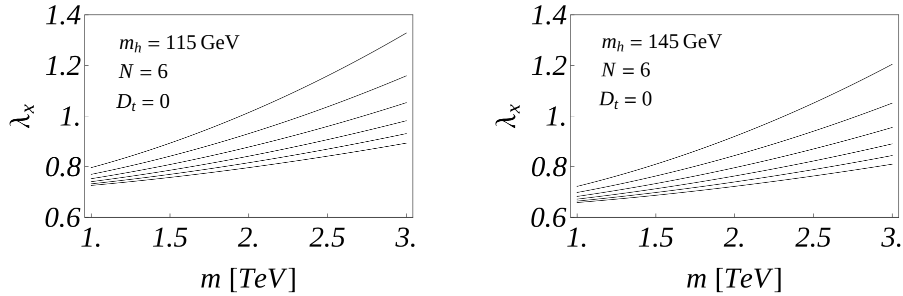

Fig. 1. Plot of λ_x corresponding to $\delta m_h^2 > 0$ as a function of m for $D_t = 0$ and $m_h = 115$ GeV (left panel), and $m_h = 145$ GeV (right panel). The various curves correspond to $\Lambda = 5,\ 6,\ 7,\ 8,\ 9,\ 10$ TeV (starting with the uppermost curve).

Since we consider $\lambda_x \sim \mathcal{O}(1)$, effects of higher order corrections [6] to (1) should be considered as well (see also [7]). In general, the fine tuning condition reads (m_h was chosen as a renormalization scale)

$$\left|\delta^{(\mathrm{SM})} m_h^2 + \delta^{(\varphi)} m_h^2 + \Lambda^2 \sum_{n=1} f_n(\lambda_x, \ldots) \left[\ln(\Lambda/m_h)\right]^n \right| = D_t m_h^2 \,, \qquad (5)$$

where the coefficients $f_n(\lambda_x, \ldots)$ can be determined recursively [6], with the leading contributions being generated by loops containing powers of λ_x: $f_n(\lambda_x, \ldots) \sim \left[\lambda_x/(16\pi^2)\right]^{n+1}$. To estimate these effects we can consider the case where $\delta^{(\mathrm{SM})} m_h^2 + \delta^{(\varphi)} m_h^2 = 0$ at one loop then, keeping only terms $\propto \lambda_x^2$, we find (using [6]), at 2 loops, $D_t \simeq (\Lambda/(4\pi^2 m_h))^2 \ \ln(\Lambda/m_h)$ (note that $N\lambda_x \simeq 4$). Requiring $D_t \lesssim 1$ implies $\Lambda \lesssim 3$ TeV for $m_h \simeq 130$ GeV.

It must be emphasized that in the model proposed here the hierarchy problem is softened (by lifting the cutoff) only if λ_x, Λ and m are appropriately fine-tuned; this fine tuning, however, is significantly less dramatic than in the SM.

3. Dark Matter

The singlets φ_i also provide a natural source for Dark Matter (DM) (for $N = 1$ see [8]). Using standard techniques for cold DM [9] we estimate its present abundance Ω_{DM}, assuming for simplicity that all the φ_i are equally abundant (*e.g.* as in the $O(N)$ limit). Ω_{DM} is determined by the thermally averaged cross-section for φ annihilation into SM final states $\varphi_i\varphi_i \to$ SM SM, which in the non-relativistic approximation, and for $m \gg m_h$, reads

$$\langle \sigma_i v \rangle \simeq \lambda_x^2 / \left(8\pi m^2\right) + \lambda_x^2 v^2 \Gamma_h(2m) / \left(8 m^5\right) \simeq \left[1.73/(8\pi)\right] \lambda_x^2 / m^2 \,. \qquad (6)$$

The first contribution in (6) originates from the hh final state (keeping only the s-channel Higgs exchange; the t and u channels can be neglected since $m \gg m_h$) while the second one comes from all other final states; $\Gamma_h(2m) \simeq 0.48$ TeV$(2m/1\text{ TeV})^3$ is the Higgs boson width calculated for its mass equal $2m$.

From this the freeze-out temperature $x_\mathrm{f} = m/T_\mathrm{f}$ is given by

$$x_\mathrm{f} = \ln\left[0.038\, m_\mathrm{Pl}\, m \left\langle \sigma_i v\right\rangle / (g_\star x_\mathrm{f})^{1/2}\right] , \tag{7}$$

where $g_\star$ is the number of relativistic degrees of freedom at annihilation and m_Pl denotes the Planck mass. In the range of parameters relevant for our purposes, $x_\mathrm{f} \sim 12$–50 and $m \sim 1$–2 TeV, so that this is indeed a case of cold DM. Then the present density of φ_i is given by

$$\Omega_\varphi^{(i)} h^2 = 1.06 \times 10^9 x_\mathrm{f} / \left(g_\star^{1/2} m_\mathrm{Pl} \langle\sigma_i v\rangle \text{ GeV}\right) . \tag{8}$$

The condition that the φ_is account for the observed DM abundance, $\Omega_\mathrm{DM} h^2 = \sum_{i=1}^N \Omega_\varphi^{(i)} h^2 = 0.110 \pm 0.018$ [10], can be used to fix $\langle\sigma_i v\rangle$, which implies a relation $\lambda_x = \lambda_x(m)$ through (6). Using this in the condition $|\delta m_h^2| = D_t m_h^2$, we find a relation between m and Λ (for a given D_t), which is plotted in Fig. 2 for $N = 6$. It should be emphasized that it is possible to find Λ, λ_x and m such that *both* the hierarchy is ameliorated to the desired level *and* such that $\Omega_\varphi h^2$ agrees with the DM requirement (we use a 3σ interval). It also is instructive to mention that the singlet mass (as required by the DM) scales with their multiplicity as $N^{-3/2}$, therefore growing N implies smaller scalar mass, *e.g.* changing N from 1 to 6 leads to the reduction of mass by a factor ~ 15.

4. Natural 2HDM

In this section, we are going to discuss a model that not only ameliorates the hierarchy problem, but also allows for extra sources of CP violation. A simple illustration is provided by the 2-Higgs Doublet Model (2HDM) with softly broken $\mathbb{Z}_2$ symmetry. The scalar potential then reads

$$\begin{aligned} V(\phi_1,\phi_2) \;=\; & -\tfrac{1}{2}\left\{m_{11}^2 \phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 + \left[m_{12}^2\phi_1^\dagger\phi_2 + \text{h.c.}\right]\right\} \\ & + \tfrac{1}{2}\lambda_1\left(\phi_1^\dagger\phi_1\right)^2 + \tfrac{1}{2}\lambda_2\left(\phi_2^\dagger\phi_2\right)^2 + \lambda_3\left(\phi_1^\dagger\phi_1\right)\left(\phi_2^\dagger\phi_2\right) \\ & + \lambda_4\left(\phi_1^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_1\right) + \tfrac{1}{2}\left[\lambda_5\left(\phi_1^\dagger\phi_2\right)^2 + \text{h.c.}\right] . \end{aligned} \tag{9}$$

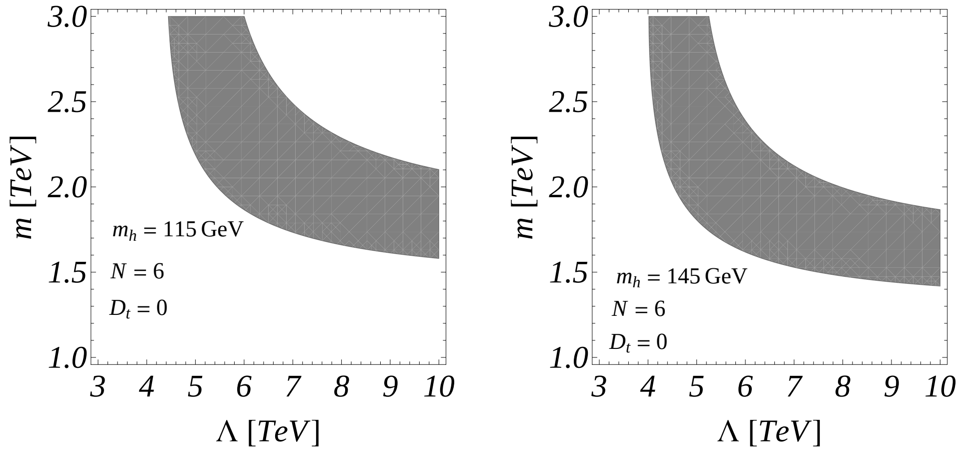

Fig. 2. Allowed regions (gray) in the (m, Λ) plane for $m_h = 115$ GeV and $m_h = 115$ GeV for $D_t = 0$, $N = 6$ and $\sum_{i=1}^{N} \Omega_\varphi^{(i)} h^2 = 0.110 \pm 0.018$ at the 3σ level.

Both doublets develop vacuum expectation values: $\langle \phi_1^0 \rangle = v_1/\sqrt{2}$ and $\langle \phi_2^0 \rangle = v_2/\sqrt{2}$. We assume here that ϕ_1 and ϕ_2 couple to down- and up-type quarks, respectively (the so-called 2HDM II). Then the cancellation of quadratic divergences implies that the following relations must hold [11]

$$\tfrac{3}{2}m_W^2 + \tfrac{3}{4}m_Z^2 + \tfrac{v^2}{2}\left(\tfrac{3}{2}\lambda_1 + \lambda_3 + \tfrac{1}{2}\lambda_4\right) = 3\frac{m_b^2}{c_\beta^2}\,, \tag{10}$$

$$\tfrac{3}{2}m_W^2 + \tfrac{3}{4}m_Z^2 + \tfrac{v^2}{2}\left(\tfrac{3}{2}\lambda_1 + \lambda_3 + \tfrac{1}{2}\lambda_4\right) = 3\frac{m_t^2}{s_\beta^2}\,, \tag{11}$$

where $v^2 \equiv v_1^2 + v_2^2$, $\tan\beta \equiv v_2/v_1$ and we adopt the notation: $s_\theta \equiv \sin\theta$ and $c_\theta \equiv \cos\theta$. We note that when $\tan\beta$ is large, the two right-hand sides can be comparable, implying $\lambda_1 \simeq \lambda_2$.

The cancellation relations (10)–(11) severely restricts the parameter space (see [12] and also [13]), however it turns out that substantial amount of CP violation survives all the constraints imposed. In order to parametrize the magnitude of CP violation, we adopt the U(2) invariants introduced by Lavoura and Silva [14] (see also [15]). We shall here adopt the basis-invariant formulation of these invariants J_1, J_2 and J_3 proposed by Gunion and Haber [16]. As is proven there (theorem no. 4) the Higgs sector is CP-conserving if and only if all J_i are real. On the basis adopted here the invariants read [17]

$$\mathrm{Im}\, J_1 = -\frac{v_1^2 v_2^2}{v^4}(\lambda_1 - \lambda_2)\mathrm{Im}\,\lambda_5\,, \tag{12}$$

$$\begin{aligned}\mathrm{Im}\, J_2 = & -\frac{v_1^2 v_2^2}{v^8}\left[\left((\lambda_1-\lambda_3-\lambda_4)^2-|\lambda_5|^2\right)v_1^4 + 2(\lambda_1-\lambda_2)\mathrm{Re}\,\lambda_5 v_1^2 v_2^2\right.\\ & \left. -\left((\lambda_2-\lambda_3-\lambda_4)^2-|\lambda_5|^2\right)v_2^4\right]\mathrm{Im}\,\lambda_5\,,\end{aligned} \tag{13}$$

$$\mathrm{Im}\, J_3 = \frac{v_1^2 v_2^2}{v^4}(\lambda_1 - \lambda_2)(\lambda_1+\lambda_2+2\lambda_4)\mathrm{Im}\,\lambda_5\,. \tag{14}$$

It is seen that there is no CP violation when $\mathrm{Im}\,\lambda_5 = 0$, see [17] for more details. For a quantitative illustration we plot in Fig. 3 maximal values of the invariants in the $\tan\beta$–$M_{H^\pm}$ plane with all the necessary constraints imposed (see [12] for details), looking for regions which allow for substantial CP violation in spite of the cancellation conditions imposed. As it is seen from the figure for large $\tan\beta$, $\tan\beta \sim 20$–40 such regions exist.

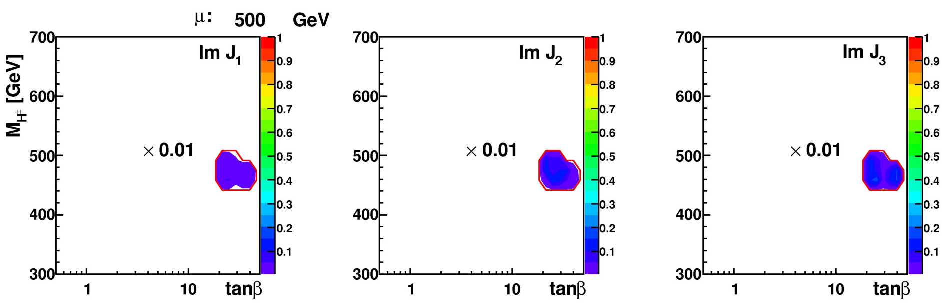

Fig. 3. Absolute values of imaginary parts of the U(2) invariants $|\mathrm{Im}\, J_i|$, for $\Lambda = 4.5$ TeV and $\mu = 500$ GeV. The color (grey-scale) coding in units 10^{-2} is given along the right vertical axis.

Unfortunately, introducing CP violation in the scalar potential we have eliminated the chance for DM candidate within just two Higgs doublet[2], a possible remedy is an extra singlet as discussed in [12].

5. Conclusions

It has been shown that the addition of real scalar singlets φ_i to the SM may soften the little hierarchy problem (by lifting the cutoff Λ to the multi TeV range). At the same time, the scalars serve as realistic candidates for Dark Matter, in that case a consistent setup within present LHC bounds

[2] For a CP-conserving 2HDM with DM candidate being a doublet component see [18].

for the Higgs-boson mass is provided *e.g.* by $N = 6$ scalars with masses $m \sim 1.5$–3 TeV and the UV cutoff $\Lambda \sim 4.5$–10 TeV. In order to accommodate also extra sources of CP violation, a 2 Higgs Doublet Model with cancellation of quadratic divergences has been discussed. It has been shown that experimental and theoretical constraints implemented by the cancellation conditions are consistent with a substantial amount of CP violation originating from the scalar potential. However, in order to provide a candidate for Dark Matter, the model must be supplemented by at least one extra scalar singlet.

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