A THEORETICAL PROGRESS ON THE CALCULATION OF NNLO LEPTONIC AND HADRONIC CORRECTIONS TO BHABHA SCATTERING AND THEIR IMPLEMENTATION INTO BABAYAGA MONTE CARLO GENERATOR*

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Tests of the accuracy of NNLO corrections in BabaYaga MC generator are presented for real experimental event selections at meson factories.

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1. Introduction

Today tests of theoretical models require an inclusion of higher order corrections and precise experimental data. The experimental measurements need values of luminosity determined with very high accuracy. To achieve this, precise calculations of higher order corrections for the process of Bhabha scattering are necessary [1,2]. The luminosity, in turn, is used to determine the low energy cross-section $\sigma(e^+e^- \to \text{hadrons})$, which is very important for calculations of the hadronic part of the anomalous magnetic moment a_μ^{had} [1] and the electromagnetic fine structure constant $\alpha_{\text{QED}}(M_Z^2)$ [3,4].

2. The theoretical framework

A Monte Carlo generator BabaYaga [5,6,7,8] is used as a tool at meson factories for their luminosity measurement. It allows the users to calculate the Bhabha scattering cross-section with the accuracy at the level of 1‰. Nevertheless, some NNLO corrections implemented in BabaYaga are approximated and were not independently tested.

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We report here shortly the results obtained in [9], which is a step forward to fill this gap. We add also further, supplementary details not presented there. To make the test realistic, event selections close to real experimental conditions were used.

The goal was to tests the NNLO corrections with leptonic and hadronic vacuum polarisation insertions, and the corresponding contributions coming from real leptons (hadrons). This part of the BabaYaga generator was not tested before, as the calculations/computer codes were not available.

The stability of the accuracy of the obtained results is very important for sound luminosity predictions. We have investigated how the accuracy changes, when varying acolinearity and angular cuts.

Software used in tests

For numerical calculations the state-of-the-art software was used. Calculations of the virtual part and the soft photon emission was performed using the upgraded package `bha_nnlo_hf` [10]. For the real, hard photon emission the upgraded Monte Carlo generator `BHAGHEN-1PH` was used [11,12]. The real lepton pairs emission was generated using `HELAC-PHEGAS` [13, 14, 15, 16] and the real charged pion pairs emission was generated with `EKHARA` [17,18,19,20]. Vacuum polarisation was calculated with `VPHLMNT` program [21, 22].

3. Numerical results

For numerical calculations experimental cuts close to real event selections from four low energy experiments: KLOE (Φ factory Dafne), BaBar (PEP-II), BELLE (KEK), BES III (BEPC II, Beijing) were chosen.

In Tables I–IV results for exact NNLO corrections and the approximate ones realized with BabaYaga MC generator are presented. The difference between them divided by the full BabaYaga result $\sigma_{\rm BY}$ (without vacuum polarisation) determines the relative error of the generator, coming from the considered NNLO corrections.

For the reaction $e^+e^- \to e^+e^- +$ hadrons, we are able to present only the charged pion pairs contribution. The reason is that no MC generator exists for these processes which is able to give a reliable result for the event selections used in the studies. However, as the pions are the lightest produced hadrons and the highest energy of the meson factories is only about 10 GeV, they are expected to give the largest hadronic contribution.

As we can see from Tables I–IV, for all experiments the electron pair correction is the largest of the considered NNLO corrections. Next are the muon pair and the hadronic corrections. The tau pair contribution is negligible for the energies of meson factories.

TABLE I

Comparison between exact NNLO results and the approximate ones from BabaYaga for KLOE experiment cuts at $\sqrt{s} = 1.02$ GeV.

KLOE $\sigma_{\rm BY} = 455.71(5)$ nb				
particles	σ_h [nb]	σ_{v+s} [nb]	$\sigma_{\rm pairs}$ [nb]	sum [nb]
EXACT NNLO				
electron	9.5021(2)	−11.5666	0.2712(15)	−1.793(2)
muon	1.49406(3)	−1.7356(2)	0.246(7)*10^{-7}	−0.2415(2)
tau	0.0201637(4)	-0.023412(2)	0	−0.003248(2)
leptons sum: $\sigma_{\rm lep}^{\rm NNLO}$				**−2.038(2)**
hadrons	1.5248(6)	−1.062(8)	0	0.463(8)
BabaYaga NNLO				
electron	9.5022(8)	−11.0721(4)	—	−1.5699(9)
muon	1.4942(2)	−1.7441(2)	—	−0.2499(3)
tau	0.020166(3)	−0.023704(2)	—	−0.003538(4)
leptons sum: $\sigma_{\rm BYlep}^{\rm NNLO}$				**−1.823(1)**
hadrons	1.5247(5)	−1.126(2)	—	0.399(2)
leptons relative difference: $\|\sigma_{\rm lep}^{\rm NNLO} - \sigma_{\rm BYlep}^{\rm NNLO}\|/\sigma_{\rm BY}$				**0.471(4)‰**
hadrons relative difference: $\|\sigma^{\rm NNLO} - \sigma_{\rm BY}^{\rm NNLO}\|/\sigma_{\rm BY}$				**0.14(2)‰**

TABLE II

Comparison between exact NNLO results and the approximate ones from BabaYaga for BaBar experiment cuts at $\sqrt{s} = 10.56$ GeV.

BaBar $\sigma_{\rm BY} = 5.195(2)$ nb				
particles	σ_h [nb]	σ_{v+s} [nb]	$\sigma_{\rm pairs}$ [nb]	sum [nb]
EXACT NNLO				
electron	0.202439(7)	−0.223667	0.01355(8)	−0.00768(8)
muon	0.075789(2)	−0.079231(2)	0.000451(2)	−0.002991(3)
tau	0.0138398(4)	−0.0144654(2)	0.120(3)*10^{-8}	−0.0006257(5)
leptons sum: $\sigma_{\rm lep}^{\rm NNLO}$				**−0.01130(8)**
hadrons	0.17995(2)	−0.1888(4)	0.000029(3)	−0.0088(4)
BabaYaga NNLO				
electron	0.20244(2)	−0.20971(5)	—	−0.00727(5)
muon	0.07580(1)	−0.07872(2)	—	−0.00292(2)
tau	0.013847(4)	−0.014541(4)	—	−0.000694(6)
leptons sum: $\sigma_{\rm BY}^{\rm NNLO}$				**−0.01088(5)**
hadrons	0.17984(2)	−0.18760(4)	—	−0.00776(5)
leptons relative difference: $\|\sigma_{\rm lep}^{\rm NNLO} - \sigma_{\rm BYlep}^{\rm NNLO}\|/\sigma_{\rm BY}$				**0.08(2)‰**
hadrons relative difference: $\|\sigma^{\rm NNLO} - \sigma_{\rm BY}^{\rm NNLO}\|/\sigma_{\rm BY}$				**0.23(8)‰**

TABLE III

Comparison between exact NNLO results and the approximate ones from BabaYaga for BELLE experiment cuts at $\sqrt{s} = 10.58$ GeV.

BELLE $\sigma_{\rm BY} = 5.501(5)$ nb				
particles	σ_h [nb]	σ_{v+s} [nb]	$\sigma_{\rm pairs}$ [nb]	sum [nb]
NNLO				
electron	0.21572(7)	−0.25596	0.01310(5)	−0.02714(9)
muon	0.080377(8)	−0.09009(1)	0.000759(1)	−0.00895(2)
tau	0.014428(4)	−0.01602(1)	0.0000321(1)	−0.00156(1)
leptons sum: $\sigma_{\rm lep}^{\rm NNLO}$				**−0.03765(9)**
hadrons	0.18969(1)	−0.2124(5)	0.00015(1)	−0.0226(5)
BabaYaga NNLO				
electron	0.21563(2)	−0.23994(2)	—	−0.02431(3)
muon	0.080376(6)	−0.08948(2)	—	−0.009104(2)
tau	0.014423(1)	−0.016091(7)	—	−0.001668(7)
leptons sum: $\sigma_{\rm BYlep}^{\rm NNLO}$				**−0.03508(3)**
hadrons	0.18964(3)	−0.21089(5)	—	−0.02125(6)
leptons relative difference: $\lvert\sigma_{\rm lep}^{\rm NNLO} - \sigma_{\rm BYlep}^{\rm NNLO}\rvert/\sigma_{\rm BY}$				**0.47(2)‰**
hadrons relative difference: $\lvert\sigma^{\rm NNLO} - \sigma_{\rm BY}^{\rm NNLO}\rvert/\sigma_{\rm BY}$				**0.27(9)‰**

TABLE IV

Comparison between exact NNLO results and the approximate ones from BabaYaga for BES III experiment cuts at $\sqrt{s} = 3.65$ GeV.

BES III $\sigma_{\rm BY} = 116.41(2)$ nb				
particles	σ_h [nb]	σ_{v+s} [nb]	$\sigma_{\rm pairs}$ [nb]	sum [nb]
NNLO				
electron	3.19544(9)	−3.55544	0.188856(997)	−0.171(1)
muon	0.83245(2)	−0.88149(1)	0.002003(6)	−0.04704(1)
tau	0.058674(2)	−0.0633(1)	0	−0.0046(1)
leptons sum: $\sigma_{\rm lep}^{\rm NNLO}$				**−0.223(1)**
hadrons	1.66065(8)	−1.81(1)	0.000539(7)	−0.15(1)
BabaYaga NNLO				
electron	3.1960(3)	−3.3730(2)	—	−0.1770(4)
muon	0.83252(7)	−0.88041(9)	—	−0.0479(1)
tau	0.058679(7)	−0.06323(2)	—	−0.00455(2)
leptons sum: $\sigma_{\rm BY}^{\rm NNLO}$				**−0.2295(4)**
hadrons	1.6613(3)	−1.7860(2)	—	−0.1247(4)
leptons relative difference: $\lvert\sigma_{\rm lep}^{\rm NNLO} - \sigma_{\rm BYlep}^{\rm NNLO}\rvert/\sigma_{\rm BY}$				**0.057(9)‰**
hadrons relative difference: $\lvert\sigma^{\rm NNLO} - \sigma_{\rm BY}^{\rm NNLO}\rvert/\sigma_{\rm BY}$				**0.21(9)‰**

For KLOE, leptons and hadrons give errors with opposite signs, so the total error is equal to 0.33‰. Similar situation occurs for BES III. Here the total error is equal to about 0.15‰.

Despite that BaBar and BELLE experiments run at similar energies, the difference in event selections cause different effects, especially for leptons. For BaBar's experimental point the error for leptons is small — 0.08‰ and the total error is equal to 0.31‰. On the contrary, for BELLE, leptons give almost six times larger error — 0.47‰, and the total error is equal to 0.74‰. This number is very close to the accuracy of 1‰ for BabaYaga MC generator, so the question about the stability of the error near the experimental point is crucial.

Figure 1 presents the total error for all four experiments for various event selections. We can see that the accuracy is stable for all of them and does not change dramatically. Only for BELLE we obtain the error larger than 1‰, however, only for very tight acollinearity cuts which are far from the ones used in the experiment.

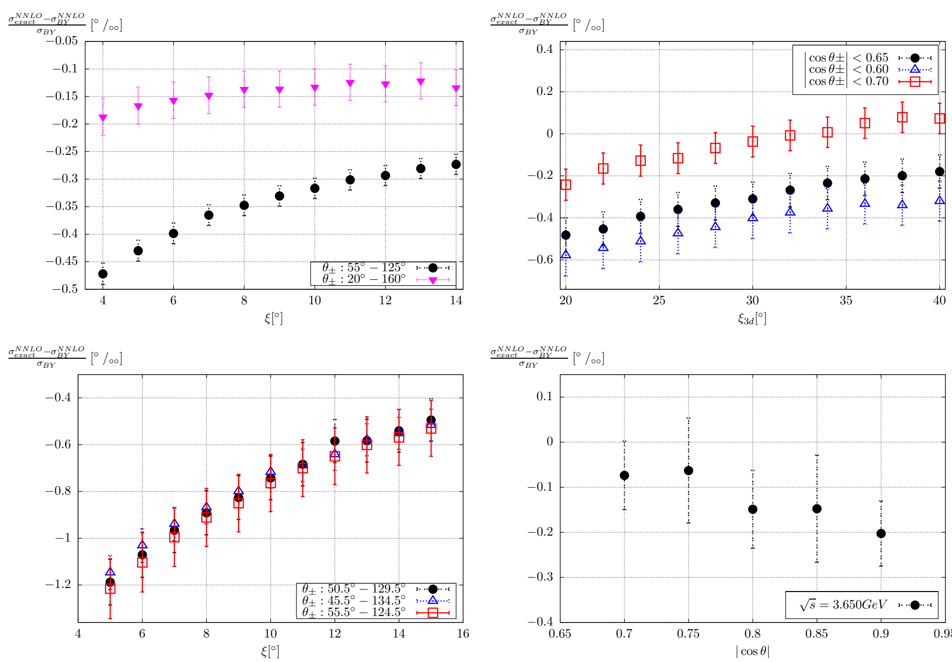

Fig. 1. The relative difference of NNLO massive corrections, as a function of: 2d acollinearity (KLOE — left top and BELLE — left bottom), 3d acolinearity (BaBar — right top) and $\cos\theta$ (BES III — right bottom).

Narrow resonances in the BES III

In Table IV results for BES III experimental cuts were presented for energy of $\sqrt{s} = 3.65$ GeV. However, this experiment works also at narrow resonances energies. Preliminary calculations for resonances: J/Ψ ($\sqrt{s} = 3.097$ GeV) and $\Psi(2S)$ ($\sqrt{s} = 3.686$ GeV) were performed.

In Table V we can see how big are the discussed NNLO corrections as compared to the full BabaYaga results. These huge contributions indicate that the behaviour of the missing corrections has to be studied more carefully. A careful treatment of beam spread effects is necessary, and it will be further investigated [23].

TABLE V

The ratio of the NNLO corrections and full BabaYaga result $\sigma_{\rm BY}$ at J/Ψ and $\Psi(2S)$ resonances: $S_x = \frac{\sigma_x^{\rm NNLO}}{\sigma_{\rm BY}}$ with $x = e^+e^-$, lep, had, tot.

$\sqrt{s}$			$\sigma_{\rm BY}$	$S_{e^+e^-}$‰	$S_{\rm lep}$‰	$S_{\rm had}$‰	$S_{\rm tot}$‰
BES	3.097	BabaYaga	158.23	−2.019(3)	−2.548(3)	558.7(7)	556.1(7)
BES	3.686	BabaYaga	114.27	−1.502(4)	−1.947(4)	−59.42(1)	−61.36(1)

4. Conclusions

Exact calculations of the NNLO massive corrections to the Bhabha scattering were presented and compared with the approximate ones from BabaYaga MC generator. The accuracy of these corrections in the generator was tested. For real experimental cuts the biggest error is at the level of 0.7‰. Only for very tight acollinearity cuts the sum of the missing pieces can reach 1‰ (BELLE).

The stability of the results against changing of the event selections was examined. No big changes of the accuracy between event selections and event selections close to the experimental ones was observed.

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