MAJORANA NEUTRINO MASS MATRIX WITH CP SYMMETRY BREAKING*

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From the new existing data with not vanishing θ_{13} mixing angle we determine the possible shape of the Majorana neutrino mass matrix. We assume that CP symmetry is broken and all Dirac and Majorana phases are taken into account. Two possible approaches "bottom–up" and "top–down" are presented. The problem of unphysical phases is examined in detail.

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1. Introduction

It is commonly believed that determination of the shape of neutrino mass matrix could shed light on a mass generation mechanism and give us some information about model lying behind it.

Many attempts have been made in literature (*e.g.* [1, 2, 3, 4]) to restrict the form of neutrino mass matrices. In general, we can divide them into two categories which are called "top–down" — where the neutrino masses, the mixing angles and the CP violation phases are predicted from a given mass matrix, and "bottom–up" method — where the existing neutrino data determine possible shape of the mass matrix.

In the "top–down" method the neutrino mass matrix, its textures [5] and symmetries are predicted from some theory beyond the Standard Model (SM). And the other way, in "bottom–up" approach, from neutrino mass matrix we can find all physical neutrino parameters as well as unphysical phases. For three and four neutrino states we can do everything analytically. For a larger number of neutrinos, only a numerical method can be used. We use the base where the charged lepton mass matrix is diagonal, so the unitary

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matrix which diagonalize neutrino mass matrix is also, with an accuracy to non-physical phases, the ordinary Maki–Nakagawa–Sakata–Pontecorvo (MNSP) mixing matrix.

In this paper both methods are presented. We will focus on the simplest, but still realistic, three dimensional case. For the "bottom–up" case we generalize the approach given in paper [6] were only CP conserving problems were considered. From this paper, among other things, we have learned that textures of $\mathcal{M}_\nu$ with number of zeros $n \geq 3$ do not reproduce experimental data (at 3σ C.L.), there are seven two-zero textures which give results in agreement with present data, some of them can produce normal, inverse and degenerate mass hierarchies.

The important goal of present considerations is to check if the CP breaking case is able to change the properties of the mass matrix which still predicts the correct neutrino parameters (Section 2). In the "top–down" approach we give exact formulas for the neutrino masses, mixing angles, Dirac and Majorana mixing phases. We show how the unphysical phases depend on the parametrization of the MNSP mixing matrix (Section 3). At the end we give some conclusion (Section 4).

2. Bottom–up method

For Majorana neutrinos, which we consider, the mass matrix $\mathcal{M}_\nu$ must be symmetric and to have CP symmetry breaking in the lepton sector, must be also complex. In general, N dimensional symmetric matrix can be described by $\frac{N^2+N}{2}$ independent parameters. In our case ($N = 3$) we can have 12 parameters — six modulus and six phases. Such a matrix can be diagonalized by unitary transformation

$$m_{\rm diag} = U^T \mathcal{M}_\nu U\,, \tag{1}$$

where unitary matrix U is parametrized by

$$U = f U_{\rm MNSP} P\,. \tag{2}$$

$U_{\rm MNSP}$ is the standard MNSP [7,8] mixing matrix as for Dirac neutrino

$$\begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - c_{12}e^{i\delta}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta}s_{13} & -c_{23}e^{i\delta}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix}, \tag{3}$$

where, as usually, we use abbreviation, *e.g.* $s12 = \sin(\theta_{12})$ and so on, and

$$f = \begin{pmatrix} e^{\imath\beta_1} & 0 & 0 \\ 0 & e^{\imath\beta_2} & 0 \\ 0 & 0 & e^{\imath\beta_3} \end{pmatrix}, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\imath\alpha_1/2} & 0 \\ 0 & 0 & e^{\imath\alpha_2/2} \end{pmatrix}. \tag{4}$$

For 3-dimensional case we have nine physical parameters: three masses (m_1, m_2, m_3), three mixing angels $(\theta_{12}, \theta_{13}, \theta_{23})$ and three phases δ — Dirac phase, and two Majorana phases: α_1, α_2.

Matrix f is composed of 3 non-physical and unmeasurable phases $\beta_i,\ i = 1, 2, 3$.

Using the reverse relation to (1) we can express all elements (separately imaginary and real parts) of the mass matrix as a function of

$$(\mathcal{M}_\nu)_{ik} = f_{ik}\left(\theta_{12}, \theta_{13}, \theta_{23}, m_1, m_2, m_3, \delta, \alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3\right) ,$$

and find their minimal and maximal values for current [9] experimental data. In such way, for the given neutrino mass hierarchy we are able to show each possible area as a function of the lightest neutrino mass. Such a distribution shows us, for example, possible texture zeros regions.

Only one plot for $(\mathcal{M}_\nu)_{ee}$ as a function of the lightest neutrino mass m_1 for normal mass hierarchy, separately for the absolute value and the phase is presented in Fig. 1.

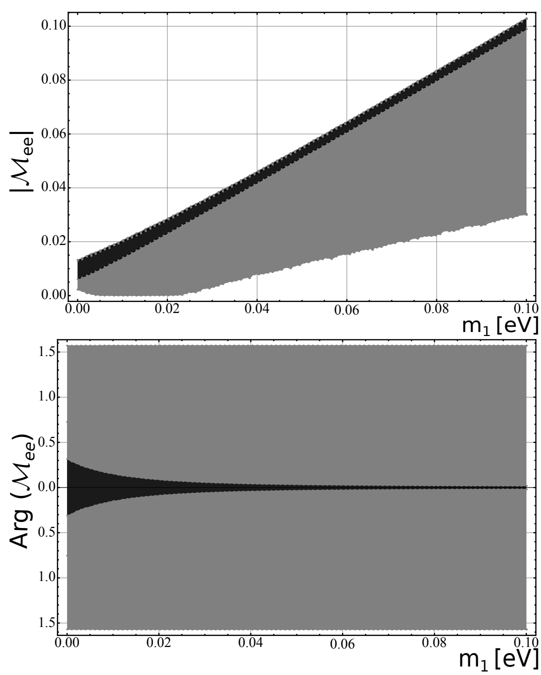

Fig. 1. Allowed values of modulus and phase (upper and lower figure respectively) of $\mathcal{M}_{ee}$ for normal mass hierarchy as a function of lightest neutrino mass m_1. Darker region shows, part with $\alpha_1,\ \alpha_2 = 0$ and $\delta \neq 0$. Lighter one shows, part with $\alpha_1,\ \alpha_2,\ \delta \neq 0$. Plot was made for 10^6 randomly generated oscillation parameters at 2σ C.L.

For the other elements see our page [10]. This plot reconstructs the results obtained in neutrinoless double beta experiments [11]. It is clear that the absolute value of $|(\mathcal{M}_\nu)_{ee}|$ is larger than zero (i) everywhere, for vanishing Majorana phase, and (ii) for $m_1 > 0.02$ eV, independently of the values of Majorana phases. The modulus and phase φ_{ee} of that element do not depend on unphysical phases β_i.

3. Top–down method

Here, we present some simple method of finding unambiguous analytical relations between oscillation parameters and mass matrix elements. First, for any Majorana neutrino mass matrix $\mathcal{M}_\nu$ we diagonalize the hermitian matrix

$$\mathcal{H} = {\mathcal{M}_\nu}^\dagger \mathcal{M}_\nu \,. \tag{5}$$

Such a matrix is diagonalized by unitary transformation

$$\mathcal{W}^\dagger \mathcal{H} \mathcal{W} = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} , \tag{6}$$

where the unitary matrix $\mathcal{W}$ is built from the eigenvectors of $\mathcal{H}$

$$\mathcal{W} = \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{pmatrix} \tag{7}$$

and eigenvalues m_i^2, $(i = 1, 2, 3)$ are squares of neutrino masses. The normalized eigenvectors are set out with an accuracy of phase. We can use that freedom in order to find matrix U which diagonalize $\mathcal{M}_\nu$ as in Eq. (1) with real and positive eigenvalues m_i

$$U = \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & e^{i\chi_3} \end{pmatrix} \begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{pmatrix} . \tag{8}$$

The new phases χ_i, $(i = 1, 2, 3)$ also depend on the element of $\mathcal{M}_\nu$. Now comparing Eq. (2) and (8) we can find relations

$$\sin\theta_{13} = |x_3| \,, \qquad \cos\theta_{13} = \sqrt{1 - |x_3|^2} \,, \tag{9}$$

$$\sin\theta_{23} = \frac{|y_3|}{\sqrt{1 - |x_3|^2}} \,, \qquad \cos\theta_{23} = \frac{|z_3|}{\sqrt{1 - |x_3|^2}} \,, \tag{10}$$

$$\sin\theta_{12} = \frac{|x_2|}{\sqrt{1 - |x_3|^2}} \,, \qquad \cos\theta_{12} = \frac{|x_1|}{\sqrt{1 - |x_3|^2}} \,, \tag{11}$$

$$e^{-i\delta} = \frac{|x_1||x_3||z_3| + |z_1|e^{i(\tau_1+\omega_3-\tau_3-\omega_1)}}{1-|x_3|^2}\,, \tag{12}$$

$$\frac{\alpha_1}{2} = 2\omega - \omega_1\,, \tag{13}$$

$$\frac{\alpha_2}{2} = \omega_3 + \delta - \omega_1\,, \tag{14}$$

$$\beta_1 = \chi_1 + \omega_1\,, \tag{15}$$

$$\beta_2 = \chi_2 + \eta_3 - \omega_2 + \omega_1\,, \tag{16}$$

$$\beta_3 = \chi_3 - \delta - \omega_3 + \tau_3 + \omega_1\,, \tag{17}$$

where $\omega_i = \mathrm{Arg}(x_i)$, $\eta_i = \mathrm{Arg}(y_i)$, $\tau_i = \mathrm{Arg}(z_i)$ respectively and ω_1, $\tau_3 = 0 \vee \pi$.

Analytical details for presented equations are given in the Appendix.

4. Conclusions and results

We have presented two possible approaches of studying neutrino mass matrix. For the "bottom–up" method we have got possible values of $\mathcal{M}_\nu$ matrix elements from current experimental data. As a example, we have presented $(M_\nu)_{ee}$ element which agrees with the neutrinoless double beta decay observations.

We have learned that Majorana phases are crucial to get some texture for neutrino mass matrix *e.g.* zero texture. Any other symmetry imposed on $\mathcal{M}_\nu$ can be studied in the same way. Whole set of plots and the computer program used for calculations can be seen on web-page [10]. From the "top–down" method we have learned how to find all physical neutrino parameters from given neutrino mass matrix which follow from any physics beyond the SM. This knowledge is useful in the future plan context. We would like to enlarge our analytical solutions for the $3+1$ mass matrix case and numerical solutions for 6×6 dimensional $\mathcal{M}_\nu$ (*i.e.* like presented in [12]).

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Appendix

We would like to give here formulas which express the physical neutrino parameters by elements of the mass matrix. Let us parametrize the $\mathcal{H}$ matrix in the following way

$$\mathcal{H} = \begin{pmatrix} A & Be^{i\phi_1} & Ce^{i\phi_2} \\ Be^{-i\phi_1} & D & Ee^{i\phi_3} \\ Ce^{-i\phi_2} & Ee^{-i\phi_3} & F \end{pmatrix}. \tag{18}$$

From Eq. (5) each element of the $\mathcal{H}$ can be easily expressed by the modulus and phases of $(\mathcal{M}_\nu)_{a,b} = m_{a,b}\, e^{i\varphi_{a,b}}$, $(a,b = e,\mu,\tau)$. The matrix $\mathcal{H}$ eigenvalues are given by

$$m_1^2 = \frac{2}{3}p\cos(\phi) - \frac{a}{3}\,, \tag{19}$$

$$m_2^2 = -\frac{a}{3} - \frac{1}{3}p\left(\cos(\phi) - \sqrt{3}\sin(\phi)\right)\,, \tag{20}$$

$$m_3^2 = -\frac{a}{3} - \frac{1}{3}p\left(\cos(\phi) - \sqrt{3}\sin(\phi)\right)\,, \tag{21}$$

where

$$p = \sqrt{a^2 - 3b}\,, \qquad \phi = \frac{1}{3}\arccos\left(-\frac{1}{p^2}\left(a^3 - \frac{9}{2}ab + \frac{27}{2}c\right)\right)\,, \tag{22}$$

and

$$a = -\operatorname{Tr}[\mathcal{H}]\,, \tag{23}$$

$$b = AD + AF + DF - B^2 - C^2 - E^2\,, \tag{24}$$

$$c = AE^2 + DC^2 + FB^2 - ADF - 2BCE\cos\left(\phi_1 + \phi_3 - \phi_2\right)\,. \tag{25}$$

The normalized $\mathcal{H}$ eigenvectors are given by

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \frac{1}{\sqrt{X_1^2 + |Y_1|^2 + |Z_1|^2}}\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}\,, \tag{26}$$

$$\begin{aligned} X_1 &= \left(D - m_1^2\right)\left(F - m_1^2\right) - E^2\,, \\ Y_1 &= CEe^{-i(\phi_2 - \phi_3)} - Be^{-i\phi_1}\left(F - m_1^2\right)\,, \\ Z_1 &= BEe^{-i(\phi_1 + \phi_3)} - Ce^{-i\phi_2}\left(D - m_1^2\right)\,, \end{aligned} \tag{27}$$

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \frac{1}{\sqrt{|X_2|^2 + Y_2^2 + |Z_2|^2}}\begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix}\,, \tag{28}$$

$$\begin{aligned} X_2 &= CEe^{i(\phi_2 - \phi_3)} - Be^{i\phi_1}\left(F - m_2^2\right)\,, \\ Y_2 &= \left(A - m_2^2\right)\left(F - m_2^2\right) - C^2\,, \\ Z_2 &= BCe^{i(\phi_1 - \phi_2)} - Ee^{-i\phi_3}\left(A - m_2^2\right)\,, \end{aligned} \tag{29}$$

$$\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = \frac{1}{\sqrt{|X_3|^2 + |Y_3|^2 + Z_3^2}} \begin{pmatrix} X_3 \\ Y_3 \\ Z_3 \end{pmatrix}, \tag{30}$$

$$\begin{aligned} X_3 &= BEe^{i(\phi_1+\phi_3)} - Ce^{i\phi_2}\left(D - m_3^2\right), \\ Y_3 &= BCe^{-i(\phi_1-\phi_2)} - Ee^{i\phi_3}\left(A - m_3^2\right), \\ Z_3 &= \left(A - m_3^2\right)\left(D - m_3^2\right) - B^2 . \end{aligned} \tag{31}$$

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