COSMOLOGY OF MULTI-SINGLET-SCALAR EXTENSIONS OF THE STANDARD MODEL*

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An extension of the Standard Model by extra scalar singlets was considered. Theoretical (unitarity, vacuum stability, triviality) and cosmological (dark matter relic abundance, direct detection experiments, constraints on dark matter self-interaction) constraints were discussed.

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1. Introduction

The Standard Model (SM) of electro-weak interactions does not provide a viable candidate for Dark Matter (DM) [1]. To unravel the DM mystery we need to consider its extensions. Among many existing proposals there is the simplest DM model, constructed by adding a real singlet scalar field to the SM particles. We would like to propose a variation of the singlet scalar extension consisting of an addition of the N extra scalar singlets transforming according to the fundamental representation of $\mathrm{O}(N)$ symmetry group.

In this paper, we confront our model with theoretical bounds (unitarity, stability, triviality) and cosmological data (the amount of DM in the Universe [1] and direct detection XENON 100 experiment [2]). We also note, that conventional collisionless Cold Dark Matter has problems explaining the observed structure of galaxies and significant self-interaction rate provides possible cure (see Sec. 3.4 and [3]).

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2. Model and theoretical constraints

We shall consider the SM extended by addition of N scalars φ_i which are singlets under the SM gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. We assume that the N-component vector $\vec{\varphi}$ transforms according to the fundamental representation of $\mathrm{O}(N)$ — an exact symmetry of the model. All SM fields are singlets under $\mathrm{O}(N)$. For the purpose of providing a DM candidate we impose an additional Z_2 symmetry, under which $\vec{\varphi}$ is odd: $\vec{\varphi} \to -\vec{\varphi}$. The most general, symmetric and renormalizable potential is

$$\begin{aligned} V(H,\vec{\varphi}) \;=\; & -\mu_H^2 H^\dagger H + \lambda_H \left(H^\dagger H\right)^2 + 1/2\mu_\varphi^2 \vec{\varphi}^{\,2} \\ & + 1/4!\lambda_\varphi \left(\vec{\varphi}^{\,2}\right)^2 + \lambda_x H^\dagger H \vec{\varphi}^{\,2}\,. \end{aligned} \tag{1}$$

The Higgs field is a SM doublet with a vacuum expectation value (VEV), $\langle H \rangle = v/\sqrt{2}$ for $v = 246$ GeV. After the symmetry breaking the Higgs-boson mass is $m_h^2 = -\mu_H^2 + 3\lambda_H v^2 = 2\mu_H^2$, as in the SM. The singlet's masses also get a VEV contribution, $m_\varphi^2 = \mu_\varphi^2 + \lambda_x v^2$.

Singlets do not develop a VEV — we would like them to be stable, so the Z_2 must be unbroken with $\mu_\varphi^2 \geq 0$. That implies

$$m_\varphi^2 \geq \lambda_x v^2 \tag{2}$$

which is a significant constraint — light scalars ($m_\varphi \ll v$) must couple very weakly to the SM. Quartic couplings are constrained by the unitarity arguments for longitudinal W boson- and scalar-scattering [4, 5]

$$m_H^2 < (8\pi)/3v^2\,, \qquad \lambda_\varphi < 8\pi\,, \qquad |\lambda_x| < 4\pi\,. \tag{3}$$

Tree level vacuum stability of the scalar potential (1) implies that either all the quartic couplings are positive: λ_H, λ_φ, $\lambda_x > 0$ or λ_x is negative and $\lambda_x^2 < \lambda_\varphi \lambda_H/6 = \lambda_\varphi m_h^2/(12v^2)$ [5].

2.1. Triviality

We require that the during the renormalization group equation (RGE) running, the quartic couplings λ_H, λ_x, λ_φ remain finite up to the cut-off scale Λ. That implies limits on the model parameters, in particular an upper limit on m_h (so-called 'triviality bound').

The RGE for running couplings in our model can be found in [6]. We solve those equations with initial conditions: $\lambda_H(\mu = m_W) = \lambda_{H\,0}$, $\lambda_x(\mu = m_W) = \lambda_{x\,0}$ and $\lambda_\varphi(\mu = m_W) = \lambda_{\varphi\,0}$.

We assume that for a given Λ there is no pole in the evolution of the scalar quartic couplings at energies below Λ. That gives us constraints in the (m_h, Λ) plane depending on initial parameters $\lambda_{x\,0}, \lambda_{\varphi\,0}$ and N — see

the left panel of Fig. 1. Note that the allowed region shrinks as λ_{x0} grows and the upper bound on m_h is getting lower. There is also an asymmetry between the negative and positive branch of λ_{x0} initial conditions, see the right panel of Fig. 1.

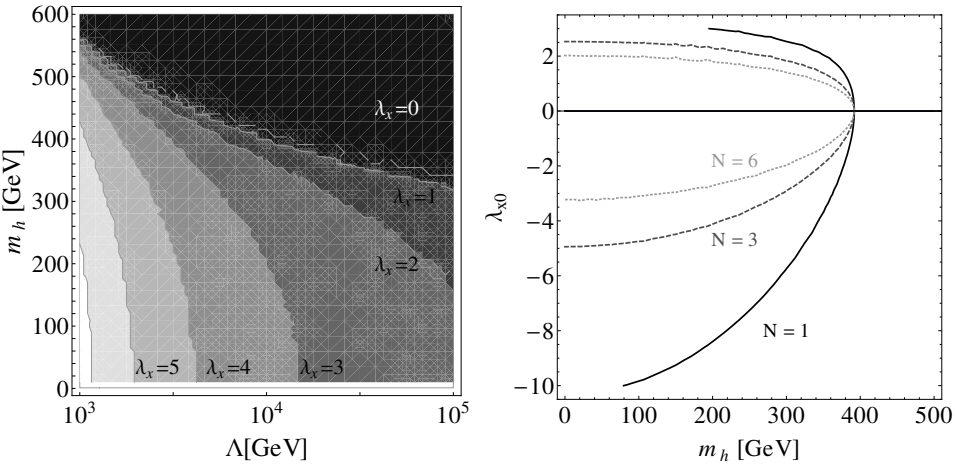


Fig. 1. Left: Triviality upper bound for the Higgs boson as a function of the cut-off Λ for different λ_{x0}, $N = 1$ and $\lambda_{\varphi 0} = 0.1$. Regions above each curve are forbidden. Right: Regions (between curves of the same type) allowed for λ_{x0} by triviality for $\Lambda = 10^4$ GeV, $\lambda_{\varphi 0} = 0.5$.

3. Experimental constraints

3.1. DM relic abundance — Cold Dark Matter (CDM)

Stability of the singlets $\vec{\varphi}$ makes them good DM candidates. To calculate DM relic abundance in the case of compound DM we need in general a set of Boltzmann equations for all component. Because of the O(N) symmetry it actually simplifies to one equation [8]

$$\frac{df}{dT} = \frac{\langle \sigma v \rangle}{K} \left(f^2 - f_{\mathrm{EQ}}^2 \right) , \qquad K(T) = \sqrt{\frac{4\pi^3 g_\star(T)}{45 m_{\mathrm{Pl}}^2}} , \tag{4}$$

where $f \equiv n/T^3$, n is the number density of DM, f_{EQ} is the equilibrium distribution, $g_\star(T)$ — number of relativistic degrees of freedom, m_{Pl} — Planck mass, $\langle \sigma v \rangle$ — the thermally averaged cross-section for DM + DM $\rightarrow$ SM + SM processes [9, 10]. Total abundance of DM, Ω_{DM}, reads

$$\Omega_{\mathrm{DM}}^N = \sum_i \Omega_{\mathrm{DM}}^i = N \Omega_{\mathrm{DM}}^1 , \tag{5}$$

where Ω_{DM}^i is the dark matter relic density from i-th scalar field φ_i.

We solve (4) in the standard CDM case [8] using MicrOMEGAs [11]. From the WMAP data [1] we know that $\Omega_{\rm DM}^{(\rm exp)}h^2 = 0.110 \pm 0.018$ (allowing for 3σ uncertainty) and for a given choice of N, m_h and m_φ we seek λ_x such that this constraint is satisfied, see Fig. 2. Note that in the vicinity of resonance, the annihilation cross-section is enhanced ($m_h \sim 2m_\varphi$), therefore λ_x is suppressed to reach the desired DM abundance.

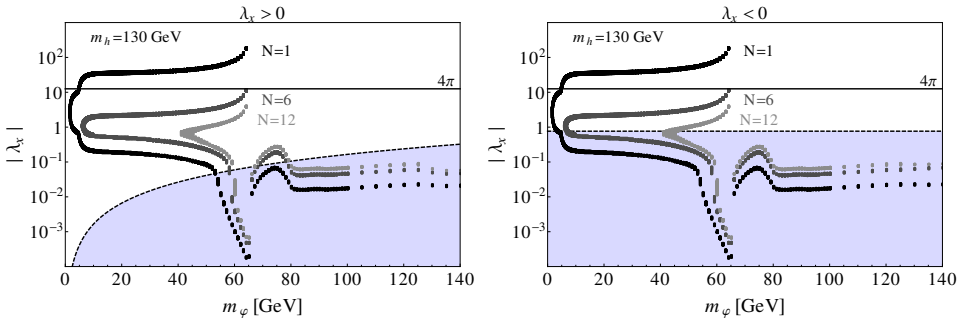


Fig. 2. $|\lambda_x|$ as a function of m_φ for CDM, $m_h = 130$GeV, $\lambda_x > 0$ (left panel) and $\lambda_x < 0$ (right panel). The dark gray, gray and light gray points correspond to $N = 1, 6$ and 12, respectively. The gray (blue) areas in the left and right panels show regions allowed by the consistency condition (2) and the vacuum stability for $\lambda_\varphi = 8\pi$, respectively. The thick black lines show the unitarity limit (3) $|\lambda_x| = 4\pi$.

3.2. DM relic abundance — Feebly Interacting Dark Matter (FIDM)

In the CDM model we achieve equilibrium between certain particles and the SM species just to lose it ('freeze-out') while the Universe cools. What happens if the DM particles interact with SM so feebly ($\lambda_x < 10^{-9}$), that equilibrium with the SM species is never achieved [12]?

In the following, we will assume the number density f was negligible at the Big Bang: $\lim_{T\to\infty} f(T) = 0$. Having f determined by the Boltzmann equation (4) (see Fig. 3, left panel) one can get the DM relic abundance

$$\Omega_{\rm DM}h^2 = m_\varphi n_\varphi/\rho_{\rm crit} = m_\varphi T_\gamma^3 f_\varphi/\rho_{\rm crit}\,, \tag{6}$$

where T_γ is the present photon temperature, $\rho_{\rm crit}$ is the critical density. For solutions satisfying (6) see Fig. 3, right panel.

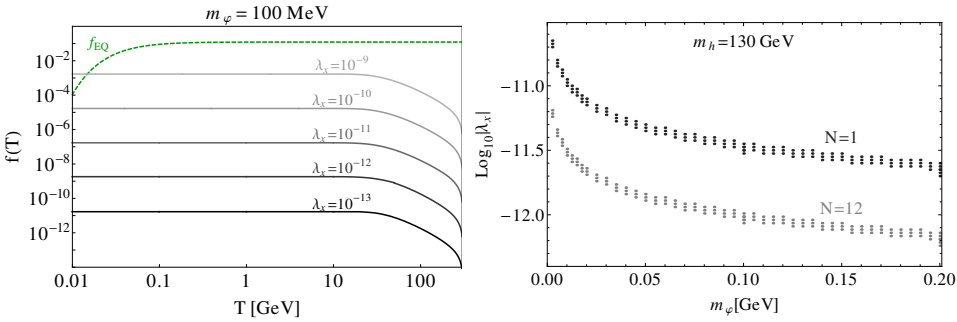


Fig. 3. Left: Solutions to the Boltzman equation (4) in the case of FIDM for a single scalar of mass $m_\varphi = 100$ MeV, $\lambda_x = 10^{-13}$ (bottom curve), ... , 10^{-9} (top curve) for $m_h = 130$ GeV. Dashed (green) curve is the equilibrium distribution f_{EQ}. Right: FIDM solutions satisfying the relic abundance condition for $m_h = 130$ GeV, $N = 1$ (darker points), $N = 12$ (lighter).

3.3. DM direct detection

The direct detection rate of $\vec{\varphi}$ is determined by the cross-section of $\vec{\varphi}$ scattering off nuclei (see the Feynman diagram in Fig. 4), which can be found *e.g.* in [9] for $N = 1$ and in [7] for compound DM case. In Fig. 4 we show allowed regions in the (m_h, m_φ) plane that remain after imposing limits on the elastic scattering of DM particles off nucleons from XENON100 (the strongest limits on $\sigma_{\mathrm{DM}\,N\to\mathrm{DM}\,N}$ in the mass range of our interest, [2]). The white band corresponds to the resonance region seen in Fig. 2 for $m_h \sim 2m_\varphi$ in which the annihilation is amplified and the coupling λ_x is suppressed. Since the XENON100 data start at $m_\varphi = 5$ GeV, therefore, the vertical strip of masses below 5 GeV is also allowed.

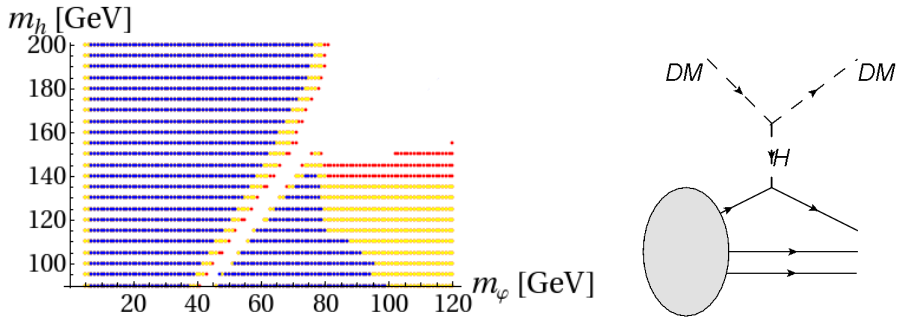


Fig. 4. Left: XENON100 constraint on (m_h, m_φ) combined with DM abundance in the case of CDM (black (blue) region forbidden for $N = 1$, black (blue) and gray (yellow) for $N = 6$, black (blue), gray (yellow) and dark gray (red) for $N = 12$). Right: Elastic scattering of $\vec{\varphi}$ off a nucleon.

3.4. Self-interacting DM

The standard ΛCDM model is facing some difficulties if compared with observations. High-resolution N-body simulations have shown that the model generates cusps of the DM density distribution in central regions of galaxies [13] and the number of subhalos predicted by the model turns out to be larger than the observed [14] number. Self-interacting DM (SIDM) was proposed by Spergel and Steinhardt [3] to cure those problems.

The key feature of SIDM is that the mean free path of DM particles should be between 1 kpc and 1 Mpc in regions where the dark matter density is about 0.4 GeV/cm^3. In terms of the unit mass cross-section the Steinhardt and Spergel hypothesis requires that

$$2.05 \times 10^3 \text{ GeV}^{-3} \leq \sigma_{\text{DM+DM}\to\text{DM+DM}}/m_{\text{DM}} \leq 2.57 \times 10^4 \text{ GeV}^{-3}\,. \quad (7)$$

For different N this condition implies a relation between $\lambda_x, \lambda_\varphi$ and m_φ illustrated in Fig. 5 as an allowed region in the $(\lambda_\varphi, m_\varphi)$ space. In the plots λ_φ varies form 0 up to its maximal value allowed by unitarity, *i.e.* 8π. Similar results were obtained in other versions of scalar DM models [15,16]. As observed from Fig. 2, m_φ consistent with the Spergel and Steinhardt condition is so small that it is not compatible with the CDM case. The only viable option is the FIDM, therefore $\lambda_x \sim 0$ (only the first panel in Fig. 5 is consistent with the DM abundance). In this case, from Fig. 3 one can see that $m_\varphi \sim 0.01$–0.15 GeV corresponds to $\lambda_x \sim 10^{-10}$–10^{-12} for $N = 1$–12.

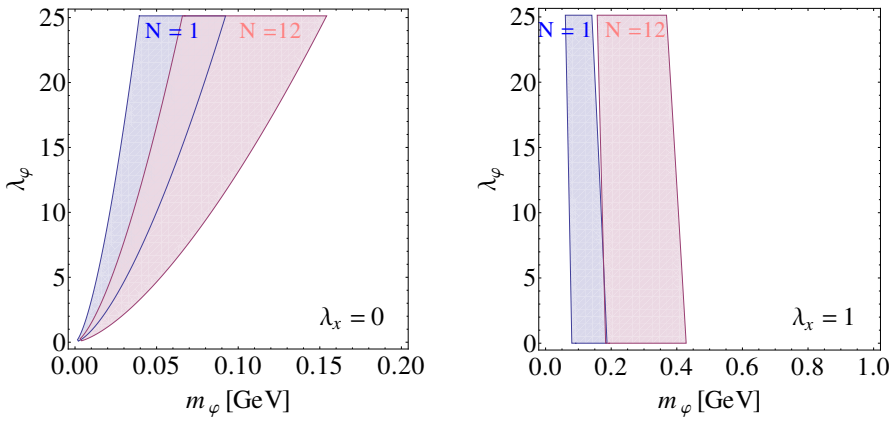


Fig. 5. Regions in $(\lambda_\varphi, m_\varphi)$ space allowed by the Steinhardt and Spergel constraint for $N = 1$ (darker region) and $N = 12$ (lighter region) for $\lambda_x = 0$ (left), 1 (right).

4. Summary and conclusions

We have considered an extension of the Standard Model by an addition of N real scalar singlets φ with O(N) symmetry that are candidates for Dark Matter. We have discussed theoretical and experimental (cosmological) constraints on the model parameters. The XENON100 direct DM detection experiment gives no constraints on the model in the FIDM case (too small coupling of DM to the SM), but constrain strongly the CDM solution (see Fig. 4). We have shown that Steinhardt and Spergel solution of the DM density distribution problem within the singlet scalar SM extension requires feebly interacting DM.

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