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NON-STANDARD INTERACTIONS, DENSITY MATRIX AND NEUTRINO OSCILLATIONS*

Robert Szafron, Marek Zralek

Institute of Physics, University of Silesia
Uniwersytecka 4, 40-007 Katowice, Poland

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We present an analysis of a neutrino production and detection processes, necessary in order to describe the oscillation phenomena in any model of neutrino interaction. We derive an oscillation probability in the presence of neutrino non-standard interactions and compare the result with the standard approach. Our results are applicable in a very wide class of New Physics models.

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1. Introduction

In the new era of precise neutrino experiments, neutrino oscillations need precise description in a wide class of models. The Standard Model (SM) is a well established theory but it is generally believed that it is only an effective theory and therefore physicists are still searching for a New Physics (NP), which may manifest itself as a Non-Standard Interactions (NSI). One of possible places where we can look for a NP effects is a neutrino sector [1, 2]. In this paper, we present a general formalism which enables us to describe neutrino oscillations in a wide class of models with NSI. This can help us to answer the question what we can learn about NP from neutrino oscillations experiments and how to precisely describe all possible effects in neutrino production, propagation and detection. The aim of this work is to concentrate on a proper description of neutrino production and detection states. We will not discuss the impact of NSI on matter oscillation which is a well known problem [3]. Our formalism is based on density matrix approach [4, 5, 6, 7]. The general idea is following: we assume that neutrino

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production process is described by some Quantum Field Theory such that we can calculate an amplitude for considered process. Then we construct a density matrix using the prescription described in Section 2 of this article. Then we propagate the neutrinos from the place of production to the detection using effective Hamiltonians [3] (Section 3). Finally, in Section 4 we calculate final detection cross-section.

2. Production process

Let us consider some generic production process

$$i \to f + \nu(\lambda, k) \tag{1}$$

with initial state i (for example a muon) and final state f (other particles that accompany neutrino). For a given initial pure state the Quantum Mechanical (QM) final state is given by

$$|\mathrm{final}\rangle = N \sum_{k,f,\lambda} \int\limits_{\Omega} d\mu\left(\vec{p}, p_f\right) A_k\left(E, \vec{p}, \lambda; i, f, p_f\right) |\nu_k(\vec{p}, \lambda), f\rangle\,, \tag{2}$$

where N is a normalisation factor, and $d\mu(\vec{p}, p_f)$ is some measure which depends on kinematics of considered process (1). $A_k(E, \vec{p}, \lambda; i, f, p_f)$ is an amplitude which for the process (1) describes the production of k neutrino with mass m_k, energy E, momentum $\vec{p}$ and helicity λ. However, usually initial state is not pure (*e.g.* for initial not polarised muon) and then also the final state of (1) is not pure and is described by a density matrix

$$\begin{aligned} \varrho_f &= N \sum_{i,i'} (\varrho_i)_{i,i'} \sum_{k,k',\lambda,\lambda'} \sum_{f,f'} \int\limits_{\Omega} d\mu\left(\vec{p}, p_f\right) \int\limits_{\Omega} d\mu\left(\vec{p'}, p'_f\right) A_k\left(E, \vec{p}, \lambda; i, f, p_f\right) \\ &\quad \times \Big|\nu_k\left(\vec{p}, \lambda\right), f\Big\rangle \Big\langle \nu'_k, \left(\vec{p'}, \lambda'\right), f'\Big| A^*_{k'}\left(E', \vec{p'}, \lambda'; i', f', p'_f\right)\,. \end{aligned} \tag{3}$$

In general, the question of purity of the final state depends on the initial state as well as on the dynamics *i.e.* considered model. The state (3) contains maximum information about all the particles that are produced in process (1), but we are interested only in a neutrino state. Therefore, we have to take the partial trace over all not important degrees of freedom. Then the neutrino state is given by a following density matrix

$$\begin{aligned} \varrho &= N \sum_{i,i'} (\varrho_i)_{i,i'} \sum_{k,k',\lambda,\lambda'} \sum_{f} \int\limits_{\Omega} d\mu\left(\vec{p}, p_f\right) \int\limits_{\Omega} d\mu\left(\vec{p'}, p'_f\right) \delta\left(p_f - p'_f\right) \\ &\times A_k\left(E, \vec{p}, \lambda; i, f, p_f\right) \Big|\nu_k\left(\vec{p}, \lambda\right)\Big\rangle \Big\langle \nu_k, \left(\vec{p'}, \lambda'\right)\Big| A^*_{k'}\left(E', \vec{p'}, \lambda'; i', f, p'_f\right)\,. \end{aligned} \tag{4}$$

In general, this state is mixed, however there can be special cases when it is pure. For example, in a theory with only left-handed neutrinos if and only if an amplitude can be written in form $A_k(E, \vec{p}, -1; i, f, p_f) = B_k \times C(E, \vec{p}; i, f, p_f)$ then the neutrino state is pure and given by the following formula

$$|\nu\,(\vec{p}, -1)\rangle = N' \sum_k B_k |\nu_k(\vec{p}, -1)\rangle\,. \tag{5}$$

This situation obviously appears in SM, where

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha\,k} |\nu_k\rangle\,. \tag{6}$$

In general case, our density matrix ϱ (4) contains all available information about flavour structure of neutrino state, but it also carries informations about neutrino spectrum. Therefore, we can define a new quantity

$$\tilde{\varrho}(\vec{p}) = \frac{\varrho}{\mathrm{Tr}(\varrho)}\,, \tag{7}$$

where Tr() denotes a trace over all discrete variables only. It represents a mass-flavour structure of neutrino state. For completeness we also introduce a quantity which represents a probability of finding a neutrino with momentum in the interval $(\vec{p}, \vec{p} + d\vec{p})$

$$\frac{dj}{d\vec{p}} = \mathrm{Tr}(\varrho) \tag{8}$$

which basically determines the neutrino spectrum produced in (1). With those definitions the following relation holds

$$\varrho = \tilde{\varrho}(\vec{p}) \times \frac{dj}{d\vec{p}}\,. \tag{9}$$

2.1. An example: Muon decay

Now, we will apply presented formalism in the simple but not trivial example of a muon decay with NSI of vector and scalar type. We assume that only left-handed neutrinos are present in our model and an interaction Lagrangian is described by

$$\mathcal{L}_\mathrm{I} = -2\sqrt{2} G_\mathrm{F} \left[g^\mathrm{S}_{ij} \left(\overline{\nu}_i P_\mathrm{R} e\right) \left(\overline{\mu} P_\mathrm{L} \nu_j\right) + g^\mathrm{V}_{ij} \left(\overline{\nu}_i \gamma^\alpha P_\mathrm{L} e\right) \left(\overline{\mu} \gamma_\alpha P_\mathrm{L} \nu_j\right) \right] + \mathrm{h.c.} \tag{10}$$

First of all, we will analyse the case where there are no scalar interactions $g^{\rm S}_{ij} = 0$, it is the usual case of charged current NSI widely discussed in literature. Neglecting electron mass we obtain

$$\frac{dj}{dx} = 2x^2(3-2x) \tag{11}$$

with $x = \frac{2E}{M}$ and

$$\tilde{\varrho}(\vec{p}) = \frac{\left(g^{\rm V}\right)^\dagger g^{\rm V}}{{\rm Tr}\left[\left(g^{\rm V}\right)^\dagger g^{\rm V}\right]}\,. \tag{12}$$

This density matrix describes a pure state if and only if $g^{\rm V}$ is a rank one matrix *i.e.* can be written in a form $g^{\rm V} \sim vu^\dagger$, where u and v are some unit vectors. Of course, this situation appears in SM but may not be the case when also NP contributes to the muon decay.

In SM in flavour base we have $(g^{\rm V})_{\alpha\beta} = \delta_{e\beta}\delta_{\mu\alpha}$. If we are interested only in linear approximation we assume that $(g^{\rm V})_{\alpha\beta} = \delta_{e\beta}\delta_{\mu\alpha} + \varepsilon_{\alpha\beta}$ and then the density matrix can be calculated

$$\begin{aligned}(\tilde{\varrho}(\vec{p}))_{\alpha\alpha'} &= (\delta_{e\beta}\delta_{\mu\alpha} + \varepsilon_{\alpha\beta})\left(\delta_{e\beta}\delta_{\mu\alpha'} + \varepsilon^*_{\alpha'\beta}\right) \\ &\approx \delta_{\mu\alpha}\delta_{\mu\alpha'} + \delta_{\mu\alpha}\varepsilon^*_{\alpha' e} + \delta_{\mu\alpha'}\varepsilon_{\alpha e}\,.\end{aligned} \tag{13}$$

Normalisation factor in linear approximation can be also calculated $N' = 1 - 2{\rm Re}(\varepsilon_{\mu e})$. Neglecting a second order terms this is equivalent to a pure state usually used in the literature [8]

$$|\nu\rangle = \sum_\alpha (\delta_{\alpha\mu} + \varepsilon_{\alpha e})\,|\nu_\alpha\rangle\,. \tag{14}$$

Second order terms in (13) and (14) are different, however in practice the difference is much below present experimental accuracy but it may be important in future, for more precise neutrino oscillations experiments.

If we assume that also the scalar part of (10) gives contribution *i.e.* $g^{\rm S}_{ij} \neq 0$ we obtain

$$\frac{dj}{dx} = \frac{4x^2\left(2\,{\rm Tr}\left[\left(g^{\rm V}\right)^\dagger g^{\rm V}\right](3-2x) + 3\,{\rm Tr}\left[\left(g^{\rm S}\right)^\dagger g^{\rm S}\right](1-x)\right)}{{\rm Tr}\left[\left(g^{\rm V}\right)^\dagger g^{\rm V} + 4\left(g^{\rm S}\right)^\dagger g^{\rm S}\right]}$$

and

$$\tilde{\varrho}(\vec{p}) = \frac{2\left(g^{\rm V}\right)^\dagger g^{\rm V}(3-2x) + 3\left(g^{\rm S}\right)^\dagger g^{\rm S}(1-x)}{2\,{\rm Tr}\left[\left(g^{\rm V}\right)^\dagger g^{\rm V}\right](3-2x) + 3\,{\rm Tr}\left[\left(g^{\rm S}\right)^\dagger g^{\rm S}\right](1-x)}\,. \tag{15}$$

If we are interested only in the lowest order approximation then the density matrix can be written as

$$\tilde{\varrho}(\vec{p}) = \frac{\left(g^{\mathrm{V}}\right)^{\dagger} g^{\mathrm{V}}}{\mathrm{Tr}\left[\left(g^{\mathrm{V}}\right)^{\dagger} g^{\mathrm{V}}\right]} + \frac{3(1-x)}{2\,\mathrm{Tr}\left[\left(g^{\mathrm{V}}\right)^{\dagger} g^{\mathrm{V}}\right](3-2x)} \times \left(\left(g^{\mathrm{S}}\right)^{\dagger} g^{\mathrm{S}} - \left(g^{\mathrm{V}}\right)^{\dagger} g^{\mathrm{V}} \frac{\mathrm{Tr}\left[\left(g^{\mathrm{S}}\right)^{\dagger} g^{\mathrm{S}}\right]}{\mathrm{Tr}\left[\left(g^{\mathrm{V}}\right)^{\dagger} g^{\mathrm{V}}\right]}\right) + O\left(\left[\left(g^{\mathrm{S}}\right)^{\dagger} g^{\mathrm{S}}\right]^{2}\right) . \tag{16}$$

The scalar term gives no linear contribution when neglecting the electron mass. Also in this case the neutrino state, in general, is mixed in QM sense.

3. Neutrino propagation

Neutrino propagation can be calculated with the use of displacement operator in time and space

$$\varrho(x) = e^{-iP_\mu x^\mu} \varrho(0) e^{iP_\mu x^\mu} , \tag{17}$$

where P_μ is four-momentum operator. Its zero component may contain a term responsible for matter effects [8]. The calculations should be performed in a LAB frame *i.e.* Lorentz transformations have to be applied which can be reduced to usual Wick rotation (see *e.g.* [4, 9] for notation and detailed discussion)

$$[\varrho'(p')]_{\lambda' i,\chi' k} = \mathcal{D}^s_{\lambda\lambda'}\left(r\left(\Lambda,\vec{p}\right)\right)\left(\mathcal{D}^s_{\chi\chi'}\left(r\left(\Lambda,\vec{p}\right)\right)\right)^{\dagger}[\varrho(p)]_{\lambda i,\chi k} \tag{18}$$

with, in our case,

$$\mathcal{D}^s_{\lambda\lambda'}\left(r(l_z(\beta),\vec{p})\right) = d^s_{\lambda\lambda'}\left(\theta_{\mathrm{Wick}}\right) . \tag{19}$$

In practice, because neutrino mass can be neglected comparing to its energy, following approximation holds

$$d^{1/2}_{\lambda\lambda'}\left(\theta_{\mathrm{Wick}}\right) \approx \delta_{\lambda\lambda'} , \tag{20}$$

so there is no change of neutrino helicity due to the Lorentz transformations.

4. Detection process

In order to describe a detection process, we chose some reaction and calculate an amplitude $B_i(\lambda, \vec{p}, x)$ describing detection of i neutrino mass eigenstate with helicty λ and momentum $\vec{p}$. x denotes all degrees of freedom of other particles participating in a detection process, that need to be summed or integrated in order to calculate detection cross-section. Then the final cross-section is given by

$$\sigma(\vec{p}, L) = \int \tfrac{1}{F} \sum_{i,i',\lambda,\lambda'} B_i\left(\lambda, \vec{p}, x\right) [\tilde{\varrho}(\vec{p}, L)]_{\lambda i;\lambda' i'} B^*_{i'}(\lambda', \vec{p}, x) d\mathrm{Lips}(x)\,. \quad (21)$$

This quantity contains also information about oscillation probability. In order to factorise the final cross-section (21) on part which describes the oscillation probability and detection cross-section, we define an operator D describing detection process. Its matrix elements are given by

$$[D]_{i,\lambda;i',\lambda'} = \int \tfrac{1}{F} B^*_i\left(\lambda, \vec{p}, x\right) B_{i'}(\lambda', \vec{p}, x) d\mathrm{Lips}(x)\,, \quad (22)$$

such that now, detection cross-section (21) can be calculated as an expectation value of D

$$\sigma(\vec{p}, L) = \mathrm{Tr}\left(\tilde{\varrho}(\vec{p}, L) D^\dagger\right)\,. \quad (23)$$

We can further simplify our considerations by introducing, similarly as before in the case of ϱ, two quantities so that we can write D as a product of two factors

$$D = \tilde{D}\sigma_N \quad (24)$$

with $\tilde{D} = \frac{D}{\mathrm{Tr}(D)}$ and $\sigma_N = \mathrm{Tr}(D)$. Then we can define probability

$$P(L, E) = \mathrm{Tr}\left(\tilde{D}^\dagger \tilde{\varrho}(\vec{p}, L)\right) \quad (25)$$

which, in general, is not universal but process dependent. With these definitions, number of events is, similarly as in SM, proportional to product of detection cross-section and oscillation probability

$$\sigma(\vec{p}, L) = \sigma_N P(L, E)\,. \quad (26)$$

In SM the $P(L, E)$ is reduced to a standard oscillation probability $P^{\mathrm{SM}}(L, E)$.

Now we will show two sample calculations of our oscillation probability (25). Assuming that detection process is sensitive only to muon neutrino and that there is no NSI contribution to detection, we have $[\tilde{D}]_{ij} = \delta_{i\alpha}\delta_{j\alpha}$ and introducing operator $[X^\mu]_{\beta,\beta'} = U^*_{\mu,\beta} U_{\mu\beta'}$ ($U = \exp(-iP^\mu x_\mu)$), in the linear approximation (13) we obtain

$$P(L, E) = P^{\mathrm{SM}}(L, E) + 2\mathrm{Re}\left(\left[\varepsilon^T X^\mu\right]_{e\mu}\right)\,. \quad (27)$$

However, using a pure state (14) $|\nu\rangle = \sum_\alpha (\delta_{\alpha\mu} + \varepsilon_{\alpha e})|\nu_\alpha\rangle$ we obtain different results in second order *i.e.* pure state (14) leads to the appearance of term $[\varepsilon^* X^\mu \varepsilon^T]_{ee}$ while exact calculation with density matrix leads to a term $\mathrm{Tr}[\varepsilon^* X^\mu \varepsilon^T]$. The difference is, therefore, not of practical importance in present experiments but it can matter in future experiments. The present bound on NSI [10] enables us to estimate the difference to be not greater than one per mille.

Let us now calculate the oscillation probability (25) in a second case when we assume that NSI are also present in detection process. We have such case, if for example, inverse muon decay is used as a detection process. Then in a linear approximation up to a normalisation constant we obtain

$$P(L,E) = P^{\mathrm{SM}}(L,E) + 2\mathrm{Re}\left(\left[\varepsilon^T X^\mu\right]_{e\mu}\right) + 2\mathrm{Re}\left(\left[\varepsilon^T \tilde{X}^\mu\right]_{e\mu}\right) , \quad (28)$$

where we have introduced a notation $[\tilde{X}^\mu]_{\beta\beta'} = U^*_{\beta\mu} U_{\beta'\mu}$. Second order corrections reads $\mathrm{Tr}[\varepsilon^* X^\mu \varepsilon^T] + 2\mathrm{Re}(U_{\mu\mu}[\varepsilon^T U \varepsilon]_{ee} + [U^* \varepsilon]_{\mu e}[\varepsilon^T U]_{e\mu}) + \mathrm{Tr}[\varepsilon \tilde{X}^\mu \varepsilon^\dagger]$. Also higher order terms appear but we can safely neglect them.

5. Conclusions

We presented a general formalism describing neutrino oscillations in the most of models beyond SM. Neutrino production, propagation and detection are described by a density matrix instead of pure QM states as usually it is done in literature. Although in some specific conditions (a simple model or specific production process) it is possible to justify a pure state approximation, NSI nevertheless in a general case cause the neutrino state to be mixed in QM sense.

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