# COSMIC TOPOLOGY AFFECTS DYNAMICS*

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The role of global topology in the dynamics of the Universe is poorly understood. Along with observational programmes for determining the topology of the Universe, some small theoretical steps have recently been made. Heuristic Newtonian-like arguments suggest a topological acceleration effect that differs for differing spatial sections. A relativistic space-time solution shows that the effect is not just a Newtonian artefact.

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## 1. Rem quoque præcipuam, hoc est mundi formam

The shape of the Universe has long been considered to be an important subject of study: *Rem quoque præcipuam, hoc est mundi formam, ac partium eius certam symmetriam non potuerunt invenire, vel ex illis colligere* [1] (third page in preface, Ref. [1].) One of the aspects of "shape", the global geometrical property *geometria situs* [2] is now known as "topology". Non-trivial topology of spatial sections of the Universe has been discussed prior to [3] and since the beginnings of relativistic cosmology (*e.g.*, [5, 6, 7, 8, 10]), along with the curvature of the spatial sections. However, topology has generally been considered to be unrelated to dynamics. For example, Robertson [10] "tacitly assumed" multiple connectedness for positively curved space while commenting that "we are still free to restore" simple connectedness and that the topology should be determined empirically. More recent work has focused on observational estimates of global topology where observations are interpreted within the family of exactly homogeneous Friedmann–Lemaître–Robertson–Walker (FLRW) models (*e.g.* [11, 12, 13, 14, 15, 16] and references therein). One of the models that has gained particular interest in order

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to match the cosmic microwave background temperature fluctuations observed in the Wilkinson Microwave Anisotropy Probe (WMAP) data, is the Poincaré dodecahedral space model [17, 18, 19, 20, 21, 22, 23, 24].

## 2. Topological acceleration: Newtonian-like derivations

Neverthless, a Newtonian-like argument, motivated as a weak-field approximation of would-be relativistic space-time models, shows that the addition of a massive particle to a homogeneous background implies an acceleration effect dependent on the global topology of the spatial section [25]. A negligible mass test particle displaced from the massive particle is subject to accelerations from images of the massive particle seen in approximately opposite directions in the covering space, with approximately equal amplitudes (Fig. 1). These nearly cancel, but not quite. For a $\mathrm{T}^1 := \mathbb{R}^2 \times \mathrm{S}^1$ spatial section, the resulting *topological acceleration* is linear to first order in $x/L$, where $x$ is the test particle's displacement and $L$ is the length of a closed spatial geodesic in the $\mathrm{S}^1$ direction.

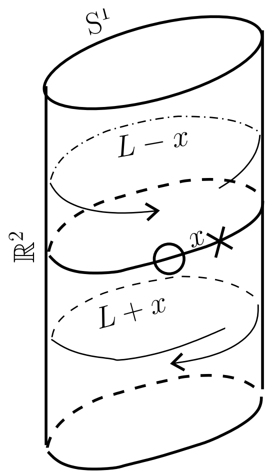


Fig. 1. $\mathrm{T}^1$ spatial section, embedded and projected for convenience, showing how a negligible mass test particle ("X") at distance $x$ from a massive particle ("O") is subject to Newtonian accelerations from the two nearest topological images of the "immediately" nearby copy of the massive object (Sec. 3). See Fig. 1 of Ref. [26] for the compact Schwarzschild-like, relativistic $\mathrm{T}^1$ model (Sec. 3.).

For a test particle displaced in a random direction from the massive particle in an exact $\mathrm{T}^3$ spatial section, the linear effects from the topological images of the massive particle in many different directions cancel [25], leaving a topological acceleration effect that is cubical in $x/L$ [27]. The effect is again linear when a $\mathrm{T}^3$ spatial section has varying side lengths [25].

Similar Newtonian-like calculations in positively curved spaces are less trivial. For the linear effect to cancel, the fundamental domain presumably needs to have several closed spatial geodesics of the same length and in different directions, in such a way that they can cancel the main components of each other's topological accelerations. There are three well-proportioned, multiply connected, positively curved spaces: the octahedral space $S^3/T^*$, the truncated cube space $S^3/O^*$, and the Poincaré dodecahedral space favoured observationally, $S^3/I^*$. The linear component of the topological acceleration effect again cancels in the octahedral space and the truncated cube space [27]. In the Poincaré space, not only does the linear component cancel, but the cubical component cancels too, leaving a topological acceleration effect that is fifth order in $x/R_C$ ($R_C$ is the radius of curvature) [27].

Thus, the topological acceleration effect appears to mark the Poincaré space — previously selected observationally as one giving one of the best matches to the WMAP data — as being unique from a theoretical point of view.

## 3. Topological acceleration: relativistic

Are the heuristic, Newtonian-like derivations of the topological acceleration effect valid relativistically? A first step in investigating this question is to study the compact Schwarzschild-like solution of the Einstein equations found by Korotkin and Nicolai [28]. Outside of the event horizon, this space-time has $T^1$ spatial sections. Consider a low-velocity test particle that is far $x \gg GM$ from the black hole's event horizon $GM$ (in Weyl coordinates, not Schwarzschild coordinates; $G$ is the gravitational constant), in a model where the spatial geodesic length $L$ is also much greater than the test particle's distance from the black hole centre, *i.e.* $0 < GM \ll x \ll L$. If the heuristic, Newtonian-like derivation is correct, then the test particle in this case should be subject to a four-acceleration whose spatial component gives the result found earlier. This is indeed the case [26]. Numerically, for low-velocity test particles, the linear expression $4\zeta(3)GML^{-3}\,x$, where $\zeta(3)$ is Apéry's constant, is a good approximation, to within $\pm10\%$, over $3\,\mathrm{h}^{-1}$ Mpc $\lesssim x \lesssim 2\,\mathrm{h}^{-1}$ Gpc, if the massive particle is at a cluster scale, $M \sim 10^{14} M_\odot$, and the closed spatial geodesic length is $L \sim 10$ to $20\,\mathrm{h}^{-1}$ Gpc [26].

## 4. Prospects

It would be good to perform relativistic calculations of topological acceleration in expanding, FLRW-like models of $T^3$, $S^3/T^*$ $S^3/O^*$, and $S^3/I^*$ spatial sections. This is not just needed to compare with astronomical observations, but also for theoretical work in early universe studies. This is needed independent of space-time theories that extend beyond four dimensions. Exact solutions of the Einstein equations for $S^3/T^*$, $S^3/O^*$, and $S^3/I^*$ models containing a massive particle above a homogeneous background seem unlikely to exist, so this would probably require numerical work. Numerical experience with $N$-body simulations calculated using Newtonian gravity on an FLRW background expanding universe model (*e.g.* [29]) shows that these are not easy. Equivalent numerical calculations that are relativistic and background-free are unlikely to be easier, but may be unavoidable.

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