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# DISTANCE DUALITY IN DIFFERENT COSMOLOGICAL MODELS*

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At cosmological scales, one can actually measure two types of distances: luminosity distance $d_{\mathrm{L}}$ and angular diameter distance $d_{\mathrm{A}}$. Within General Relativity, providing there are no processes eliminating photons from the beam, these two distances are related by the so-called distance duality relation. In this paper we used the measurements of the angular diameter distance of 38 cluster of galaxies by Bonamente *et al.* together with our own fits on the latest Union2 compilation of supernovae to test the distance duality relation in different cosmological models invoked to explain accelerating expansion of the Universe. Our results demonstrate that distance duality violation parameter $\eta(z)$ does not depend on the cosmological model assumed, but considerably depends on assumptions about mass density distribution profile of the cluster. Maximum likelihood estimates of $\eta$ might be interpreted as the distance duality violation. However, this effect is more pronounced for isothermal models of clusters than for the models based on hydrostatic equilibrium. This suggests that more sophisticated and accurate modeling of clusters mass density profiles is needed before the X-ray + SZ technique becomes competitive to other methods of measuring distances.

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## 1. Introduction

Last few decades brought a real revolution in cosmology. It is most remarkably manifested by discovery of the accelerating expansion of the Universe, which means that about 70% of the content of the Universe is in the form of the so-called dark energy. More than a decade of intensive research focused on the accelerating expansion of the Universe brought the consensus that the puzzle is real. This phenomenon was first discovered in Hubble diagram for the Type Ia supernovae (SNIa) [1] and now is strongly supported by the data from a large number of independent studies of SNIa [2], cosmic microwave background radiation (CMBR) anisotropies and baryon acoustic oscillations imprinted in the large scale structure power spectrum [4]. The fact that 70% of the content of the Universe is completely unknown to us has far reaching consequences to both cosmology and fundamental physics — it is really a revolution. However, this revolution was only possible because of the advances in cosmic distance determinations going far beyond classical distance tracers such like cepheids. The story has additional complication because, at cosmological scales, space-time can no longer be treated as Minkowski space but is rather the Friedman–Robertson–Walker space-time. As a consequence, there is some ambiguity in the notion of the distance. Namely, besides the distance measure suggested by the metric (often called the comoving distance), which is not directly observable, one uses (observationally motivated) luminosity distance $d_{\rm L}$ and angular diameter distance $d_{\rm A}$. Fortunately, these distances are related to each other by the so-called reciprocity [5] relation $d_{\rm L}(z) = (1+z)^2 d_{\rm A}(z)$. Bassett and Kuntz [6] were the first to pose the question: what if the reciprocity relation is violated? The relation could be violated if there exist processes eliminating photons from the beam on the way between the source and the observer, or — which is the most extreme case — if gravity is not described by a metric theory and photons do not follow null geodesics. Contemplating such cases is not purely academic since the first concern that comes to ones mind is whether the apparent supernovae dimming might be caused by unaccounted extinction or axion–photon mixing. Similarly, there are suggestions that perhaps gravity should be modified at galactic and cosmological scales (*e.g.* MOND and its relativistic extensions [7]).

After [6] various tests of Etherington reciprocity relation have been performed: by Uzan *et al.* [8] and then by other authors [9]. Interpretation of the disagreement between $d_{\rm L}(z)$ and $d_{\rm A}(z)$ seen in the data can be obscured, however, by time varying cosmic equation of state. Therefore, in this paper we test the reciprocity relation in the framework of several cosmological scenarios invoked to explain accelerating expansion. In the similar spirit [10] discussed sensitivity of Lorentz invariance violating effects on the cosmological model assumed.

## 2. Cosmological models tested

The ΛCDM model is a standard reference point in modern cosmology. It is also called the concordance model since it fits rather well to independent data (such as CMBR data, Large Scale Structure considerations, supernovae data). There are, however, reasons why we are not fully satisfied with the concordance scenario. They can be summarized as the fine-tuning problems.

Therefore the next, popular explanation of the accelerating Universe is to assume the existence of a negative pressure component called dark energy. One can heuristically assume that this component is described by hydrodynamical energy-momentum tensor with (effective) cosmic equation of state: $p = w\rho$ where $w < -1/3$. In such case this component is called "quintessence". Usually the quintessence is attributed to some sort of a scalar field. Another scalar field invoked by cosmologists is the inflaton, which in order to accomplish its role as driving the inflation and creating particles at the reheating epoch, clearly had its own dynamics. Therefore, thinking about quintessence as having origins in the evolving scalar field would lead to a natural expectation that $w$ coefficient should vary in time, *i.e.* $w = w(z)$. Bearing in mind that the scale factor $a(t)$ is a real physical degree of freedom instead of the redshift $z$, the parametrization of $w(z) = w_0 + w_a \frac{z}{1+z}$ developed by [11] turned out to be well suited for such case. Two more different models deserve a mention: the Chaplygin gas and brane-world model of Dvali, Gabadadze and Porrati. The formulae for the expansion rates in these models are given in Table I.

TABLE I

Expansion rates $H(z)$ in cosmological models representative to various dark energy scenarios.

| Model | Cosmological expansion rate $H(z)$ (the Hubble function) |
|---|---|
| ΛCDM | $H^2(z) = H_0^2 \left[\Omega_m\,(1+z)^3 + \Omega_\Lambda\right]$ |
| Quintessence | $H^2(z) = H_0^2 \left[\Omega_m\,(1+z)^3 + \Omega_{\rm Q}\,(1+z)^{3(1+w)}\right]$ |
| Chevalier–Polarski–Linder | $H^2(z) = H_0^2 \left[\Omega_m\,(1+z)^3 + \Omega_{\rm Q}\,(1+z)^{3(1+w_0+w_a)}\,\exp(\frac{-3w_a z}{1+z})\right]$ |
| Chaplygin Gas | $H^2(z) = H_0^2 \left[\Omega_m(1+z)^3 + \Omega_{\rm Ch}\left(A_0 + (1-A_0)(1+z)^{3(1+\alpha)}\right)^{\frac{1}{1+\alpha}}\right]$ |
| Braneworld | $H^2(z) = H_0^2 \left[(\sqrt{\Omega_m(1+z)^3 + \Omega_{r_c}} + \sqrt{\Omega_{r_c}})^2\right]$ |

In the class of generalized Chaplygin gas models matter content of the Universe consists of pressure-less gas with energy density $\rho_m$ representing baryonic plus Cold Dark Matter (CDM) and of the generalized Chaplygin gas with the equation of state $p_{\rm Ch} = -\frac{A}{{\rho_{\rm Ch}}^{\alpha}}$ representing dark energy responsible for acceleration of the Universe. Values of $\alpha$ exponent close to zero mean that the model is equivalent to $\Lambda$CDM case. Chaplygin models have been confronted with supernovae data *e.g.* in [12, 13]. At last, the brane-world models belong to the class of theories which seek the solution of presently accelerating expansion of the Universe not in an exotic material component, but in modifications of gravity. According to this picture, our 4-dimensional Universe is a surface (a brane) embedded into a higher dimensional bulk space-time in which gravity propagates. Therefore, there exists a certain cross-over scale $r_{\rm c}$ above which an observer will detect higher dimensional effects. See [13] and the references therein for more details.

## 3. Method and results

In order to test reciprocity relation, one should have independent data on luminosity and angular diameter distances over a range of redshifts.

For more than a decade now, supernovae Ia have been used as standard candles of cosmology. The latest data set, comprising $n = 557$ supernovae, comes from the compilation given in [3] also known as Union2. In [13] we have used this set to fit cosmological parameters for the models described in the previous section. These best fitted models will serve as a reference point for calculating theoretical distances. Then, the observed angular diameter distances $d_A^{\rm obs}$ will be taken from Bonamente *et al.* [14] (Tables 2, 4 and 5). They combined the X-ray data from *Chandra* with Sunyaev–Zel'dovich (SZ) effect measurements for 38 clusters to obtain the angular diameter distances $d_{\rm A}(z)$. Essentially, the idea is analogous to the original Alcock–Paczyński test where the transverse size of the cluster is inferred from X-ray data, and the radial one from the SZ effect. The results depend on the assumed mass (baryonic and Dark Matter) distribution in cluster, hence three cases were considered in [14]: hydrostatic equilibrium model (Table 2), spherical isothermal model (Table 4) and spherical cored isothermal model (Table 5). Because the X-ray surface brightness depends on the luminosity distance as well, the angular diameter distances obtained that way are affected by potential deviations from the Etherington's relation.

Following [8] we define the measure of violation of the reciprocity relation

$$\eta(z) := \sqrt{\frac{d_{\rm A}^{\rm th}(z)}{d_{\rm A}^{\rm data}(z)}}\,, \tag{1}$$

where: $d_{\rm A}^{\rm th}(z)$ denotes theoretical angular diameter distance. However, we do not know what the true, theoretical angular diameter distance is. Therefore, we assume it to be the one calculated for each cosmological models tested, by using respective expansion functions $H(z)$ from the Table I with parameters (like $\Omega_m$, $w$, $w_0$, $w_a$, *etc.*) taken as best fitted to the Union2 supernovae (values from Table 3 in [13]). In other words, cosmological model parameters obtained from standard candles serve as input for calculating theoretical $d_{\rm A}^{\rm th}$, which are then compared with measured $d_{\rm A}^{\rm data}$.

For each of the 38 clusters we have calculated individual values of $\eta(z)$ given different assumptions on cosmological model and the cluster mass distribution. Fig. 1 shows $\eta(z)$ error bars for the quintessence scenario under hydrostatic equilibrium model of [14]. It turns out to be representative to all other cosmological models. The likelihood function for $\eta$ is shown in Fig. 2.

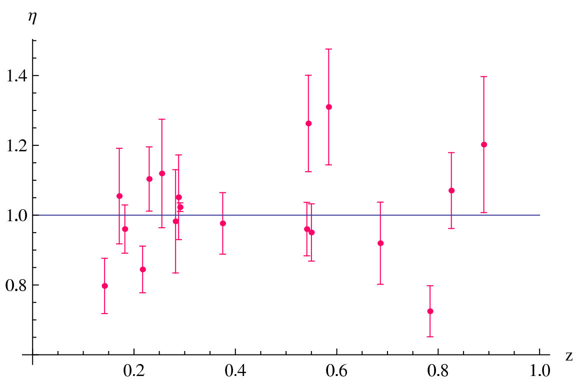


Fig. 1. $\eta(z)$ error bar plot for 38 Bonamente clusters in quintessence scenario under the hydrostatic equilibrium assumption.

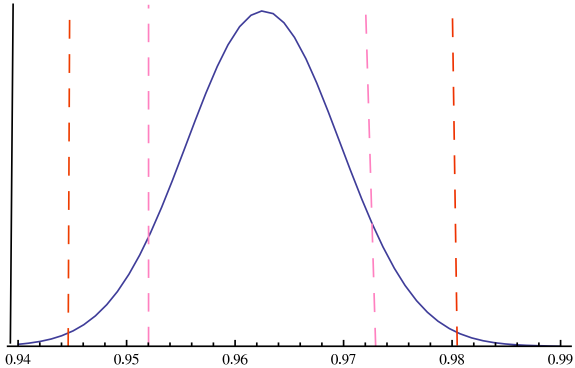


Fig. 2. Joint likelihood function for $\eta$.

Maximum-likelihood estimates for $\eta$ are summarized in Table II. One can see that the measure of potential reciprocity violation $\eta(z)$ does not depend on cosmological model. However the dependence on the assumption concerning cluster's mass profile is significant.

TABLE II

Values of $\eta(z)$ in five different cosmological scenarios for three models for galaxy clusters mass distribution.

| Cosmological model | Hydrostatic equilibrium model | Isothermal cored model | Spherical isothermal model |
|---|---|---|---|
| ΛCDM | $0.962 \pm 0.007$ | $0.878 \pm 0.018$ | $0.873 \pm 0.017$ |
| Quintessence | $0.964 \pm 0.007$ | $0.879 \pm 0.018$ | $0.874 \pm 0.016$ |
| Var Quintessence | $0.964 \pm 0.007$ | $0.879 \pm 0.018$ | $0.874 \pm 0.017$ |
| Chaplygin gas | $0.962 \pm 0.007$ | $0.878 \pm 0.018$ | $0.873 \pm 0.017$ |
| Braneworld | $0.972 \pm 0.007$ | $0.887 \pm 0.018$ | $0.881 \pm 0.017$ |

## 4. Conclusions

In this paper we used the measurements of the angular diameter distance of 38 cluster of galaxies by Bonamente *et al.* [14] together with our fits [13] on the latest Union2 compilation of supernovae to test the distance duality relation in different cosmological models invoked to explain accelerating expansion of the Universe. Our results demonstrate that distance duality violation parameter $\eta(z)$ does not depend on the cosmological model assumed, but considerably depends on assumptions about mass density distribution profile of the cluster. The maximum likelihood estimates of $\eta$ from the Bonamente sample (Table II) are all more than $2\sigma$ away from the value of $\eta = 1$ which might suggest the distance duality violation. However, this effect is more pronounced for isothermal models than for models based on hydrostatic equilibrium, and one can expect that it simply tells us that more sophisticated and accurate modeling of clusters mass density profiles is needed before the X-ray + SZ technique becomes competitive to other methods of measuring distances.

## REFERENCES

[1] A.G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998); S. Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999).

[2] T.M. Davis *et al.*, *Astrophys. J.* **666**, 716 (2007); M. Kowalski *et al.*, *Astrophys. J.* **686**, 749 (2008).

[3] R. Amanullah *et al.*, *Astrophys. J.* **716**, 712 (2010).

[4] D.N. Spergel *et al.*, *Astrophys. J. Suppl.* **148**, 175 (2003); D.J. Eisenstein *et al.*, *Astrophys. J.* **633**, 560 (2005).

[5] J.M.H. Etherington, *Phil. Mag.* **15**, 761 (1933).

[6] B.A. Bassett, M. Kunz, *Phys. Rev.* **D69**, 101305 (2004).

[7] M. Milgrom, *Astrophys. J.* **270**, 365 (1983); M. Milgrom, *New Astronomy Rev.* **51**, 906 (2008).

[8] J.P. Uzan, N. Aghanim, Y. Mellier, *Phys. Rev.* **D70**, 083533 (2004).

[9] F. De Bernardis, E. Giusarma, A. Melchiorri, *Int. J. Mod. Phys.* **D15**, 759 (2006).

[10] M. Biesiada, A. Piórkowska, *Class. Quantum Grav.* **26**, 125007 (2009).

[11] M. Chevallier, D. Polarski, *Int. J. Mod. Phys.* **D10**, 213 (2001); E.V. Linder, *Phys. Rev.* **D68**, 083503 (2003).

[12] M. Biesiada, W. Godłowski, M. Szydłowski, *Astrophys. J.* **622**, 28 (2005).

[13] M. Biesiada, B. Malec, A. Piórkowska, *Res. Astron. Astrophys.* **11**, 641 (2011).

[14] M. Bonamente *et al.*, *Astrophys. J.* **647**, 25 (2006).