THE EFFECT OF EXTRA MATTER ON UNIFICATION OF YUKAWA COUPLINGS IN MSSM*

A.F. Kord, M. Jalali, H. Shahfar

Department of Physics, Sabzevar Tarbiat Moallem University
P.O.Box 397, Sabzevar, Iran

(Received October 18, 2011)

In this paper we investigate the effect of extra matter on unification of the third Yukawa couplings in the context of the Minimal Supersymmetric Standard Model using two-loop β-functions.

DOI:10.5506/APhysPolB.42.2445
PACS numbers: 11.30.Pb, 12.60.Jv, 12.10.Dm

1. Introduction

The Minimal Supersymmetric Standard Model (MSSM) is a very attractive extension of the Standard Model (SM). It provides solutions to some unanswered theoretical points such as the hierarchy problem, the unification of strong and electroweak interactions and the inconsistency of the SM as a QFT (due to the existence of a Landau pole) [1]. In the minimal supersymmetric extension of the Standard Model, we just construct a supermultiplet for each existing basic field. However, for a satisfactory realistic theory we need two Higgs supermultiplets, with opposite values of the weak hypercharge. There are two reasons why we need at least two chiral Higgs multiplets. First, we need two Higgs superfields to give masses to up and down quarks. The second reason is related to chiral anomalies. One knows that chiral anomalies spoil the gauge invariance and the renormalizability of the theory. In the SM, they are canceled between quarks and leptons. However, in supersymmetry if we have just one chiral Higgs superfield, it contains higgsinos, which are fermions, and lead to anomalies. To cancel them we need to add the second Higgs doublet with the opposite hypercharge.

Thus the MSSM is a $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ invariant theory with vector supermultiplets for the gauge fields and chiral supermultiplets for the quarks, leptons and two Higgses.

* Presented at the XXXV International Conference of Theoretical Physics "Matter to the Deepest", Ustroń, Poland, September 12–18, 2011.

1.1. The MSSM Lagrangian

We know supersymmetry is not an exact symmetry because superpartners of ordinary particles have not been observed at accessible energies. It must be broken in order to give the superpartners large masses. The MSSM Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\mathrm{SUSY}} + \mathcal{L}_{\mathrm{soft-breaking}}\,, \tag{1}$$

$$\mathcal{L}_{\mathrm{SUSY}} = \mathcal{L}_{\mathrm{Gauge}} + \mathcal{L}_{\mathrm{Yukawa}}\,, \tag{2}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{Gauge}} = & \sum_{\mathrm{SU(3),SU(2),U(1)}} \frac{1}{4}\left(\int d^2\theta\,\mathrm{Tr}\,W_\alpha W^\alpha + \int d^2\bar{\theta}\,\mathrm{Tr}\,\bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}}\right) \\ & + \sum_{\mathrm{Matter}} \int d^2\theta d^2\bar{\theta} \varPhi^+ e^{g_3\hat{V}_3+g_2\hat{V}_2+g_1\hat{V}_1}\varPhi\,,\end{aligned} \tag{3}$$

$$\mathcal{L}_{\mathrm{Yukawa}} = \int d^2\theta W + \mathrm{h.c.}\,, \tag{4}$$

where W is the superpotential for the Supersymmetric Standard Model. The superpotential of the MSSM with minimal particle content contains only one supersymmetric Higgs mass term (the μ-term) and the supersymmetric Yukawa couplings generating masses for the quarks and charged leptons. It is given by

$$W = \epsilon_{ab}\left((Y_U)_{ij}Q_i^bU_j^cH_2^a + (Y_D)_{ij}Q_i^bD_j^cH_1^a + (Y_L)_{ij}L_i^bE_j^cH_1^a - \mu H_1^aH_2^b\right)\,, \tag{5}$$

here there is a summation over the generation indices $i,j,k = 1,2,3$, and gauge indices $a,b = 1,2$; color indices are suppressed, also ϵ_{ab} is a totally anti-symmetric tensor.

1.2. Soft supersymmetry breaking Lagrangian

Since supersymmetry is necessarily broken, and since none of the fields of the MSSM can break supersymmetry spontaneously without spoiling the gauge invariance, it is supposed that supersymmetry breaking takes place via some other fields. We parameterize the mechanism that breaks supersymmetry in a soft way (*i.e.* maintaining the cancellation of quadratic divergences). The possible soft terms were classified by Girardello and Grisaru [2]. They consist of mass terms for the gauginos, analytic trilinear terms for the scalar fields known as "A-terms", an analytic quadratic term for the Higgs fields known as the "B-term", and scalar mass terms. Therefore, the soft supersymmetry breaking Lagrangian density is given by

$$
\begin{aligned}
-L_{\mathrm{soft}} = & \left(m^2_{\widetilde{Q}}\right)_{ij} \widetilde{Q}^{\dagger}_i \widetilde{Q}_j + \left(m^2_{\widetilde{Q}}\right)_{ij} \widetilde{L}^{\dagger}_i \widetilde{L}_j + \left(m^2_{\widetilde{U}^c}\right)_{ij} \widetilde{U}^{\dagger c}_i \widetilde{U}^c_j \\
& + \left(m^2_{\widetilde{D}^c}\right)_{ij} \widetilde{D}^{\dagger c}_i \widetilde{D}^c_j + \left(m^2_{\widetilde{E}^c}\right)_{ij} \widetilde{E}^{\dagger c}_i \widetilde{E}^c_j + m^2_{H_2} \hat{H}^{\dagger}_2 \hat{H}_2 + m^2_{H_1} \hat{H}^{\dagger}_1 \hat{H}_1 \\
& + \left((h_U)_{ij} \hat{H}_2 \widetilde{Q}_i \widetilde{U}^c_j + (h_D)_{ij} \hat{H}_1 \widetilde{Q}_i \widetilde{D}^c_j + (h_E)_{ij} \hat{H}_1 \widetilde{L}_i \widetilde{E}^c_j + \mathrm{h.c.} \right) \\
& - \left(B \hat{H}_1 \hat{H}_2 + \mathrm{h.c.} \right) + \tfrac{1}{2} \left(M_1 \widetilde{B} \widetilde{B} + M_2 \widetilde{W^3} \widetilde{W^3} \right) + M_2 \widetilde{W^+} \widetilde{W^+} \\
& + \tfrac{1}{2} M_3 \widetilde{g}^a \widetilde{g}^a \,, \qquad (6)
\end{aligned}
$$

where we have suppressed SU(2) indices. Here $\widetilde{B}$, $\widetilde{W}$ and $\widetilde{g}$ are gaugino fields, $\widetilde{Q}$, $\widetilde{U}$, $\widetilde{D}$ and $\widetilde{L}$, $\widetilde{E}$ are squark and slepton fields, respectively, and $\hat{H}_{1,2}$ are SU(2) doublet Higgs fields.

Of course, we are faced with a lot of free parameters in the soft MSSM Lagrangian; far more than in the Standard Model. A careful count shows that there are 105 masses, phases and mixing angles which cannot be rotated away by redefining the phases, angles and flavor basis for the quark and lepton supermultiplets. To reduce this huge number of free parameters, we need another assumption that is called the universality of the soft terms. We assume that at the unification scale all the spin 0 particle masses are set equal to m_0, all the spin 1/2 particle masses are equal to $M_{1/2}$ and all the cubic and quadratic terms repeat the structure of the Yukawa superpotential with constants of proportionality A, b respectively.

2. The scenario of semi-perturbative unification(SPU)

One of the most successful aspects of the MSSM is the unification of the $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ gauge couplings around the scale 2×10^{16} GeV. In the framework of the MSSM, the value of the unified gauge coupling α_{G} is obtained to be $\alpha_{\mathrm{G}} \sim 0.04$, for which perturbative physics should work well near M_{G}. However, the predictions for α_{G} and M_{G} from the perspective of string theory are more different from the MSSM [3]. Moreover, we know only string theory has the potential to unify all gauge interactions with gravity in a consistent way.

It is possible to raise the values of α_{G} and M_{G} to be consistent with M_{st}(string unification mass scale) and α_{st} by adding some extra matter. It is an inevitable consequence of adding additional matter to the MSSM that α_{G} increases. Specifically, we will follow the scenario of semi-perturbative unification (SPU) that was presented in Ref. [4]. In this scenario matter in complete SU(5) multiplets is added at some intermediate scale $M_n < M_{\mathrm{G}}$ such that $\alpha_{\mathrm{G}} > \alpha_3$; however, we need α_{G} to stay perturbative in the sense of quantum field theory.

We assume these additional contributions from matter are in SU(5) multiplets and only consider 5 and 10 representations of SU(5). It seems that an effective number (n_5, n_{10}) of 5 or 10 representation added to the model at the weak scale may be a good parameterization for studying the effects of extra matter. n_5 and n_{10} represent not only the number of extra representations but also their mass scale and other effects, so it is not necessary that they are integer [4]. The coefficients of the MSSM β-functions will be changed in the case of additional n_5 5 representation and n_{10} 10 representations. The full two and three loop β-functions with n_5 and n_{10} numbers have been presented in Refs. [5, 6]. In previous paper, we have investigated the effects of the presence of extra matter on the evolution of the gauge couplings and susy spectrum. Here we examine the effect of extra mater on the unification of Yukawa couplings.

In some SO(10) GUT models the top quark Yukawa coupling Y_t is unified with Y_b and Y_τ at the GUT scale. Imposing this constraint selects a unique value for $\tan\beta$ and m_t. However, the problem with these models is that m_t is very small, so it is inconsistent with experimental measurement. One could also consider the unification of the Yukawa couplings at some scale other than that at which the gauge couplings unify [7, 8, 9].

In order to investigate the effects of extra matter, we run RGEs up two loop corrections. We will focus on the standard treatment with universal boundary conditions at gauge unification, often termed CMSSM or MSUGRA (the minimal super-gravity). We run all gauge couplings and third family Yukawa couplings to GUT scale ($M_{\rm G}$), and apply the boundary conditions on the soft terms at $M_{\rm G}$. The whole system of the MSSM parameters are evolved to M_Z scale. We iterate the entire procedure to determine a self-consistent solution for Yukawa and gauge couplings.

In our calculation we choose different values for m_t and $\tan\beta$ and then for each pairs of them we change n_5 or n_{10} and investigate the effects of the variation n_5 or n_{10} over unification of the third generation Yukawa couplings. At $M_{\rm G}$, the third generation Yukawa couplings can be examined to check how well they unify. The measure of unification adopted in this paper is given by

$$R_1 = \frac{h_b\,(M_{\rm G})}{h_t\,(M_{\rm G})}\,, \qquad R_2 = \frac{h_\tau\,(M_{\rm G})}{h_t\,(M_{\rm G})}\,, \tag{7}$$

where h_t, h_b, and h_τ are the third generation Yukawa couplings, and R_1 and R_2 indicate the unification if they rich to 1.

In Figs. 1 and 2 we show the plot of R_1 and R_2 for $n_{10} = -2$, using two-loop β-functions for all couplings. They are plotted against n_5, and we set $m_t = 175$ GeV. The results show R_2 and R_1 increase; however R_2 only reaches to 1 for large value of n_5. It means we have unification only for h_τ and h_t.

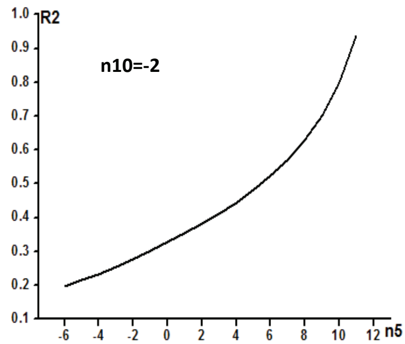

Fig. 1. Plot of R_2 against n_5 for $m_t = 175$ GeV.

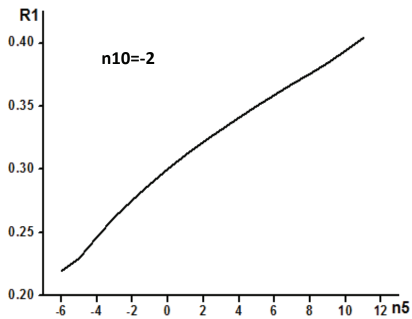

Fig. 2. Plot of R_1 against n_5 for $m_t = 175$ GeV.

In order to obtain unification for the third generation Yukawa couplings, we choose the special values for n_5 and n_{10}. Figs. 3, 4, and 5 show our results for different values of m_t and $\tan\beta$. The results show it is possible to obtain unification for Yukawa couplings for large values of m_t.

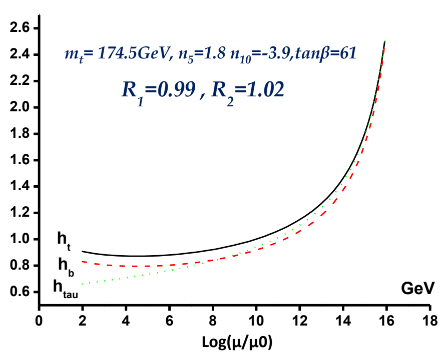

Fig. 3. Plot of the third generation Yukawa couplings. Solid line (black), dashed line (red), and dotted line (green) correspond to h_t, h_b, and h_τ respectively.

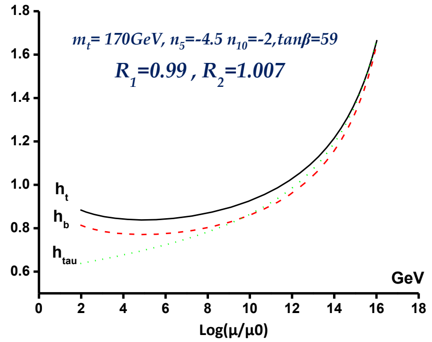

Fig. 4. Plot of the third generation Yukawa couplings. Solid line (black), dashed line (red), and dotted line (green) correspond to h_t, h_b, and h_τ respectively.

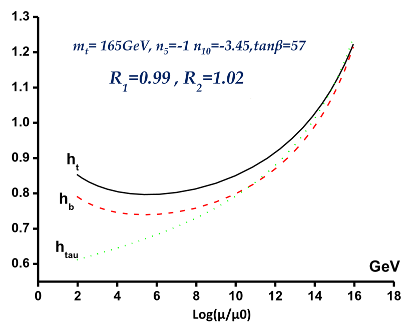

Fig. 5. Plot of the third generation Yukawa couplings. Solid line (black), dashed line (red), and dotted line (green) correspond to h_t, h_b, and h_τ respectively.

We also investigate the effect of variation of n_5 and n_{10} on the mass of neutralino. Figs. 6 and 7 show the effects of extra matter on the mass of neutralino.

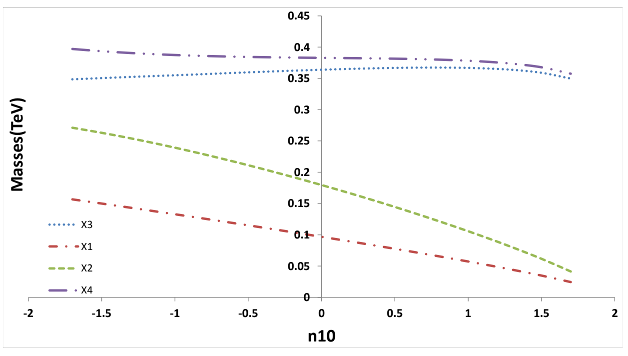

Fig. 6. Plot of the neutralino masses against n_{10} for $n_5 = 0$ and $m_t = 170$ GeV.

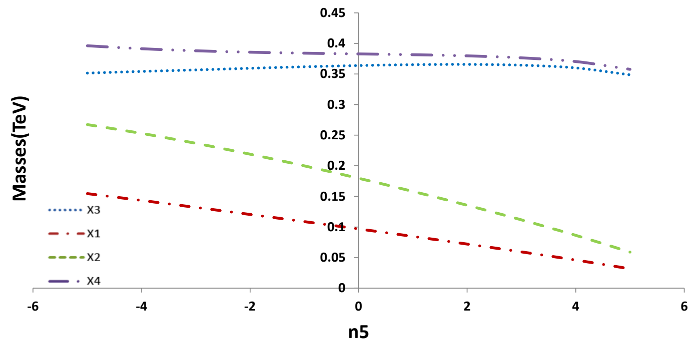

Fig. 7. Plot of the neutralino masses against n_5 for $n_{10} = 0$ and $m_t = 170$ GeV.

3. Conclusion

We have investigated the effects of additional matter on the third Yukawa couplings and neutralino masses. In our calculation we have used the scenario of semi-perturbative unification (SPU) which in the extra matter can be added at intermediate scale. We have given a number of examples of the effects of additional matter on prediction of the Yukawa coupling unification, and shown it is possible to obtain the unification of the third generation of Yukawa couplings for large value of top quark mass.

We have found the impact of two loop corrections became even more dramatic on the sparticle spectrum in the semi-perturbative unification scenario.

REFERENCES

[1] D.I. Kazakov, `arXiv:hep-ph/0012288v2`.

[2] L. Girardello, M.T. Grisaru, *Nucl. Phys.* **B194**, 65 (1982).

[3] K. Babu, J. Pati, *Phys. Lett.* **B384**, 140 (1996).

[4] C. Kolda, J.M. Russell, *Phys. Rev.* **D55**, 4252 (1997).

[5] I. Jack, D.R.T. Jones, A.F. Kord, *Phys. Lett.* **B579**, 180 (2004).

[6] I. Jack, D.R.T. Jones, A.F. Kord, *Ann. Phys.* **316**, 213 (2005).

[7] H. Arason *et al.*, *Phys. Rev. Lett.* **67**, 2933 (1991).

[8] S. Kelley, J.L. Lopez, D.V. Nanopoulos, *Phys. Lett.* **B274**, 387 (1992).

[9] S. Kelley, D.V. Nanopoulos, *Nucl. Phys.* **B373**, 55 (1992).