FULLY NLO PARTON SHOWER IN QCD*

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The project of constructing a complete NLO-level Parton Shower Monte Carlo for the QCD processes developed in IFJ PAN in Kraków is reviewed. Four issues are discussed: (1) the extension of the standard inclusive collinear factorization into a new, fully exclusive scheme; (2) reconstruction of the LO Parton Shower in the new scheme; (3) inclusion of the exclusive NLO corrections into the hard process and (4) inclusion of the exclusive NLO corrections into the evolution (ladder) part.

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1. Introduction

Precise calculations of the QCD cross-sections for the LHC are based on the factorization procedure. It allows to divide the entire collision process into separate parts: (1) the non-perturbative initial distribution of partons in the incoming protons, to be taken from experiments in form of the Parton Density Functions (PDFs); (2) the actual Parton Showers (PSs), both in the initial and final states, described by the evolution equations resulting from the resumed perturbative calculations; (3) the hard scattering calculated perturbatively to a fixed order and (4) the non-perturbative hadronization of partons, described by phenomenological models. There are a few approaches to the factorization, amongst which the collinear factorization, well founded in the field theory, is the most popular one. Let us recall a few milestones in its development. The first, LO, resummations, dated to the early 1970s, are

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due to Gribov, Lipatov, Altareli, Parisi and Dokshitzer [1]. The general theorem has been addressed in the papers of Ellis, Georgi, Machacek, Politzer and Ross [2] with the help of the axial gauge in which the appealing ladder-like structure of the singularities has been proven. The practical calculations of the NLO kernels were done by Floratos, Ross and Sachrajda [3, 4] and then by Kalinowski, Konishi and others [5, 6], based on the operator product expansion as well as by Curci, Furmanski and Petronzio [7, 8] in diagrammatic approach in dimensionally regularized MS scheme. Further reformulations of the factorization scheme were due to Collins, Soper, Sterman [9], Bodwin [10] and others in 1980s. Finally, twenty years later, the NNLO kernels have been calculated by Moch, Vermaseren and Vogt [11,12]. All the above papers concern the analytical results. The Monte Carlo (MC) implementations, *i.e.* the PS programs, used in the actual data analysis were developed in a different pace. The first implementations: PYTHIA [13] and HERWIG [14], based on the (improved) LO calculations, were created in mid-1980s and systematically developed since then. The next step — the complete NLO-based simulations have not been constructed yet, despite the fact that the analytical results have been known for more than 30 years! There are two half-way solutions: MC@NLO [15] and POWHEG [16] which combine the NLO hard matrix element with the LO-type PS (LO evolution). So far a LO accuracy of the MC programs was satisfactory in the data analysis, as compared to the experimental precision of the hadronic experiments. However, with the start-up of the LHC the situation has changed. The generic expected experimental precision of LHC results is of the order of 1%, and to match this precision the MC simulations must be of the NLO type at least.

The goal of the KRKMC project [17,18] developed in Kraków is to fill in precisely this gap and to construct for the first time the complete NLO-level PS program for QCD. In the following, after a short introduction to the collinear factorization, we will briefly describe the four main ingredients of the KRKMC project: the extended, exclusive collinear factorization scheme, the new LO PS scheme, exclusive NLO corrections to the hard process and exclusive corrections to the ladder part.

In the results presented here some simplifications are temporarily present: only non-singlet kernels are included in the evolution, only $q\bar{q} \to W/Z$ hard process is included and non-running α_S is used.

2. Extended exclusive collinear factorization scheme

The standard collinear factorization rearranges the squared matrix element into a sum of two-particle-irreducible universal kernels K_0, and a process-dependent function C_0, as shown in Fig. 1 for the Drell–Yan type

process (use of the physical gauge is instrumental here!). The collinear singularities are located in the K_0 functions. At this point, one introduces the projection operator $\mathbb{P}$ which separates the singular part $\mathbb{P}K_0$ from the non-singular rest $(\mathbb{1}-\mathbb{P})K_0$, leading to the rearrangement

$$|M|^2 \;=\; C_0 \cdot \frac{1}{1-K_0} = C \otimes (1 + K + K \otimes K + K \otimes K \otimes K + \dots)\,, \quad (1)$$

$$C \;=\; C_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}\,, \qquad K = \mathbb{P}K_0 \cdot \frac{1}{1-(1-\mathbb{P})\cdot K_0}\,, \quad (2)$$

where the reorganized kernel and the hard process, K and C, are to be calculated up to a requested accuracy: LO, NLO, *etc.* The dot $(\cdot)$ denotes four-dimensional integration, whereas the $\otimes$ is a one-dimensional integral (convolution) over the longitudinal momentum (lightcone) variable x^+. The transverse momentum degrees of freedom are integrated out.

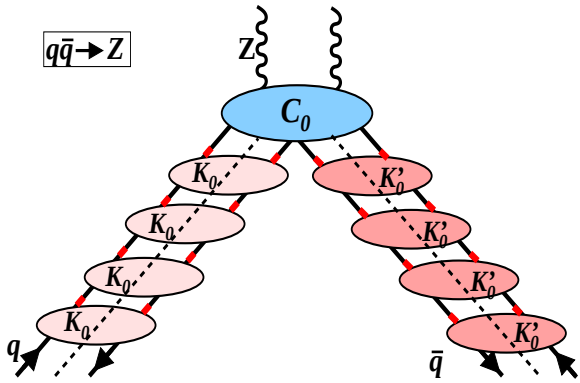


Fig. 1. The collinear factorization for the Drell–Yan type process.

Why is the above scheme not good for precision MC simulation? The three main reasons are: (1) It does not conserve four momenta. In order to reduce $\cdot$ into $\otimes$, the $\mathbb{P}$ operator allows for unphysical configurations of momenta. (2) There are very strong over-subtractions (cancellations) present. Eq. (1) is a geometrical series whereas we expect the final result to be of the exponential form. (3) The scheme is defined in dimensional regularization (pole-part extraction), whereas the MC simulation must be done in four dimensions. How can one cure these drawbacks? (1) The projection operator $\mathbb{P}$ has to be redefined. (2) The time-ordered exponential must follow directly from the construction of the modified factorization expression (1). (3) The geometrical regularization has to be introduced for the real emissions instead of the dimensional one.

We define a new $\overleftarrow{\mathbb{P}}'$ operator and also introduce a finite-part operator $\mathbb{B}$. To the lowest order we have $\overleftarrow{\mathbb{B}}_\mu[K_0] = K_0 - \overleftarrow{\mathbb{P}}'_\mu\{K_0\}$, as expected. In higher orders $\overleftarrow{\mathbb{B}}$ is defined recursively: $\overleftarrow{\mathbb{B}}_\mu[K_0 \cdot K_0] = K_0 \cdot K_0 - \overleftarrow{\mathbb{P}}'_\mu\{^{s_2}K_0\} \cdot \overleftarrow{\mathbb{P}}'_{s_2}\{^{s_1}K_0\} - \overleftarrow{\mathbb{P}}'_\mu\{^{s_2}K_0 \cdot \overleftarrow{\mathbb{B}}_{s_2}[K_0]\} - \overleftarrow{\mathbb{B}}_\mu[K_0] \cdot \overleftarrow{\mathbb{P}}'_\mu\{K_0\}$, and so on.

$\overleftarrow{\mathbb{P}}'_\mu$ sets the upper limit μ on the phase space for all real partons towards the hadron using the kinematical variable $s(k_1, \ldots, k_n) < \mu$, where s stands for virtuality, maximal rapidity, maximal k^{T}, *etc.* The notation $\{^{s_i}A\}$ defines $s = s_i$, *e.g.* $\overleftarrow{\mathbb{P}}'_\mu\{^{s_3}A\} \cdot \overleftarrow{\mathbb{P}}'_{s_3}\{^{s_2}A\} \cdot \overleftarrow{\mathbb{P}}'_{s_2}\{^{s_1}A\}$ means $\theta_{\mu>s_3>s_2>s_1}$ instead of $\overleftarrow{\mathbb{P}}_\mu\{^{s_3}A\} \cdot \overleftarrow{\mathbb{P}}_\mu\{^{s_2}A\} \cdot \overleftarrow{\mathbb{P}}_\mu\{^{s_1}A\}$ corresponding to $\theta_{\mu>s_3}\ \theta_{\mu>s_2}\ \theta_{\mu>s_1}$ (CFP-like). Also, $\overleftarrow{\mathbb{P}}'_\mu(A)$ extracts a singular part from an integrand A (not from the integral $\int A$ like CFP!).

An exclusive parton density function is then defined as the integrand in

$$D(\mu) = \exp_{\mathrm{TO}}\left(\overleftarrow{\mathbb{P}}'_\mu\left\{^s\ K_0 \cdot \overleftarrow{\mathbb{B}}_s\ \left[\frac{1}{1-K_0}\right]\right\}\right), \quad \partial_\mu D(\mu, x) = P \otimes D(\mu)(x) \tag{3}$$

and at the inclusive level it fulfills the standard DGLAP equation.

3. LO Parton Shower revisited

Having established the general factorization framework, we now turn to the LO MC. It is the basis of the project. All the NLO effects will be added on top of it by means of the rejection techniques. The strategy is standard — one combines twice the formula (3) (a Drell–Yan case)

$$\sigma\left(C_0^{(0)} I_{\mathrm{F}}^{(1)} I_{\mathrm{B}}^{(1)}\right) = \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \left\{\sigma\left[C_0^{(0)} \left(\overleftarrow{\mathbb{P}}' K_{0\mathrm{F}}^{(1)}\right)^{n_1} \left(\overleftarrow{\mathbb{P}}'' K_{0\mathrm{B}}^{(1)}\right)^{n_2}\right]\right\}_{\mathrm{TO}} \tag{4}$$

and parametrizes it in terms of the Sudakov variables (α_i, β_i for emitted partons and x_i for virtual, "ladder", ones) that can be directly generated. There are two technical problems to be resolved: (1) the constraint on the value of the final $x_{\mathrm{F/B}} = \prod z^i_{\mathrm{F/B}}$ (necessary for resonant processes) and (2) complete coverage of the phase space without any gaps. The standard solution of the problem (1) is the "backward evolution" [13], which uses a pretabulated grid of PDFs. In order to avoid this complication, we proposed a different algorithm which imposes the constraint on top of the normal "forward" evolution, see [19, 20] for details. The problem (2) is solved by a kinematical mapping of the original phase space to the "tangent space" (k_i, α_i, $\beta_i \to \bar{k}_i$, $\hat{\alpha}_i$, $\hat{\beta}_i$). The mapping we proposed in Ref. [21] is a plain

rescaling, although defined in a recursive way

$$k_{\pi_i} = \lambda_i \bar{k}_{\pi_i}\,, \quad \lambda_i = \frac{s(\bar{x}_{i-1} - \bar{x}_i)}{2\left(P - \sum_{j=1}^{i-1} k_{\pi_j}\right)\cdot \bar{k}_{\pi_i}}\,, \quad i = 1, 2, \ldots, n_1 + n_2\,. \tag{5}$$

The rescaling factors λ_i are chosen such that

$$\bar{s}_i = s\bar{x}_i = s\prod_{\pi_j \in \mathrm{F}}^{i} \hat{z}_{\mathrm{F}\pi_j} \prod_{\pi_j \in \mathrm{B}}^{i} \hat{z}_{\mathrm{B}\pi_j} = \left(P - \sum_{j=1}^{i} k_{\pi_j}\right)^2 = \left(P - \sum_{j=1}^{i} \lambda_j \bar{k}_{\pi_j}\right)^2 . \tag{6}$$

This mapping has three important features: (1) it preserves angles, *i.e.* rapidity ordering, and the upper limit of the phase space (integration) in rapidity; (2) it preserves soft factors ($d\alpha/\alpha \ldots$), *i.e.* the original behavior in the soft limit; (3) it covers the phase space completely, without any gaps.

The complete LO MC algorithm looks as follows:

1. the variables $\hat{z}_{\mathrm{F}}$ and $\hat{z}_{\mathrm{B}}$ are generated by the FOAM MC Sampler [22, 23],
2. the four-momenta $\bar{k}_i^{\mu}$ are generated separately in the F and B parts of the phase space with the constraints $\sum_{j\in \mathrm{F}} \hat{\alpha}_j = 1 - \hat{z}_{\mathrm{F}}$ and $\sum_{j\in \mathrm{B}} \hat{\beta}_j = 1 - \hat{z}_{\mathrm{B}}$,
3. the double-ordering permutation π is established,
4. the rescaling parameter λ_1 is calculated; $k_{\pi_1} = \lambda_1 \bar{k}_{\pi_1}$ is set, such that $(P - k_{\pi_1})^2 = sx_1$,
5. the parameter λ_2 is calculated and $k_{\pi_2} = \lambda_2 \bar{k}_{\pi_2}$ is set, such that $(P - k_{\pi_1} - k_{\pi_2})^2 = sx_2 = sz_{\pi_1} z_{\pi_2}$ and so on,
6. in the rest frame of $\hat{P} = P - \sum_j k_{\pi_j}$ four-momenta q_1^{μ} and q_2^{μ} are generated according to the Born angular distribution.

Exact analytical integration of the LO MC distributions of Eq. (4) over the multigluon phase space is possible (we use the rapidity ordering with Ξ being the rapidity of the produced boson, or equivalently the splitting point between the forward and backward hemispheres)

$$\sigma\left(C_0^{(0)} \Gamma_{\mathrm{F}}^{(1)} \Gamma_{\mathrm{B}}^{(1)}\right) = \int_0^1 d\hat{x}_{\mathrm{F}}\, d\hat{x}_{\mathrm{B}}\, D_{\mathrm{F}}\left(\Xi, \hat{x}_{\mathrm{F}}\right)\, D_{\mathrm{B}}\left(\Xi, \hat{x}_{\mathrm{B}}\right)\, \sigma_{\mathrm{B}}\left(s\hat{x}_{\mathrm{F}}\hat{x}_{\mathrm{B}}\right)\,,$$

with two PDFs obeying the DGLAP non-singlet LO evolution equation

$$\frac{\partial}{\partial \Xi} D_{\mathrm{F}}\left(\Xi, x\right) = \left[P \otimes D_{\mathrm{F}}(\Xi)\right](x)\,.$$

4. Exclusive NLO corrections to the hard process

Having described the "underlying" LO MC, we proceed now with the NLO corrections. We begin with the hard process part. The NLO correction consists of a one-parton real emission and the matching first order virtual correction. It is included by means of a MC weight. In the case of the Drell–Yan process (Fig. 1) the weight reads

$$W_{\rm MC}^{\rm NLO} = 1 + \Delta_{\rm S+V} + \sum_{j\in {\rm F}} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_{\rm F},\hat{p}_{\rm B};a_j,z_{{\rm F}j})}{\bar{P}(z_{{\rm F}j})\; d\sigma_{\rm B}\left(\hat{s},\hat{\theta}\right)/d\Omega} + \sum_{j\in {\rm B}} \frac{\tilde{\beta}_1(\hat{s},\hat{p}_{\rm F},\hat{p}_{\rm B};a_j,z_{{\rm B}j})}{\bar{P}(z_{{\rm B}j})\; d\sigma_{\rm B}\left(\hat{s},\hat{\theta}\right)/d\Omega}\,.$$

The IR/colinear-finite real emission part $\tilde{\beta}_1$ (numerators) is a fully-differential real emission matrix element minus two counter-terms (the counter-terms are LO collinear distributions generalized to off-collinear regions)

$$\tilde{\beta}_1(\hat{p}_{\rm F},\hat{p}_{\rm B};q_1,q_2,k) = \left[\frac{(1-\alpha)^2}{2}\frac{d\sigma_{\rm B}}{d\Omega_q}(\hat{s},\theta_{\rm F1}) + \frac{(1-\beta)^2}{2}\frac{d\sigma_{\rm B}}{d\Omega_q}(\hat{s},\theta_{\rm B2})\right]$$
$$-\theta_{\alpha>\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_{\rm B}}{d\Omega_q}\left(\hat{s},\hat{\theta}\right) - \theta_{\alpha<\beta}\frac{1+(1-\alpha-\beta)^2}{2}\frac{d\sigma_{\rm B}}{d\Omega_q}\left(\hat{s},\hat{\theta}\right)\,.$$

The denominators are just the LO "underlying" differential distributions: numerators of the DGLAP LO kernels $\bar{P}(z_{{\rm F}j/{\rm B}j})$ times the Born cross-section $d\sigma_{\rm B}(\hat{s},\,\hat{\theta})/d\Omega$. The sums go over all emitted partons in both the F and B hemispheres. This way all partons contribute to the hard scattering and there is no problem of defining the "last", "hardest", *etc.* one.

Similarly to the real emission part, the virtual+soft correction is defined as a difference between the complete inclusive result and two inclusive MC-type counterterms

$$\Delta_{\rm V+S} = \left(D_{\rm DY}^{\rm MS}(z) - 2C_{\rm ct}^{\rm MC}(z)\right)\Big|_{\delta-{\rm part}} = \frac{C_{\rm F}\alpha_{\rm s}}{\pi}\;\left(\tfrac{2}{3}\pi^2 - \tfrac{5}{4}\right)\,.$$

$D_{\rm DY}^{\rm MS}(z)$ can be taken from the literature (Eq. (89) in Ref. [24]) or recalculated from the Feynman graphs [25]. Details, in particular on the construction of the counterterms, can be found in [21]. Note, that the $\Delta_{\rm V+S}$ is kinematics independent (it is a constant).

As in the LO case, the exact analytical integration of the NLO MC distributions over the multigluon phase space is possible, and leads to

$$\sigma\left(C_0^{(1)}\Gamma_{\rm F}\Gamma_{\rm B}\right) = \int_0^1 d\hat{x}_{\rm F}\; d\hat{x}_{\rm B}\; dz\; D_{\rm F}(\Xi,\hat{x}_{\rm F})\; D_{\rm B}(\Xi,\hat{x}_{\rm B})\sigma_{\rm B}(sz\hat{x}_{\rm F}\hat{x}_{\rm B})$$
$$\times\left\{\delta_{z=1}(1+\Delta_{\rm S+V}) + C_{2r}^{\rm MC}(z)\right\},$$

where

$$C_{2r}^{\mathrm{MC}}(z) = -\frac{C_{\mathrm{F}}\alpha_{\mathrm{s}}}{\pi}(1-z)\,. \tag{7}$$

The above inclusive NLO correction to the hard scattering in the MC scheme differs from the MS correction (Eq. (90) in [24])

$$C_{2r}^{\mathrm{MS}}(z) = \frac{C_{\mathrm{F}}\alpha_{\mathrm{s}}}{\pi}\frac{\bar{P}(x)}{1-z}[4\ln(1-z) - 2\ln z]\,. \tag{8}$$

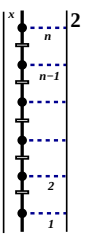
As we see, the MC correction in Eq. (7) is a simple, regular polynomial, whereas the MS one in Eq. (8) contains the singular logarithmic terms $\ln(1-z)/(1-z)$. These terms originate from the "mistreatment" of the phase space done by the k_{T} ordering implied by MS, see also [24].

To summarize, the new NLO MC factorization scheme has the following interesting features: (1) The NLO corrections are added on top of the LO MC with a simple, positive weight. (2) There is no need to correct for the difference in the collinear counter-terms of the PSMC and MS schemes provided PDFs are in the PSMC scheme. (3) The virtual+soft corrections are completely kinematics independent — all the complicated $d\Sigma^{\mathrm{c}\pm}$ contributions of the MC@NLO scheme are absent. (4) There is a built-in resummation of the $\frac{\ln^n(1-x)}{1-x}$ terms (demonstrated for $n=1$).

5. Exclusive NLO corrections to the ladders

The NLO corrections to the multi-emission ladder part have not been included in any of the existing PSs yet, except for a partial proposal of Ref. [26]. The scheme presented here is the first complete solution for the non-singlet evolution. Our scheme is based on the reweighting technique. Once again, we begin with the LO "underlying" differential distribution for the single ladder

$$\bar{D}^{\mathrm{LO}}(x,Q) = \sum_{n=0}^{\infty}$$



$$= e^{-S}\sum_{n=0}^{\infty}\prod_{i=1}^{n}\frac{d^3k_i}{k_i^0}\theta_{Q>a_i>a_{i-1}}\rho_1^{(0)}(k_i)\,,$$

$$\rho_1^{(0)}(k_i) = \frac{2C_{\mathrm{F}}^2\alpha_{\mathrm{s}}}{\pi}\frac{1}{k_i^{\mathrm{T}2}}\frac{1+z^2}{2}\,.$$

The NLO corrected distribution looks as follows

$$\bar{D}^{\mathrm{NLO}}(x,Q) =$$

$$= e^{-S}\sum_{n=0}^{\infty}\Bigg\{ \Bigg| \begin{array}{c} n \\ n-1 \\ p \\ 2 \\ 1 \end{array} \Bigg|^2 + \sum_{p_1=1}^{n}\sum_{j_1=1}^{p_1-1} \Bigg| \begin{array}{c} n \\ p_1 \\ j_1 \\ 1 \end{array} \Bigg|^2 + \sum_{p_1=1}^{n}\sum_{p_2=1}^{p_1-1}\sum_{\substack{j_1=1 \\ j_1\neq p_2}}^{p_1-1}\sum_{\substack{j_2=1 \\ j_2\neq p_1,j_2}}^{p_2-1} \Bigg| \begin{array}{c} n \\ p_1 \\ p_2 \\ j_1 \\ j_2 \end{array} \Bigg|^2 + \ldots \Bigg\}$$

$$= e^{-S}\left\{ \delta_{x=1} + \sum_{n=1}^{\infty}\left(\prod_{i=1}^{n} \int\limits_{Q>a_i>a_{i-1}} \frac{d^3k_i}{k_i^0}\, \rho_1^{(0)}(k_i)\beta_0^{(1)}(z_i) \right) \delta_{x=\prod_{j=1}^{n} x_j} \right.$$

$$\left. \left[1 + \sum_{p=1}^{n}\sum_{j=1}^{p-1} W(\tilde{k}_p,\tilde{k}_j) + \sum_{p_1=1}^{n}\sum_{p_2=1}^{p_1-1}\sum_{\substack{j_1=1 \\ j_1\neq p_2}}^{p_1-1}\sum_{\substack{j_2=1 \\ j_2\neq p_1,j_2}}^{p_2-1} W(\tilde{k}_{p_1},\tilde{k}_{j_1})W(\tilde{k}_{p_2},\tilde{k}_{j_2}) + \ldots \right] \right\}.$$

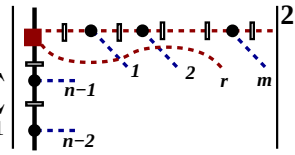
The above formula might look complicated, but in fact its structure is simple. The LO ladder is multiplied by appropriate non-singular NLO weights: in the first term by the virtual weights $\beta_0^{(1)}$ (in the picture dots are replaced by squares); in the second term one NLO real correction, $W(\tilde{k}_p,\ \tilde{k}_j)$, is added with all possible choices of the first and second momentum (two sums are visible); the third contribution is identical to the second one but with two NLO corrections and so on. The actual definitions of the weights are intuitively obvious — they are ratios of the NLO to LO distributions (with counterterms if needed) and can be graphically represented as

$$\beta_0^{(1)} = \frac{\left| \begin{array}{c} \\ \\ \end{array} \right|^2}{\left| \begin{array}{c} z \\ 1-z \end{array} \right|^2} = 1 + 2\Re\left(\Delta_{\mathrm{ISR}}^{(1)}\right), \qquad W(k_2,k_1) = \frac{\left| \begin{array}{c} 2 \\ 1 \end{array} \right|^2}{\left| \begin{array}{c} 2 \\ 1 \end{array} \right|^2} = \frac{\left| \begin{array}{c} 2 \\ 1 \end{array} + \begin{array}{c} 2 \\ 1 \end{array} \right|^2}{\left| \begin{array}{c} 2 \\ 1 \end{array} \right|^2} - 1\,.$$

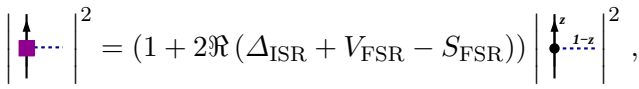
We have recalculated the real emission weight $W(k_2,k_1)$ in the new factorization scheme [27,28] and we are in the process of recalculating the virtual weight [25]. We have tested numerically the above algorithm by comparing the inclusive distribution $\bar{D}^{\mathrm{NLO}}(x,Q)$ from our exclusive MC with the standard inclusive result. We obtained three-digit agreement limited by the statistics, see [17,18] for details.

In the above we have ignored the contribution to $W(k_2, k_1)$ from the gluon-pair production. This contribution has additional singularity in the limit of a vanishing mass of the emitted gluon pair. In the inclusive approach this singularity is canceled by the corresponding virtual one. In the MC exclusive simulation this is not possible and the singularity has to be included into the "underlying" distribution, otherwise it would ruin the convergence of the algorithm. Fortunately, as it is well known, this singularity is a part of the LO corrections to the final state and can be included into the algorithm at the expense of a few additional combinatorial sums over the final-state emissions. Graphically it looks as follows

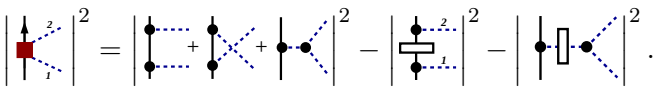
$$e^{-S_{\rm ISR}-S_{\rm FSR}} \sum_{n,m=0}^{\infty} \sum_{r=1}^{m}$$



where the Sudakov form-factor $S_{\rm FSR}$ is subtracted in the virtual part



and the FSR real counterterm is subtracted together with the ISR one



Both the virtual and real corrections (weights) are now regular. All singularities have canceled separately in each weight, allowing for construction of the efficient MC algorithm. The exact analytical integration is possible in this case and we have used this result to perform numerical tests of the MC program with the percent-level precision.

6. Summary

We have briefly reviewed the most important results of the first ever, complete NLO, fully exclusive, PS project under development in Kraków: (1) Extension of the collinear factorization, better suited for the MC implementation, is defined. (2) LO PSMC is (re-)constructed from scratch, in a way compatible with the new factorization scheme. (3) The NLO parts of the hard process and the evolution kernels are recalculated in the new scheme (so far the non-singlet NLO exclusive kernels only). (4) The differences between the new MC and standard MS schemes are understood, keeping the universality (process independence) in mind. (5) The proposed

solution has advantages as compared to the other techniques of adding the NLO corrections to the hard process and it is completely new for the ladder parts. (6) Implementation in the MC has been tested at the prototype level with the relative precision of 10^{-3}.

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