

## COSMOLOGICAL MODELS AND MISUNDERSTANDINGS ABOUT THEM\*

ANDRZEJ KRASIŃSKI

The Nicolaus Copernicus Astronomical Centre, Polish Academy of Sciences  
Bartycka 18, 00-716 Warsaw, Poland

*(Received October 10, 2011)*

Advantages of inhomogeneous cosmological models that are exact solutions of Einstein's equations over linearised perturbations of homogeneous models are presented. Examples of effects that can be described in the inhomogeneous ones are given: the non-repeatable light paths, the observed anisotropies in the cosmic microwave background, the redshift drift and the maximum diameter distance. Criticisms of inhomogeneous models that are based on misunderstandings or fallacious reasonings are pointed out and corrected; these include the “weak singularity”, the positivity of deceleration “theorem”, the “pathology” of redshift behaviour at the “critical point” and the alleged necessity of the bang time to be constant.

DOI:10.5506/APhysPolB.42.2263

PACS numbers: 98.80.Jk, 98.80.Es, 04.20.Cv

### 1. What is “cosmological principle”?

The “cosmological principle” derives from Copernicus' discovery that can be stated as follows: when the origin of coordinates is placed in the centre of the Sun, the description of the motions of planets becomes simpler. In later centuries further discoveries indicated that the position of the Sun in the Universe is not in any way privileged. Ultimately, this conclusion assumed a fundamentalist form: all positions in space are equivalent; every observer will see the same large-scale image of the Universe.

This “cosmological principle” is not a summary of knowledge based on observations, but a postulate. Just as all other hypotheses, it requires observational verification.

Progress in observing technology, with still farther regions coming into view, produced no justification for this principle: only more structures were becoming visible. Nevertheless, we are told that the Universe is homogeneous “at a sufficiently large scale”. The definition of this “sufficient scale”

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\* Presented at the XXXV International Conference of Theoretical Physics “Matter to the Deepest”, Ustroń, Poland, September 12–18, 2011.

is far from precise (“a few” hundred megaparsecs). This is the size of the “fundamental cell” of the Universe, which should be repetitive — but it is so large that details of mass distribution at its edges and beyond are fuzzy.

## 2. Why consider generalised cosmological models?

Traditionally, structures are described by solutions of Einstein’s equations linearised around homogeneous models. This method has problems:

1. It is impossible to determine the radius of convergence of a series of approximations when we know only the first and sometimes the second term — as is always the case in cosmology.
2. In practice, one demands that the parameter of a perturbative calculation is smaller than 1 (but this is not a sufficient condition). In cosmology, there are two such parameters: the density contrast  $\Delta\rho/\rho_b$  and the curvature contrast  $\Delta R/R_b$  ( $\rho_b$  is the mass density in the background model,  $\Delta\rho$  is the difference between the value of density at the location considered and  $\rho_b$ ,  $R_b$  is the curvature of a 3-dimensional space of constant time in the background model,  $\Delta R$  is the analogue of  $\Delta\rho$ ). *Both* must be small. The curvature contrast alone is not any measure of goodness of approximation [1]. But  $|\Delta\rho/\rho_b| < 1$  is not fulfilled in most objects considered in cosmology:

	Star	Globular cluster	Galaxy	Virgo Cluster	Great Attractor	Void
$\Delta\rho/\rho_b$	$1.5 \times 10^{29}$	$2 \times 10^5$	$6 \times 10^4$	190	0.6	−0.9

3. Among the solutions of the linearised Einstein equations there are such that are not approximations to any exact solution [2].

Inhomogeneous models are not alternatives, but *generalisations* that reduce to the traditional ones in the limit of spatial homogeneity. One should not expect that observations will tell us that one class is fine, and the other should be rejected. The relation is similar to that between a globe and a map of a region of the Earth. A globe portrays the Earth as a perfect sphere, but a map of a small area will show mountains and other features.

## 3. Geometry of the cosmological models

### 3.1. The Robertson–Walker (RW) models

The metric of this model (Fig. 1) follows from *assumed symmetries of spacetime*

$$ds^2 = dt^2 - S^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) \right]. \quad (3.1)$$

If the matter in the model has zero pressure, then  $S(t)$  obeys

$$S_{,t}^2 = 2GM/(c^2 S) - k + \Lambda S^2/3, \quad (3.2)$$

where  $k$  and  $M$  are arbitrary constants and  $\Lambda$  is the cosmological constant.

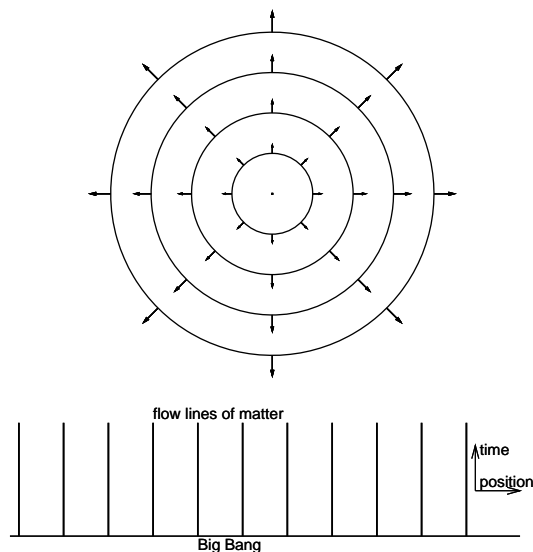


Fig. 1. Expansion in the RW models. Upper picture: The velocity of expansion is proportional to the distance from the observer at any fixed instant, but changes with time. Lower picture: The initial explosion occurs simultaneously in the coordinates of (3.1)  $\Rightarrow$  all matter particles have the same age at any later instant.

The *redshift*  $z$  is defined by

$$z = \frac{\text{emitted frequency}}{\text{observed frequency}} - 1 \equiv \frac{\nu_e}{\nu_o} - 1. \quad (3.3)$$

For the RW models the redshift is

$$z = S(t_o)/S(t_e) - 1. \quad (3.4)$$

The *luminosity distance* between an observer at  $(t, r) = (t_o, 0)$  and the source of light at  $(t_e, r_e)$  is defined as the flat-space distance to a source that would give the same observed flux of radiation, corrected for the recession velocity of the source. In the RW models this is

$$D_L = r_e S(t_e)(1 + z)^2. \quad (3.5)$$

The observable quantities are  $z$ , the Hubble coefficient at  $t_o$

$$H_0 = S_{,t}/S|_{t=t_o} \quad (3.6)$$

and three dimensionless parameters

$$(\Omega_m, \Omega_k, \Omega_\Lambda) \stackrel{\text{def}}{=} \frac{1}{3H_0^2} (8\pi G\rho_0, -3k/S_0^2, \Lambda)|_{t=t_o} \quad (3.7)$$

that obey  $\Omega_m + \Omega_k + \Omega_\Lambda \equiv 1$ . In these variables

$$D_L(z) = \frac{1+z}{H_0\sqrt{\Omega_k}} \sinh \left\{ \int_0^z \frac{\sqrt{\Omega_k} dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_k(1+z')^2 + \Omega_\Lambda}} \right\}. \quad (3.8)$$

This formula applies also with  $\Omega_k < 0$  ( $\sinh(ix) \equiv i \sin x$ ) and  $\Omega_k \rightarrow 0$ ; the latter is the current favourite model of a majority of cosmologists.

### 3.2. The Lemaître–Tolman (LT) model

The metric of the Lemaître–Tolman [3, 4] model (Fig. 2) is

$$ds^2 = dt^2 - \frac{R_{,r}^2}{1 + 2E(r)} dr^2 - R^2(t, r) (d\vartheta^2 + \sin^2 \vartheta d\varphi^2), \quad (3.9)$$

where  $R(t, r)$  obeys (from the Einstein equations)

$$R_{,t}^2 = 2E(r) + 2M(r)/R - \Lambda R^2/3. \quad (3.10)$$

The functions  $M(r)$  and  $E(r)$  are arbitrary. The integral of (3.10) contains one more arbitrary function,  $t_B(r)$  — the “timetable” of the initial explosion. For example, when  $E = 0 = \Lambda$  the solution of (3.10) is

$$R = (9M/2)^{1/3} (t - t_B(r))^{2/3}. \quad (3.11)$$

The RW limit of LT is

$$M = \text{const.} \times r^3, \quad E = -kr^2/2, \quad t_B = \text{const.}, \quad R = rS(t). \quad (3.12)$$

The LT model is spherically symmetric around one centre. It does not represent the whole Universe, but a single structure embedded in an RW background. One RW background can contain several LT regions.

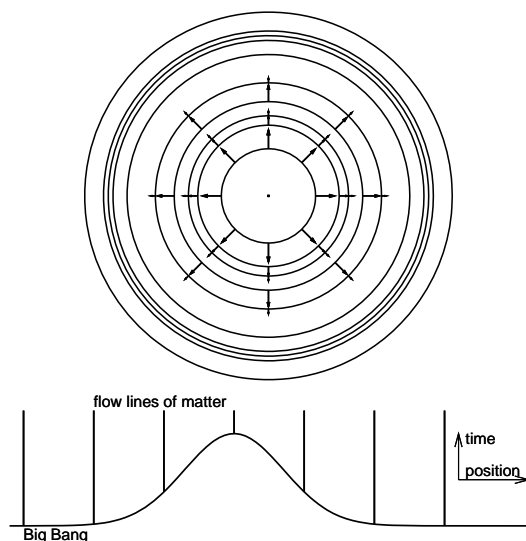


Fig. 2. Expansion in the LT model. Upper picture: The velocity of expansion is not correlated with the position of a matter shell. The spatial distribution of velocity is an arbitrary function of  $r$ . Lower picture: The initial explosion is, in the coordinates of (3.9), non-simultaneous  $\Rightarrow$  the age of matter particles depends on  $r$ . The “timetable” of the initial explosion is a second arbitrary function of  $r$ .

#### 4. Explaining away “accelerated expansion” of the Universe by inhomogeneous matter distribution

The hypothesis of accelerated expansion of the Universe arose from observations of type Ia supernovae. Such a supernova is the final stage of evolution of a white dwarf in a binary system. The maximal absolute luminosity of all supernovae of this class is assumed to be the same.

By measuring the redshift of these supernovae, one can calculate the distance to them, *assuming that the Universe we live in is RW with known parameters, and so the Hubble law is exactly fulfilled* — and then calculate the expected flux of radiation through a unit surface area to be observed on the Earth. It turned out that the actually observed maximal flux is smaller than expected, as if the supernovae were farther from us than we thought. In order to explain this discrepancy, *the previously used Universe model had to be modified. Attempts to fit various RW models to the observed luminosities led to the best fit achieved when  $\Omega_k = 0$ ,  $\Omega_m \approx 0.3$ ,  $\Omega_\Lambda \approx 0.7$ .*

A positive value of  $\Omega_\Lambda$  means that the Universe has to expand with acceleration. This gave rise to the puzzle of “dark energy” (that would propel the acceleration) and to research programs aimed at solving it. But is this the only possible explanation of the “dimming of supernovae”?

The interpretation of observations is possible only when *the background geometry of the space is pre-assumed*. The statements in italics in the text above indicate the points at which this prior assumption intervened<sup>1</sup>.

This last remark must be exactly understood because its mistaken understanding created a false legend. What we have to explain is the relation between the observed luminosity of the type Ia supernovae and their redshifts, *i.e.* we have to reproduce the function  $D_L(z)$  in our model. *The “accelerated expansion” of the Universe is not an observed phenomenon, but an element of interpretation of observations, forced upon us by the RW models.* If we can re-create the observed  $D_L(z)$  in a decelerating inhomogeneous model, then the “accelerated expansion” becomes an illusion.

This can be achieved using the LT model [5]. We assume that the observer is at the symmetry centre and that  $E/r^2 = E_0 = \text{const.}$ , (the same  $E$  as in the RW model). For  $t_B(r)$  take the implicit definition

$$D_L(z) = \frac{1+z}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}}, \quad (4.1)$$

where  $\Omega_m = 0.3$  and  $\Omega_\Lambda = 0.7$ , and  $H_0$  is the present value of the Hubble coefficient. The trick is that  $z$  and  $H_0$  are taken from observations, but the definition of  $D_L(z)$  is no longer (3.5). Instead,  $D_L(z) = (1+z)^2 R(t, z)$  is taken from an LT model with  $\Lambda = 0$ , and this defines  $t_B(r)$ . Equations for  $t_B(r)$  can now be solved numerically.

Comparison with (3.8) shows that (4.1) defines the same relation between  $D_L$  and  $z$  as in the “standard” RW model with  $\Omega_k = 0$ ,  $\Omega_m = 0.3$  and  $\Omega_\Lambda = 0.7$ . However, we achieved this with  $\Lambda = 0$ , *i.e.* with decelerated expansion — as dictated by the laws of gravitation that we all know. Had we used the LT model rather than RW to interpret the observations, there would be no need for the “dark energy” and “accelerated expansion”.

Using this LT model one can explain the “dimming of supernovae” in an intuitively clear way. With a non-constant  $t_B$  of suitable shape (see Fig. 3), each LT matter shell that intersects our past light cone is older by  $\Delta t$  than a  $\Lambda = 0$  RW shell that would intersect the light cone at the same point. Therefore, at this intersection, the LT shell expands slower than an RW shell would do. The  $\Delta t$  increases toward the past, and so does the difference between the expansion velocities. In this way the LT model imitates the acceleration of expansion relative to the  $\Lambda = 0$  RW model.

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<sup>1</sup> This is, in fact, a vicious circle. We must assume a model to interpret the observations, but then we use the observations to determine the model. Efforts to break free from this circle are under way, but so far have not led to generally usable results.

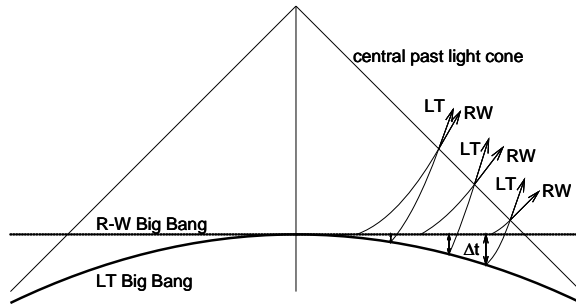


Fig. 3. One of the ways of imitating accelerated expansion in the LT model: via a non-simultaneous Big Bang. Explanation in the text.

## 5. Other results of relevance to inhomogeneous models

### • Non-repeatable light paths

Generic light rays sent from the same source at different times to the same observer pass through different sequences of intermediate matter particles [6] (Fig. 4). As a consequence, the observer should see distant objects drift across the sky. Under most favourable conditions, the drift rate would be  $10^{-7}$  to  $10^{-6}$  arc second per year, but should be detectable after a few years of monitoring a given source, using devices that are already under construction. This drift does not exist in the RW models.

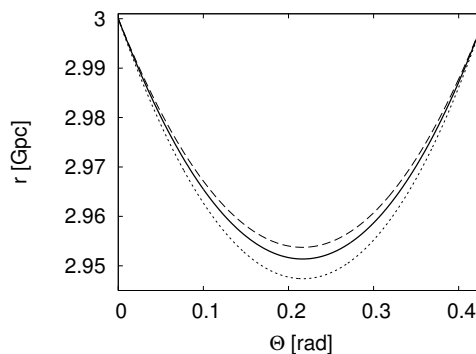


Fig. 4. Light rays sent between a given light source (at right) and a given observer (at left) at different times do not proceed through the same sequence of intermediate world lines. The graph shows three such rays projected on the same space of constant time along the flow lines of the cosmic medium.

### • Anisotropies of temperature of the CMB radiation

Inhomogeneities in mass distribution in the path of light rays cause directional variations of temperature of the CMB radiation. Several investigations [7] showed that no variations larger than  $10^{-6}$ – $10^{-5}$  should be

expected. This agrees with the measured values. Thus, the high isotropy of the CMB radiation does not imply that we live in an RW model — the interaction between mass inhomogeneities and light is simply very weak.

### • Redshift drift

As the Universe evolves, the redshifts of astronomical objects change with time. For the  $\Lambda$ CDM model  $\Delta z > 0$  for  $z < 2$ . For the giant void models (see next section)  $\Delta z < 0$  is expected for all  $z$ . Thus, a detection of a negative redshift drift for all  $z$  would be a proof against dark energy. However, the converse is not true, as there are Gpc-scale inhomogeneous models that also have  $\Delta z > 0$  for low  $z$  [8].

### • Maximum of the diameter distance

In every model that begins with a Big Bang (RW included!) the observed angular diameter of distant objects decreases with distance up to a certain location, and then increases as the observations approach the Big Bang. The position of the minimum puts constraints on a model [9] and is a consistency check that may rule out some models.

## 6. Errors and misconceptions

Astrophysicists are unusually tolerant toward a loose approach to mathematics. Papers written in such a style planted errors in the literature, which were then uncritically cited and came to be taken as established facts. In this section, characteristic examples of misconceptions are presented (marked by ■) together with their explanations (marked by \*).

■ The LT models that explain away dark energy using matter inhomogeneities contain a “*weak singularity*” at the centre [10], where the scalar curvature  $R$  has the property  $g^{\mu\nu} R_{;\mu\nu} \rightarrow \infty$ .

\*  $g^{\mu\nu} R_{;\mu\nu} \rightarrow \infty$  is not a singularity by any accepted criterion [11]. It only implies a discontinuity in the gradient of mass density — a thing common in Nature (*e.g.* on the surface of the Earth). At the centre,  $g^{\mu\nu} R_{;\mu\nu} \rightarrow \infty$  implies a conical profile of density — also a nonsingular configuration.

■ Decelerating inhomogeneous models with  $\Lambda = 0$  cannot be fitted to the  $D_L(z)$  relation that implies acceleration in  $\Lambda$ CDM. This is because the following equation prohibits  $q_4 < 0$  [12]

$$H^2 q_4 = 4\pi\rho/3 + 14\sigma^2/15 \quad (6.1)$$

( $q_4$  is the deceleration parameter,  $H$  is the Hubble parameter,  $\rho$  and  $\sigma$  are the density and shear of the cosmic medium).

\* Equation (6.1) is based on approximations that are not explicitly spelled out [11]. An approximate equation cannot determine the sign of anything.



Refs. [5, 13, 14] provide explicit counterexamples to (6.1). If the approximations are taken as exact constraints imposed on the LT model, they imply the vacuum (Schwarzschild) limit. Moreover, the  $q_4$  of (6.1), although it coincides with the deceleration parameter in the RW limit, is not a measure of deceleration in an inhomogeneous model.

■ There is a “*pathology*” in the LT models that causes the redshift-space mass density to become infinite at a certain location (called “*critical point*”) along the past light cone of the central observer [10].

\* The “critical point” is the apparent horizon (AH), at which the past light cone of the central observer begins to re-converge toward the past. This re-convergence had long been known in the RW models [15, 16], and the infinity in density is a purely numerical artefact — a consequence of trying to integrate past AH an expression that becomes 0/0 at the AH. Ways to handle this problem are known [14, 17, 18].

■ Fitting the LT model to cosmological observations, such as number counts or the Hubble function along the past light cone, results in predicting a *huge void*, at least several hundred Mpc in radius, around the centre (too many papers to be cited, literature still growing).

\* The implied void is a consequence of handpicked constraints imposed on the arbitrary functions of the LT model, for example a constant bang time  $t_B$ . With no *a priori* constraints, the giant void is not implied [14].

■ The *bang time function must be constant* because  $dt_B/dr \neq 0$  generates decaying inhomogeneities, which would have to be “huge” in the past, and this would contradict the predictions of the inflationary models (private communication from the referees of [14]).

\* The LT models are not supposed to apply prior to the emission of CMB because the pressure in them is zero. Thus, their predictions for times close to the Big Bang cannot be taken literally. Moreover, the occurrence of inflation is not in any way proven. Inflationary models are just one of hypotheses that compete for observational confirmation. Using them to justify or reject some other hypotheses is dogmatic and unscientific.

## 7. A brief conclusion

The theory of relativity has much more to offer to cosmology than the simplistic RW models found 90 years ago. Relativistic cosmology made a lot of progress since then. The inhomogeneous models allow us to explain most of the observed phenomena without introducing any “new physics” (like “dark energy”). The alleged deficiencies of the LT models follow from hastily contrived reasonings that contain errors in computation or in interpretation of the results.

This article is based on several years of collaboration with Krzysztof Bolejko, Marie-Noëlle Célérier and Charles Hellaby. The research for it was partly supported by the Polish Ministry of Education and Science grant no N N202 104 838.

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