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PARTICLE PRODUCTION AND ANGULAR CORRELATIONS AT HIGH ENERGY*

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I discuss two topics of importance to phenomenological applications of perturbative high energy evolution. In the first part I discuss a mechanism which leads to long range rapidity and angular correlations in particle production in dense environment. I argue that positive angular correlations are leading $1/N_c$ effect and may be responsible for the "ridge" structure observed in high multiplicity p–p collisions at LHC. In the second part I describe the setup for calculation of particle production at high transverse momenta and high energy, which fully takes into account the perturbative saturation effects and the leading twist physics. Here I note that recent calculations of inclusive particle production within the high energy approach are missing a term due to inelastic scattering of projectile partons. This piece has to be included in order that the results have proper perturbative limit. Its inclusion is expected to affect strongly the high momentum tail of the particle spectrum.

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1. Introduction

The last several years have seen a lot of activity applying the ideas of gluon saturation [1], or Color Glass Condensate [2] to analyze various data. The saturation based on calculational techniques have advanced considerably during the last two–three years. In particular, large part of next-to-leading corrections [3] is now taken into account in calculating the evolution of gluon density to high energy. This allowed for good fits to the HERA DIS data at low x [4]. Lately many aspects of the RHIC (and the LHC) data have been analyzed in the framework of saturation physics. These include the single inclusive particle production [5,6] and the two hadron correlations

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at forward rapidity [5, 7] in dA collisions, and more recently description of single particle spectra [8] and attempts to understand [9] ridge in p–p collisions at the LHC [10].

Although the saturation based interpretation is philosophically very simple and appealing, alternative interpretations are also available and it is important to be able to differentiate between them. One would like to understand to what extent the data really unambiguously supports, the idea of saturation. One of the problems we are faced with in this regard, is that the saturation based calculations, although in principle rooted in controlled perturbative approach to fundamental QCD physics, in practical implementations rely on phenomenological or semi-phenomenological ansatze and variety of shortcuts. As a result, it is sometimes difficult to understand what features of the theoretical results are genuine and robust predictions of saturation, and which are model dependent transient features.

This paper is based on two recent papers [11, 12] which try to contribute to clarification of these issues. The first part is based on [11]. Here I argue that the appearance of long range rapidity and angular correlations at high energy is a very generic phenomenon and qualitatively does not depend on models of high energy scattering. In the second part, based on [12] I reanalyze the basis of the calculational approach to particle production at forward rapidity, and conclude that an important contribution to particle production has been omitted in the recent numerical calculation. This contribution is in fact most sensitive to saturation in high energy evolution, and thus it would be especially interesting to include it in future calculations.

2. Angular correlations in gluon emission

The CMS observation of angular and long range rapidity correlations in the hadron spectrum, the so-called "ridge" in proton–proton collisions [10], has triggered a lot of discussions in recent literature [13, 9]. The purpose of this paper is to point out that at high energy, rapidity and angular correlations between produced particles are to be expected on very general grounds. The framework of our discussion here is similar to that of [9], but the argumentation will be quite general without referring to specific models of high energy evolution and/or hadronic wave function.

Consider high energy scattering of a hadronic projectile on a stationary target in the lab frame. Since the projectile is very energetic, its wave function is approximately boost invariant. The boost invariance is of course only approximate, since at too high energy the rapidity evolution is important, and that introduces rapidity dependence inside the wave function. However, for rapidity intervals $\Delta Y < \frac{1}{\alpha_s}$ the evolution is not important [14], and thus can be neglected if the produced particles are separated by rapidity interval which is not parametrically large.

The boost invariance leads naturally and straightforwardly to long range rapidity correlations. Simply put, the incoming wave function is the same at rapidity Y_1 and Y_2. The gluon distribution at rapidity Y_1 and Y_2 are the same, these gluons scatter exactly on the same target, and thus whatever happens at Y_1 also happens at Y_2. If for a particular target field configuration a gluon is likely to be produced at Y_1 at some impact parameter, a gluon is also likely to be produced at Y_2 at the same impact parameter: *et voilà* — correlations. This is especially true in the context of the projectile wave function dominated by the large "classical" Weizsacker–Williams field, since in this case fluctuations in the wave function are small and the gluon density *configuration by configuration* is almost the same at all rapidities. This is the property of the hadronic wave function at high energy [15]

$$|\varPsi\rangle = \exp\left\{i\int d^2x b_i^a(x)\int d\eta\left(a_i^{\dagger a}(x,\eta)+a_i^a(x,\eta)\right)\right\}B\left(a,a^\dagger\right)|\psi\rangle\,. \quad (1)$$

Here ψ is the wave function of valence charges, determining the distribution of the charge density ρ, B is a Bogolyubov-type operator of the soft gluon fields a, and the Weizsacker–Williams field b is given in terms of ρ via classical Yang–Mills equations of motion. For large projectile the WW field is parametrically large $b\sim\frac{1}{g}$, while the Bogolyubov operator B produces the fluctuations of the gluon field of the order of unity. Thus for fixed $\rho(x)$ the gluon density fluctuates very weakly around large average value determined by the classical field

$$n=\left\langle a^\dagger a\right\rangle \propto b^2\sim O\left(\frac{1}{\alpha_{\mathrm{s}}}\right)\,,\qquad \left\langle n^2\right\rangle-\langle n\rangle^2\sim 1\,. \quad (2)$$

The smallness of the fluctuations is clearly helpful. Although *the wave function* at different rapidities in a boost invariant projectile must be the same, the magnitude of the *color field* (and therefore the number of gluons) may differ at different values of Y for the same configuration of the valence color charge density ρ, if the fluctuations in this wave function are significant. Thus in the same scattering event there may be significant differences between particle production at different rapidities. Still, although the quasi-classical nature of the state Eq. (1) ensures long range rapidity correlations at large values of ρ, it is not absolutely necessary. Even in the presence of considerable fluctuations in the soft gluon wave function, one nevertheless would expect positive correlations in rapidity. The only really necessary condition is that the density of incoming partons is large enough, so that there is a large probability to produce more than one particle at a given impact parameter (we will quantify what we mean by "given impact parameter" shortly).

Thus the long range rapidity correlations come practically for free whenever the energy is high enough so that the wave function of the incoming hadron is approximately boost invariant, and there is very little in the actual dynamics of the collision that can affect this feature. But by almost exactly the same logic we must conclude that positive angular correlations are also almost unavoidable. Indeed, if two gluons hit the target at the same impact parameter, their scattering amplitude is determined by the same configuration of the target field. Thus, if the first gluon is likely to be scattered with momentum $\boldsymbol{q}$, the same is true for the second gluon. One, therefore, expects clear forward correlations for gluons that scatter at the same impact parameter. Of course, the two gluons will not scatter always with exactly the same momentum transfer even if they hit at exactly the same impact parameter, since even a fixed configuration of target fields corresponds to a nontrivial probability distribution of momentum transfer. Nevertheless, given that this distribution has a maximum at some particular momentum transfer, the angular correlations must be very generic.

To better understand why angular correlations naturally arise in the context of high energy let us briefly recap our understanding of the transverse structure of the hadron in the saturation regime. It is convenient to think of the distribution of the (color) electric field configurations in the target.

The target wave function is characterized by the saturation momentum $Q_{\rm s}$. The saturation momentum plays a dual role in the hadronic wave function. First, it measures the typical magnitude of electric field in the wave function. The scattering amplitude of a dipole on the target is given in terms of simple parton scattering amplitude $S(x) = Pe^{ig\int dx^+A^-(x)}$ as $N(r) = 1 - \frac{1}{N_c}{\rm tr}[S^\dagger(0)S(r)]$. The vector potential is simply related to the electric field as $\partial_i A^- = F^{-i}$. Let us for, convenience, define electric field integrated over the longitudinal extent of the target, $E_i = \int dx^+ F^{-i}$. The dipole scattering amplitude is then given in terms of gE, and assuming for illustrative purposes that odd powers of E average to zero in the hadronic ensemble, we have roughly

$$N(\vec{r}) \sim 1 - e^{-\frac{1}{2}\left(g\vec{r}\cdot\vec{E}\right)^2} . \tag{3}$$

This is of the order of unity for $r_{\rm s}^2 = Q_{\rm s}^{-2} = (gE)^{-2}$.

On the other hand, it is known that the field components with transverse momenta $p_{\rm T} < Q_{\rm s}$ are suppressed in the wave function [16]. This means that the electric fields in the target are correlated on the length scale $\lambda \sim Q_{\rm s}^{-1}$. Thus the saturation momentum doubles up as the inverse of the correlation length of target color fields. Typical field configurations in the target can thus be thought of having a domain-like structure of Fig. 1.

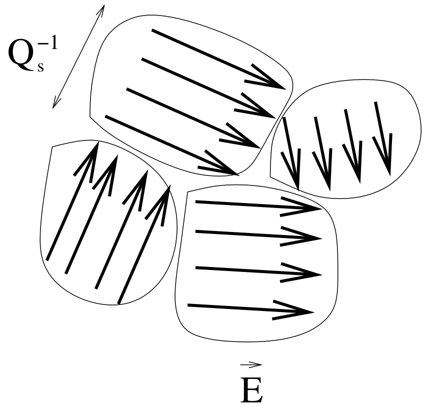

Fig. 1. Typical color electric field configuration in the target.

Now consider a projectile parton with charge q impinging on one of the domains of the target. While traversing the target field, the parton acquires transverse momentum

$$\delta\vec{P} = gq \int dx^+ \vec{F}^- = gq\vec{E}\,. \tag{4}$$

A parton at a different rapidity but with the same charge will pick up exactly the same transverse momentum if it scatters on the same "domain". This, of course, results in positive angular correlation of produced gluons.

We note that this simple picture also suggests that angular correlations at angle ϕ and $\phi + \pi$ have equal strength. At high energy, particle production is dominated by gluons. Gluons, of course, belong to real representation of the gauge group, thus it is equally probable to find an incoming gluon with charge q and charge $-q$ in the projectile wave function at any rapidity. Suppose, for example, that on a given configuration the color field in the target is in the third direction in the color space $E_i^a = E_i\delta^{a3}$, while in the incoming projectile the gluon corresponds to the vector potential in the second direction A_i^2. One can always write $A^2 = -i/2(A^+ - A^-)$, where $A^+ = A_1 + iA_2$ is positively charged with respect to color charge in the third direction, and A^- is negatively charged. Thus, necessarily equal number of gluons in the incoming projectile have opposite sign charges and are kicked in opposite directions while scattering on the target. This produces equal strength correlations at angles zero and π. This feature of equal forward and backward correlations was noted in [11]. For quarks which carry fundamental charges, this degeneracy should be absent and taking into account the projectile quarks will lead to stronger positive angular correlation than the negative one.

The previous discussion is clearly oversimplified, since it does not address some important points. For example, for a soft gluon to be produced in the final state, it is not enough for it to acquire some transverse momentum. It also must decorrelate from the valence charge that emitted it in the incoming wave function. Otherwise, it will not be produced as a particle in the final state, but rather as part of the Weizsacker–Williams field of the produced valence parton. We will, therefore, turn to an explicit formula that determines the gluon double inclusive spectrum in order to see to what extent this explicit expression is consistent with our simple discussion.

According to [17] (see also [18]) the inclusive two gluon production probability is given by

$$\frac{dN}{d^2pd^2kd\eta d\xi} = \left\langle A^{ab}_{ij}(k,p)A^{*ab}_{ij}(k,p)\right\rangle_{\mathrm{P,T}} \tag{5}$$

with the amplitude

$$\begin{aligned}
A^{ab}_{ij}(k,p) = &\int\limits_{u,z} e^{ikz+ipu}\int\limits_{x_1,x_2}\\
&\times \left\{f_i(z-x_1)\left[S(x_1)-S(z)\right]\rho(x_1)\right\}^a\left\{f_j(u-x_2)\left[S(u)-S(x_2)\right]\rho(x_2)\right\}^b\\
&-\frac{g}{2}\int\limits_{x_1} f_i(z-x_1)f_j(u-x_1)\left\{\left[S(x_1)-S(z)\right]\tilde\rho(x_1)\left[S^\dagger(u)+S^\dagger(x_1)\right]\right\}^{ab}\\
&+g\int\limits_{x_1} f_i(z-u)f_j(u-x_1)\left\{\left(S(z)-S(u)\right)\tilde\rho(x_1)S^\dagger(u)\right\}^{ab}\,.
\end{aligned} \tag{6}$$

Here

$$f_i(x-y) = \frac{(x-y)_i}{(x-y)^2} \tag{7}$$

and we have defined $\tilde\rho \equiv -iT^a\rho^a$. The charge density is normalized such that for a single gluon $\rho^a = gT^a$. In these formulae $\rho^a(x)$ is the valence color charge density in the projectile wave function, while $S^{ab}(x)$ is the eikonal scattering matrix determined by the target color fields. The average in Eq. (5) denotes averaging over the projectile and the target wave functions. We also note that in this expression the gluon with momentum p is assumed to have larger rapidity, and thus the emission of the two gluons is not completely symmetric.

The physical meaning of the three terms in Eq. (6) is straightforward. The first term corresponds to independent production of the two gluons. This term is leading in the limit of large color density $\rho \sim 1/g$. One should keep in mind, however that in this limit other terms not included in Eq. (6)

are equally important [19, 20]. The second term corresponds to production of two gluons emitted from the same color source in the incoming projectile wave-function. The third term corresponds to the process whereby the softer gluon has been emitted in the wave function by the harder one, with both gluons subsequently produced in the collision. In terms of BFKL ladders, the (square of the) first term is a part of the diagram containing two independent ladders, while the (square of the) last two terms describe emission of two gluons contained in the same BFKL ladder.

To calculate the cross-section one has to square the amplitude. This produces many terms, but in the text we only reproduce one part of this expression which arises from squaring the first term in the amplitude Eq. (6) which is responsible for independent production of the two gluons.

$$\frac{dN}{d^2pd^2kd\eta d\xi} = \left\langle\sigma^4\right\rangle_{\mathrm{P,T}} \tag{8}$$

with

$$\begin{aligned}\sigma^4 \;=\; & \int\limits_{u,z,\bar{u},\bar{z}} e^{ik(z-\bar{z})+ip(u-\bar{u})} \int\limits_{x_1,x_2,\bar{x}_1,\bar{x}_2} \\ & \times\vec{f}(\bar{z}-\bar{x}_1)\cdot\vec{f}(x_1-z)\,\vec{f}(\bar{u}-\bar{x}_2)\cdot\vec{f}(x_2-u) \\ & \times\left\{\rho(x_1)\left[S^\dagger(x_1)-S^\dagger(z)\right]\left[S(\bar{x}_1)-S(\bar{z})\right]\rho(\bar{x}_1)\right\} \\ & \times\left\{\rho(x_2)\left[S^\dagger(u)-S^\dagger(x_2)\right]\left[S(\bar{u})-S(\bar{x}_2)\rho(\bar{x}_2)\right]\right\}\,.\end{aligned} \tag{9}$$

It is very easy to see that it indeed produces angular correlations. One can write it as

$$\sigma^4(k,p) = \langle\sigma(k)\sigma(p)\rangle_{\mathrm{P,T}}\,, \tag{10}$$

where

$$\begin{aligned}\sigma(k) \;=\; & \int\limits_{z,\bar{z}} e^{ik(z-\bar{z})} \int\limits_{x_1,\bar{x}_1} \vec{f}(\bar{z}-\bar{x}_1)\cdot\vec{f}(x_1-z) \\ & \times\left\{\rho(x_1)\left[S^\dagger(x_1)-S^\dagger(z)\right]\left[S(\bar{x}_1)-S(\bar{z})\right]\rho(\bar{x}_1)\right\}\,.\end{aligned} \tag{11}$$

For *fixed configuration* of the projectile sources $\rho(x)$ and target fields $S(x)$, the function $\sigma(k)$ as a function of momentum has a maximum at some value $\boldsymbol{k}=\boldsymbol{q}$. Therefore, clearly the product in Eq. (10) is maximal for $\boldsymbol{k}=\boldsymbol{p}=\boldsymbol{q}$. The value of the vector $\boldsymbol{q}$ of course differs from one configuration to another, but the fact that momenta $\boldsymbol{k}$ and $\boldsymbol{p}$ are parallel does not. Therefore, after averaging over the ensemble $\sigma^4(k,p)$ has maximum at relative zero angle between the two momenta.

We reiterate, that even though averaged over all configuration $\langle\sigma(k)\rangle_{\mathrm{P,T}}$ must be isotropic, there is absolutely no reason for it to be isotropic for any given configuration. The strength of the maximum depends, of course, on the detailed nature of the field configurations constituting the two ensembles (the projectile and the target). We will discuss some qualitative features of these in the next section. But first, it is interesting to ask is the maximum of $\sigma(k)$ unique, or perhaps there is finite degeneracy. It is in fact easy to see that the maximum is doubly degenerate. The probability $\sigma(k)$ can be written in terms of the single gluon production amplitude $a(k)$

$$a_i^a(k) = \int dz e^{ikz} \int_{x_1} f_i(z-x_1)[S(x_1)-S(z)]^{ab}\rho^b(x_1)\,. \tag{12}$$

Since the amplitude a is real in *coordinate space*, we have

$$\sigma(k) = a(k)a^*(k) = a(k)a(-k)\,. \tag{13}$$

Configuration by configuration this is clearly symmetric

$$\sigma(k) = \sigma(-k)\,. \tag{14}$$

The "classical" contribution to the two-particle inclusive production probability is therefore symmetric under

$$\sigma^4(k,p) = \sigma^4(-k,p) \tag{15}$$

and must have two degenerate maxima — at relative angles $\Delta\phi = 0, \pi$. This degeneracy was alluded to earlier.

The third term in Eq. (6), where the gluon produced at the point z is emitted from the other observed gluon at the point u, disfavors production at the same impact parameter because of the suppression factor $S(u) - S(z)$. The two gluons when scattered at the same impact parameter do not decohere, but rather scatter as a single coherent state, with the gluon at z emerging in the final state as part of the Weizsacker–Williams field of the gluon at u. On the other hand, whenever the two gluons do decohere, since they were correlated in the incoming wave function, they emerge in the final state with large relative transverse momentum. Thus this particular term in the amplitude mostly leads to back-to-back production in the final state and is responsible for the large away side, rapidity independent maximum at relative angle π, prominently present in the data.

The second term in Eq. (6) favors production at the point u close to x_1, but z far from x_1. Thus one expects the momentum of the gluon produced at z to be uncorrelated with that of the gluon produced at u. Whenever the gluon at u is produced with significant transverse momentum, the balancing transverse momentum resides at the "valence" rapidity. This term is therefore responsible for the away side peak between one of the observed particles and another particle produced at a more forward rapidity.

One can estimate the overall magnitude of the correlation by the following simple argument. In order for two produced gluons to be correlated in the final state, they have to be close in the initial state and also scatter off the same target field. We will assume that both the target and the projectile are characterized by corresponding saturation momenta $Q_{\rm s}^{\rm P(T)}$. The inverse of the correlation momentum is the correlation length in the hadron $L \sim 1/Q_{\rm s}$. It is reasonable to expect that typical field configurations contributing to the hadronic ensemble of, say the target, have variation only on distance scale greater than $1/Q_{\rm s}$. Thus the two gluons that hit the target at distance $x < 1/Q_{\rm s}^{\rm T}$ apart from each other scatter on the same field. By the same argument, for the two incoming gluons to be in the same state they have to be located in the impact parameter plane no further than $1/Q_{\rm s}^{\rm P}$ away from each other. Thus for correlated production the two gluons need to be within the radius $\frac{1}{Q_{\rm c}^{\max}}$ of each other, where $Q_{\rm c}^{\max}$ is the larger of the two saturation momenta $Q_{\rm s}^{\rm P}$ and $Q_{\rm s}^{\rm T}$. On the other hand, the total number of produced gluons is proportional to the total transverse area of the smaller between the two objects participating in collision. Thus parametrically

$$\left[\frac{d^2N}{d^2p\,d^2k} - \frac{dN}{d^2k}\frac{dN}{d^2p}\right] \Big/ \frac{dN}{d^2k}\frac{dN}{d^2p} \sim \frac{1}{(Q_{\rm s}^{\max})^2 S_{\min}}\,. \tag{16}$$

This estimate is parametrically the same as given in [14].

We would like at this point to make contact with the recent paper [9]. The calculation of gluon production in [9] is based on simplified version of Eq. (10) supplemented with specific prescription for averaging over the projectile and target fields. Specifically, [9] expands the scattering matrix S to first order in target fields, and keeps only the leading term $S(x) \to 1 + \alpha(x)$. The expression for σ^4 then becomes a homogeneous function of the target and projectile fields

$$\sigma^4 \sim (\rho\alpha\alpha\rho)(\rho\alpha\alpha\rho)\,. \tag{17}$$

For simplicity we suppress the color indices and transverse coordinates on all the functions. One next averages over the charge densities assuming Gaussian ensemble

$$\left\langle \rho^4 \right\rangle = 3\left\langle \rho^2 \right\rangle \left\langle \rho^2 \right\rangle \tag{18}$$

and similarly for the target. And finally, the high energy evolution is included by substitution

$$\langle \rho(x)\rho(y) \rangle \to \Phi(x-y) \tag{19}$$

with Φ taken to be a solution of the Balitsky–Kovchegov equation [21]. Although the angular distribution has not actually been calculated in [9], the authors argued that the correlation should, in fact, have a maximum at collinear momenta.

Our general discussion provides an intuitive explanation for this result and also makes it clear that the presence of the correlations does not depend on the specifics of the approximation used to estimate $\frac{d^2N}{d^2kd^2p}$. The magnitude of the effect, however, may depend on the approximation quite strongly. We next want to comment on this issue.

From Eqs. (10), (11) we know that the basic averages that one needs to calculate are of the type

$$\left\langle \left[S^\dagger(x)S(z)\right]^{ab}\left[S^\dagger(y)S(u)\right]^{cd}\right\rangle_{\rm T} \tag{20}$$

and similarly for the projectile

$$\left\langle \rho^a(x)\rho^b(\bar{x})\rho^c(y)\rho^d(\bar{y})\right\rangle_{\rm P}\,. \tag{21}$$

The Gaussian averaging procedure described above is fairly restrictive, in the sense that as any Gaussian averaging it probably tends to underestimate correlations. In particular, Gaussian averaging over color singlet ensemble necessarily puts the densities in Eq. (21) pairwise into color singlet states. As pointed out in [22] this leaves out some possible configurations which are overall color singlets, but where no two factors of ρ form a color singlet separately. This for example happens, when the factors of ρ are pairwise in color octets, with the two octets forming an overall singlet. Such configurations in principle can also contribute to the correlated part of the particle production. Formally, they are suppressed in the large N_c limit. However, the correlated part of the production probability itself *when calculated with the Gaussian averaging* is also suppressed by $1/N_c^2$ relative to the uncorrelated part, and thus omission of these terms may be dangerous [22]. Physically these terms correspond to interference contributions. For example, when the two factors of ρ in Eq. (11) are in an octet, this corresponds to a situation when the charge densities in the amplitude and complex conjugate amplitude are different, but still the same gluon in the final state is produced due to the difference in the scattering factors S in the amplitude and the conjugate amplitude.

Although these $1/N_c^2$ suppressed terms are interesting, taking them properly into account requires one to go beyond the dipole model [23] and the BK equation, and in the dense region studying the full B-JIMWLK evolution [2]. However, it is not obvious that even in the leading order in $1/N_c$ the

Gaussian approximation is adequate to discuss correlated production. Here we would like to discuss only these leading order terms. We will argue that Gaussian averaging procedure is likely to miss terms in the correlated production probability which are of the same order in $1/N_c$ as the uncorrelated piece.

The leading N_c piece in Eq. (10) comes from the configuration where the charge densities in each one of the single gluon production amplitudes are in the color singlet. The relevant average to calculate is

$$\begin{aligned}&\left\langle \rho^a(x_1)\rho^a(\bar{x}_1)\rho^b(x_2)\rho^b(\bar{x}_2)\right\rangle_{\mathrm{P}} \times \\ &\left\langle \mathrm{tr}\left\{\left[S^\dagger(x_1)-S^\dagger(z)\right][S(\bar{x}_1)-S(\bar{z})]\right\} \mathrm{tr}\left\{\left[S^\dagger(x_2)-S^\dagger(u)\right][S(\bar{x}_2)-S(\bar{u})]\right\}\right\rangle_{\mathrm{T}} .\end{aligned} \tag{22}$$

Let us first concentrate on the projectile average. As mentioned above, averaging with a Gaussian weight one obtains in the leading order in $1/N_c$

$$\begin{aligned}&\left\langle \rho^a(x_1)\rho^a(\bar{x}_1)\rho^b(x_2)\rho^b(\bar{x}_2)\right\rangle_{\mathrm{Gaussian}} \\ &= \left\langle \rho^a(x_1)\rho^a(\bar{x}_1)\right\rangle_{\mathrm{Gaussian}} \left\langle \rho^b(x_2)\rho^b(\bar{x}_2)\right\rangle_{\mathrm{Gaussian}} .\end{aligned} \tag{23}$$

In this approximation therefore, clearly the correlated piece in the production probability vanishes, and only the subleading in $1/N_c$ correction resurrects the correlations. We stress, however, that this is not the result of the leading N_c approximation *per se*, but rather of the Gaussian averaging procedure.

It may be tempting to think that factorization in the large N_c limit is natural due to presence of large number of degrees of freedom, and therefore in some sense large N_c might act similarly to heavy nucleus. However, this is not the case. Even though the number of degrees of freedom is large, even in the large N_c limit the theory has legitimate states which contain small number of particles. A color dipole is an example of such state. It is a superposition of many states (different color orientations) of two particles, rather than a state with many particles. In a state like this the central limit theorem does not hold, the fluctuations in density can be large even in the large N_c limit, and it is the large fluctuations in the ensemble that break factorization of correlation functions.

In fact, a very similar question was considered a while ago in [24] in connection with factorization of dipole densities in the dipole model [23]. Indeed the observable we are interested in Eq. (22) is rather similar to the dipole density

$$n(x_1,\bar{x}_1) = \left(\rho^a(x_1)-\rho^a(\bar{x}_1)\right)^2 . \tag{24}$$

As shown in [24] within the dipole model (which is defined entirely within the large N_c limit [27]) the product of two densities does not factorize, but rather behaves as

$$\langle n(x_1,\bar{x}_1)n(x_2,\bar{x}_2)\rangle - \langle n(x_1,\bar{x}_2)\rangle\,\langle n(x_2,\bar{x}_2)\rangle \sim \langle n(x_1,\bar{x}_2)\rangle\,\langle n(x_2,\bar{x}_2)\rangle\, b^{-\lambda}\,, \tag{25}$$

where b is the transverse distance between the two dipoles and λ is a number, whose exact value is unimportant for us. This result hods in the limit where the distance between the two dipoles is much greater than their respective sizes, and thus it does not display any angular correlation between the orientations of the two dipoles. Nevertheless Eq. (25) clearly exhibits the fact that factorization is not an inherent property of the large N_c limit. Once we accept that the factorization is broken, it is natural to expect that the actual correlation function in the regime where the two dipoles overlap in space, also exhibits angular correlations in the orientation of the two dipoles.

Note that this is precisely the regime relevant to our discussion of angular correlations in emission. The *same* configuration of color charges produces the same gluons (at different rapidities), which produce correlated hadrons in the final state. Thus the most important region of the phase space is when all four points in the correlator Eq. (22) are close to each other, in the sense that they are all within the correlation length $1/Q_{\rm s}$. It is very hard to imagine that in this regime factorization holds (see [25,26] for more discussion of such correlations). Thus we indeed expect that any realistic non-Gaussian weight function for the ensemble averaging will lead to a nonvanishing contribution to the correlated piece of gluon production even in the large N_c limit.

Turning to the target averaging in Eq. (22), the terms that have to be averaged are of the type of observables described in the large N_c limit by the dipole model [23]

$$\left\langle \mathrm{tr}\left\{\left[S^\dagger(x)S(z)\right]\right\}\mathrm{tr}\left\{\left[S^\dagger(y)S(u)\right]\right\}\right\rangle_{\rm T} = \langle s(x,z)s(z,x)s(y,u)s(u,y)\rangle_{\rm T}\,, \tag{26}$$

where $s(x,y) = \mathrm{tr}[S_{\rm F}^\dagger(x)S_{\rm F}(y)]$ — is the scattering amplitude of the fundamental dipole, and the equality in Eq. (26) holds in the large N_c limit. The approximation which is frequently used in the literature to calculate the averages of this type also invokes factorization

$$\langle s(x,y)s(u,v)\rangle = \langle s(x,y)\rangle\langle s(u,v)\rangle\,. \tag{27}$$

The target averaging of [9] would follow from this approximation in the limit of weak fields. When the dipole (x,y) is far from the dipole (u,v) (much further than $1/Q_{\rm s}$), the factorization is a good approximation since the fields on which the scattering amplitude is calculated are not correlated with each other. However, as before, we are clearly interested in the case where all the points are within the distance of the order of $1/Q_{\rm s}$ or smaller. In this case, just like for the projectile, the factorization of Eq. (27) is not a property of the large N_c limit but is rather an *ad hoc* assumption, used only due to its simplicity.

Strict factorization of the type Eq. (27) is only possible if the statistical ensemble consists of a single configuration. There is, however, no reason to expect that in the large N_c limit fluctuations around some leading configurations are suppressed by powers of $1/N_c$. For example, the energy evolution of s is given by the dipole model Hamiltonian, which does not contain N_c at all. The probability distribution of the dipole model $W[s]$ evolves with rapidity according to [25, 27]

$$\frac{d}{dY}W[s] = \frac{\bar{\alpha}_{\rm s}}{2\pi}\int_{x,y,z} \frac{(x-y)^2}{(x-z)^2(z-y)^2}\,[s(x,y)-s(x,z)\,s(y,z)]\,\frac{\delta}{\delta s(x,y)}W[s] \tag{28}$$

with $\bar{\alpha}$ — the 't Hooft coupling, which is finite at infinite N_c. Thus any nontrivial initial distribution $W[s]$ evolves smoothly to higher energy and remains nontrivial.

The recent paper [28] explores the question of what happens to correlations that are present in the initial ensemble, as rapidity grows. Technically one chooses an ensemble $W_0[s]$ of initial configurations $s(x,y)$, which contains short range correlations. These correlations should be confined to within the saturation radius, that is $\langle s(x,y)s(u,v)\rangle \neq \langle s(x,y)\rangle\langle s(u,v)\rangle$ only when $|x-u|,|y-v| < 1/Q_{\rm s}$. In [28] no impact parameter dependence was included, and the results should be understood as correlations at the same impact parameter. Each configuration of the ensemble was evolved independently according to the BK equation [27]. The correlations at the final rapidity are then calculated by averaging the correlator calculated in the final ensemble over the ensemble of initial conditions. The results are that initial correlations decrease very quickly with rapidity, approximately exponentially. The physics of this decrease, as elucidated in [28], is fairly simple. It amounts to the fact that the correlations are expected to appear at distances smaller than saturation radius, and the BK equation is simply not a valid evolution in this regime. To properly include correlations one should use the KLWMIJ evolution [29] at short distances stitched to the JIMWLK (or BK) evolution at large separation. In other words, one must include the Pomeron loop contributions.

To summarize, there are good reasons to expect that the factorization of both projectile and target averages is broken *at leading order in* $1/N_c$ in the kinematical domain relevant to the correlated production of particles. To study this question one certainly has to go beyond simple rotationally invariant solutions of the BK equation and moreover, include Pomeron loops in high energy evolution. While technically challenging, it would be very interesting to understand and quantify this effect.

The angular correlations originate from configuration by configuration fluctuations of the projectile and target structure in the transverse plane away from a rotationally invariant state. The effect we have discussed here has several tell-tale features. Produced particles are correlated in angle, with forward and backward correlations being of equal strength in the case where the two colliding objects are nuclei. When the colliding objects are not dense, there is an additional contribution to particle production, from a "single ladder" which significantly enhances back-to-back correlations. This contribution is responsible for the bulk of the observed back-to-back correlated production. The correlation is present also in the magnitude of the transverse momentum and not just in the angle. The single gluon production probability $\sigma(k)$ must have a maximum at momenta of the order of $Q_{\rm s}$. Thus most of the correlated gluons are emitted with the momentum of roughly this magnitude and the correlation is maximal at $|k|, |p| \sim Q_{\rm s}$. The latter correlation in fact does not require local rotational asymmetry of the projectile/target configurations. It would be interesting to try and measure these correlations as well.

The relative magnitude of the forward correlations should initially increase with energy for p–p collisions, since the relative importance of the single ladder terms diminishes. Interestingly however, the estimate of Eq. (16) suggests that the effect decreases with energy, once the colliding systems can be treated as saturated objects with well defined saturation momentum, since the saturation momentum grows with energy. Thus at very high energies the effect should disappear. If we apply this logic also to nuclear collisions, we should conclude that the effect if observed by ALICE should be significantly smaller than that observed by PHOBOS and STAR. Our discussion, of course, disregards the effects of flow, which are generally believed to be very important for nucleus–nucleus collisions. The latest STAR data [30] support this view. It is possible therefore, that our considerations about angular correlations are not valid for nuclear collisions, in the sense that the main mechanism of collimation is indeed due to the flow. It would nevertheless be interesting to try and disentangle the flow effects from the intrinsic correlations in the initial state discussed in this note. We also note that the estimate Eq. (16) refers not only to angular correlation, but rapidity correlation in general. Thus independently of the question of radial flow, if the observed long range rapidity correlations are due production from correlated domains in the boost invariant incoming wave function, the trend should be that of decreasing correlated production going to higher energy.

We have also argued that the correlations must survive also in the leading order in $1/N_c$ expansion. Their subleading nature in current numerical implementations is due to the factorization assumption which is not valid in the region of the phase space relevant for the correlated production. We believe that improvement of this aspect of current calculations is imperative in order for the results to be quantitatively reliable.

Finally, we want to elaborate a little more on possible relation of our considerations with ridge observed by CMS [10]. The ridge is not observed in minimal bias events but only in a small fraction of all events, which have high multiplicity. This suggests that the energy of the collision is not high enough so that the "average" configurations of the proton wave function do not contain enough gluons at different rapidities and the same impact parameter for correlations to be observable. The high multiplicity events are presumably due to rare fluctuations in the proton wave function which create "hot spots" — collisions between these hot spots then produce high multiplicity final states. Such hot spots will then, naturally, also lead to enhanced correlations since more particles than average are concentrated at the same impact parameter. This picture is also qualitatively consistent with the range of transverse momenta at which the correlation is actually observed. The ridge appears at transverse momenta in the range 1 GeV $< p_{\mathrm{T}} <$ 3 GeV. Given that experimentally one observes hadrons which are products of the hadronization of the emitted gluons, the transverse momentum of the gluons emitted initially must have been in the range 3–5 GeV. This is much too high to be associated with the saturation momentum Q_{s}, which at these energies should not be higher than 2 GeV. Hot spots however have a small radius and high density, and thus have a saturation momentum significantly higher than the minimal bias configurations $Q_{\mathrm{s}}^{(\text{hot spot})} \gg Q_{\mathrm{s}}$. The correlated gluons will then naturally have momentum of order $Q_{\mathrm{s}}^{(\text{hot spot})}$ which is much higher than the expected value of Q_{s}.

If the hot spot scenario is correct, it would mean that in order to describe the effect quantitatively one needs to have knowledge not of the "standard deviation" which characterizes fluctuations (and correlations) in the bulk of the wave function, but to understand the "tail" of the distribution which contains the hot spots.

3. Particle production at high transverse momenta

This paper addresses the calculation of single particle production in dense environment. RHIC experiments observed strong suppression in the particle production in dA at forward rapidities. The "state of the art" saturation calculation of this effect appears in [6]. Although the data is described quite well, there are some peculiarities to the results of [6]. First, a very small K-factor is required to fit the overall magnitude of the production of neutral pions, while no K-factor is required to fit the charged hadron multiplicity. Secondly and perhaps more worryingly, the suppression in the theoretical curves of [6] when extended to the LHC energies persists to extremely high transverse momenta, where one expects perturbation theory to be long applicable and R_{dA} to be equal to one.

The calculations of [6] are based on the "hybrid formalism" of [31]. In this approach the wave function of the projectile at large values of x is calculated perturbatively, without soft approximation, while the scattering of the projectile partons on the target fields is treated in the eikonal approximation. The exact treatment of the projectile function is of course necessary to describe particle production at forward rapidity, since these partons cannot be in any way considered soft. It has another advantage over the soft approximation in the projectile wave function since it satisfies energy (longitudinal momentum) conservation for the incoming projectile. The energy conservation in the scattering process is still violated, since the recoil (and radiation) in the scattering event itself is not taken into account in the eikonal approximation. Although energy conservation must be very important at large values of x_{F} and its effect has to be understood to make sure the treatment is consistent, we have nothing new to add to this point. In this paper we will revisit the derivation of inclusive particle production within the hybrid formalism *per se* relaxing only the collinear approximation made in [31]. Our goal is to identify the terms which where omitted in [31] but may nevertheless be important when the transverse momentum of produced particles is significantly higher than the saturation momentum of the target. As we will show, such terms, which do not correspond to collinear emission of the incoming projectile partons do indeed exist and contribute at leading twist. These terms have a simple physical interpretation and also have a simple form amenable to numerical implementations.

The expression derived in [31] and used in [6] for particle production has a very intuitively appealing and simple form

$$\begin{aligned}\frac{dN}{d^2kd\eta} &= \frac{1}{(2\pi)^2}\int\limits_{x_{\mathrm{F}}}^{1}\frac{dz}{z^2}\left[x_1 f_g\left(x_1,Q^2\right)N_{\mathrm{A}}\left(x_2,\frac{k}{z},b=0\right)D_{h/g}(z,Q)\right.\\ &\quad\left.+\varSigma_q x_1 f_q\left(x_1,Q^2\right)N_{\mathrm{F}}\left(x_2,\frac{k}{z},b=0\right)D_{h/q}(z,Q)\right]\,, \qquad (29)\end{aligned}$$

where $N_{\mathrm{A(F)}}(k)$ is the Fourier transform of the forward scattering amplitudes of the adjoint (fundamental) dipole. It describes the process whereby incoming low p_{T} partons scatter on the target independently of each other, acquiring large momentum k_{T} in the process, and subsequently fragment into observed hadrons. This process is certainly the origin of large part of produced particles.

However, there is another physical mechanism which produces large k_{T} particles in leading twist, whereby high p_{T} particles preexisting in the wave function of the incoming projectile scatter with only a small momentum transfer from the target. The soft scattering is nevertheless enough to decohere the incoming partons from the rest of the wave function so that they materialize as on shell particles in the final state. The high p_{T} partons in the projectile wave function arise due to DGLAP splitting of very forward partons. The scattering process is essentially just the inelastic scattering of the forward projectile partons with emission of gluons (or quarks/antiquarks). As we will show explicitly within the hybrid formalism, this mechanism of production is equally important as the one taken into account in Eq. (29) when the saturation momentum of the target is small. When Q_{s} is large, this contribution is somewhat suppressed, but may still be quantitatively quite large. Parametrically, while the contribution of Eq. (29) is roughly proportional to $\ln \frac{k_{\mathrm{T}}}{\Lambda_{\mathrm{QCD}}}$, the additional inelastic scattering contribution scales like $\ln \frac{k_{\mathrm{T}}}{Q_{\mathrm{s}}}$. It is thus only suppressed for $k_{\mathrm{T}} \sim Q_{\mathrm{s}}$ when $Q_{\mathrm{s}} \ll \Lambda_{\mathrm{QCD}}$, and even then the suppression is merely logarithmic. Given that for RHIC data $Q_{\mathrm{s}}/\Lambda_{\mathrm{QCD}} \sim 5$, it seems prudent to keep this contribution in numerical calculations.

It is quite clear that taking into account the inelastic mechanism must bring the calculation of particle production into agreement with the perturbative result at large p_{T}. Thus we hope that including this contribution will bring R_{dA} close to unity at reasonable values of p_{T}. It is also interesting to note that the final states of the inelastic scattering are quite different from those of the elastic one. The elastic piece is dominated by quarks in the final state, while the inelastic one contains comparable number of quarks and gluons. Since the fragmentation functions of quarks are very different, we expect it to affect the relative magnitude of neutral pion and charged hadron production and thus be relevant to the problem of a very small K factor for neutral pion production encountered in [6]. Whether it helps or makes things worse remains to be seen. Naively one expects gluons to fragment predominantly into neutral mesons, and thus the problem of the K factor may become even more acute, since the neutral to charged hadron ratio is likely to increase after including the inelastic contribution.

We also note, that it is the inelastic term which is especially sensitive to the saturation effects. The wave function of the incoming hadron knows nothing about saturation by itself. The effects of saturation come entirely from the distributions of the target. The target fields are directly affected by saturation at momenta $k < Q_{\mathrm{s}}$. The elastic scattering probes the large momentum component of target fields, equal to the final momentum of the produced parton. Thus as long as $p_{\mathrm{T}} > Q_{\mathrm{s}}$, this part of hadron production should be less affected by the saturation effects and one could expect that its dependence on energy and atomic number stems from perturbative physics. Any nonleading twist scaling then presumably comes from effects of a possible "nonperturbative" initial condition propagated to higher momenta via perturbative evolution. The inelastic scattering contribution on the other hand probes the target fields at $k_{\rm T} \ll p_{\rm T}$ which includes the region $k_{\rm T} < O(Q_{\rm s})$. It is this region of momenta which is strongly affected by target saturation effects. Thus if one neglects the inelastic contribution, one also severely limits one's options of studying effects of saturation.

We start our discussion by deriving the expression for gluon contribution to hadron production in the hybrid formalism. We will include the quark and antiquark contributions later. Our approach in the formal sense is similar to that of [32], although like in [31] we are not approximating the gluon splitting function by its low x limit. This will give us a possibility to compare our results with the $k_{\rm T}$ factorized formula which arises very simply in the approach of [32], to get some intuition from the simple $k_{\rm T}$ factorized expression and also to see the similarities and differences between the hybrid and the $k_{\rm T}$ factorized results.

We consider a process where an energetic projectile scatters off a static target. The wave function of the incoming projectile is an eigenstate of the QCD Hamiltonian. When calculated in the perturbation theory it can be represented as

$$|\Psi\rangle_{\rm in} = \Omega|v\rangle\,, \tag{30}$$

where $|v\rangle$ is the zeroth order wave function (an eigenfunction of the free Hamiltonian), and Ω is a unitary operator which diagonalizes the QCD Hamiltonian in perturbation theory

$$\Omega^\dagger H_{\rm QCD}\Omega = H_{\rm diag}\,. \tag{31}$$

The gluonic state immediately after scattering is

$$|\Psi\rangle_{\rm out} = S|\Psi\rangle_{\rm in}\,, \tag{32}$$

where S is the eikonal scattering matrix for the projectile partons which propagate through the static target fields.

The number of produced gluons is then given by

$$\frac{dN}{d^2k dk^+} = \frac{1}{(2\pi)^3}\langle v|\Omega^\dagger S^\dagger \Omega a^\dagger\left(k,k^+\right) a\left(k,k^+\right)\Omega^\dagger S\Omega|v\rangle\,. \tag{33}$$

Our first goal is to find the operator $\varOmega$. We start with the light-cone Hamiltonian of QCD

$$H=\int\limits_{k^+>0}\frac{dk^+}{2\pi}d^2z\left(\tfrac{1}{2}\varPi_a^-\left(k^+,z\right)\varPi_a^-(-k^+,z)+\tfrac{1}{4}G_a^{ij}\left(k^+,z\right)G_a^{ij}\left(-k^+,z\right)\right)\,, \tag{34}$$

where the electric and magnetic pieces have the form

$$\begin{aligned}\varPi_a^-(x^-,x) &= -\frac{1}{\partial^+}(D^i\partial^+A_i)^a(x^-,x)\,,\\ G_a^{ij}(x^-,x) &= \partial_iA_j^a(x^-,x)-\partial_jA_i^a(x^-,x)-gf^{abc}A_i^b(x^-,x)A_j^c(x^-,x)\,.\end{aligned} \tag{35}$$

Our convention for the covariant derivative is

$$D_i^{ab}\varPhi^b=\left(\partial_i\delta^{ab}-gf^{acb}A_i^c\right)\varPhi^b\,. \tag{36}$$

We are working in the light cone gauge, hence $A^+=0$ and, as usual, other light cone component of the vector potential A^- is expressed via the solution of Maxwell's equations as $A^-=-\frac{1}{\partial^+}\partial_iA_i$. The transverse components of the vector potential A^i which are the only dynamical degrees of freedom are expanded in the standard way in terms of the creation and annihilation operators

$$A_i^a(x^-,z)=\int\limits_0^\infty\frac{dk^+}{2\pi}\frac{1}{\sqrt{2k^+}}\left\{a_i^a(k^+,z)e^{-ik^+x^-}+a_i^{a\dagger}(k^+,z)e^{ik^+x^-}\right\}\,, \tag{37}$$

where the creation and annihilation operators satisfy the canonical commutation relations

$$\left[a_i^a(k^+,x),a_j^{b\dagger}(p^+,y)\right]=2\pi\delta_{ij}^{ab}(k^+-p^+)\delta^2(x-y)\,. \tag{38}$$

We will calculate gluon production to the leading order in the coupling constant, and we therefore require to know the Hamiltonian only to first order in g. After some algebra we find

$$\begin{aligned}H &= H_0+H_1\,,\\ H_0 &= \int\limits_{k,k^+>0}\frac{k^2}{2k^+}a_j^{a\dagger}(k^+,k)a_j^a(k^+,k)\,,\end{aligned}$$

$$H_1 = -igf^{abc} \int\limits_{k,p,k^+,p^+>0} \frac{1}{\sqrt{2k^+p^+(k^++p^+)}} \times \Big\{ -\left[\frac{p^+}{k^+}k_i - p_i\right] a_i^b(k^+,k)a_j^c(p^+,p)a_j^{a\dagger}(k^++p^+,k+p) + \frac{p^+}{p^++k^+}k_j a_i^b(k^+,k)a_i^c(p^+,p)a_j^{a\dagger}(k^++p^+,k+p)\Big\} + \text{h.c.}\,, \tag{39}$$

where the integration measure is understood as $\frac{dk^+}{2\pi}$ and $\frac{d^2k}{(2\pi)^2}$.

As Ω is a unitary operator, we define Hermitian operator G by

$$\Omega = e^{-iG} = 1 - iG - \tfrac{1}{2}G^2 + \dots\,. \tag{40}$$

The unitary operator Ω as discussed above is the operator that diagonalizes the Hamiltonian. To first order in the coupling constant the eigenvalues of the Hamiltonian are those of the free Hamiltonian H_0. To this order we have

$$\Omega^\dagger H \Omega = H - i[H,G] = H_0\,. \tag{41}$$

Thus the operator G is determined from

$$i[H_0, G] = H_1\,. \tag{42}$$

This immediately gives

$$G = -gf^{abc} \int\limits_{k,p,k^+,p^+>0} \frac{1}{\sqrt{2k^+p^+(k^++p^+)}} \frac{1}{\omega_{p+k} - \omega_p - \omega_k} \times \Big\{ -\left[\frac{p^+}{k^+}k_i - p_i\right] a_i^b(k^+,k)a_j^c(p^+,p)a_j^{a\dagger}(k^++p^+,k+p) + \frac{p^+}{p^++k^+}k_j a_i^b(k^+,k)a_i^c(p^+,p)a_j^{a\dagger}(k^++p^+,k+p)\Big\} + \text{h.c.} \tag{43}$$

with

$$\omega(k) = \frac{k^2}{2k^+}\,. \tag{44}$$

The number of produced gluons to leading order in the coupling is given by

$$\frac{dN}{d^2kdk^+} = \frac{1}{(2\pi)^3}\langle v| \left[\hat{S}^\dagger G - G\hat{S}^\dagger\right] a_k^{a\dagger}(k^+,k)a_k^a(k^+,k)\left[G\hat{S} - \hat{S}G\right]|v\rangle\,. \tag{45}$$

Here the factor $\frac{1}{(2\pi)^3}$ is due to our normalization of the creation and annihilation operators Eq. (38). For simplicity of the calculation we will assume that the longitudinal momentum of the observed gluon is (at least) slightly smaller that the momentum of gluons in the state $|v\rangle$, although, in fact, our formulae will be valid in a more general case.

The calculation of the matrix element is straightforward. The S matrix operator acts as a color rotation on all gluon creation and annihilation operators in coordinate space

$$\hat{S}^\dagger a_i^a(q^+, v)\hat{S} = S^{ab}(v) a_i^b(q^+, v)\,. \tag{46}$$

Since by assumption there are no gluons with longitudinal momentum k^+ in the state $|v\rangle$, one of the creation operators in the operator G in the amplitude $\left[G\hat{S}-\hat{S}G\right]|v\rangle$ must be at momentum k^+ and is "contracted" with $a(k^+)$ in the observable. This then leaves us with (apart from the various factors of S, and omitting for simplicity transverse dependences) expectation value of the type

$$\begin{aligned}
&\langle v|a^\dagger(p^+ + k^+)a(p^+)a^\dagger(q^+)a(q^+ + k^+)|v\rangle \\
&= \delta(p^+ - q^+)\langle v|a^\dagger(p^+ + k^+)a(p^+ + k^+)|v\rangle \\
&+\langle v|a^\dagger(p^+ + k^+)a^\dagger(q^+)a(p^+)a(q^+ + k^+)|v\rangle\,.
\end{aligned} \tag{47}$$

The second term involves a two-particle density in the state $|v\rangle$. It is suppressed in the leading twist "partonic" approximation. Since we keep to this approximation in the present paper, we neglect this term. We note that in the soft approximation, where the gluon production is given by the k_{T} factorized expression of [33] this term does indeed give a nonvanishing contribution. We will make explicit connection with the soft approximation later.

Keeping only the first term in Eq. (47) and reverting to coordinate space, where the S-matrix is diagonal we obtain

$$\begin{aligned}
&\frac{dN}{d^2k dk^+} \\
&= \frac{1}{(2\pi)^3}\int e^{ik(z-\bar{z})}\langle v|\left[\hat{S}^\dagger G - G\hat{S}^\dagger\right] a_k^{a\dagger}(k^+,\bar{z}) a_k^a(k^+,z)\left[G\hat{S}-\hat{S}G\right]|v\rangle \\
&= \frac{g^2}{(2\pi)^3}\frac{1}{N_c^2-1}\int \frac{1}{k^+} e^{ik(z-\bar{z})+i\bar{p}v+i\bar{q}\bar{z}-i(\bar{p}+\bar{q})\bar{u}-ipv-iqz+i(p+q)u} \\
&\times\ \mathrm{tr}\left\{\left[S_{\bar{u}}^\dagger T^a S_{\bar{u}} - S_v^\dagger T^a S_{\bar{z}}\right]\left[S_u^\dagger T^a S_u - S_z^\dagger T^a S_v\right]\right\}\left\langle a_j^{b\dagger}\left(\frac{k^+}{\xi},\bar{u}\right) a_j^b\left(\frac{k^+}{\xi},u\right)\right\rangle \\
&\times\ \frac{2}{(1-\xi)}\left[(1-\xi)^2+\xi^2+(1-\xi)^2\xi^2\right]\frac{[\xi\bar{p}_i-(1-\xi)\bar{q}_i]}{[\xi\bar{p}-(1-\xi)\bar{q}]^2}\frac{[\xi p_i-(1-\xi)q_i]}{[\xi p-(1-\xi)q]^2}\,.
\end{aligned} \tag{48}$$

To arrive at this expression we assumed that the projectile state is color and rotationally invariant, so that

$$\left\langle a_i^{a\dagger}(p^+,\bar u)a_j^b(p^+,u)\right\rangle = \frac{1}{2(N^2-1)}\delta^{ab}\delta_{ij}\left\langle a_k^{c\dagger}(p^+,\bar u)a_k^c(p^+,u)\right\rangle\,. \tag{49}$$

Changing the integration variables

$$\begin{aligned} \xi\bar p-(1-\xi)\bar q &= \bar\omega\,, \qquad \bar p+\bar q=\bar\kappa\,,\\ \xi p-(1-\xi)q &= \omega\,, \qquad p+q=\kappa \end{aligned} \tag{50}$$

and integrating over ω, $\bar\omega$, κ, $\bar\kappa$, u and $\bar u$ one obtains

$$\begin{aligned} \frac{dN}{d^2k dk^+} &= \frac{\alpha_{\rm s}}{2\pi^2}\frac{1}{(2\pi)^2}\frac{1}{N_c^2-1}\int_x^1 \frac{d\xi}{\xi}\frac{1}{k^+}e^{ik(z-\bar z)}\\ &\times\frac{2}{(1-\xi)}\left[(1-\xi)^2+\xi^2+(1-\xi)^2\xi^2\right]\frac{(v-\bar z)_i}{(v-\bar z)^2}\frac{(v-z)_i}{(v-z)^2}\\ &\times{\rm tr}\left\{\left[S^\dagger((1-\xi)v+\xi\bar z)T^aS((1-\xi)v+\xi\bar z)-S_v^\dagger T^aS_{\bar z}\right]\right.\\ &\times\left.\left[S^\dagger((1-\xi)v+\xi z)T^aS((1-\xi)v+\xi z)-S_z^\dagger T^aS_v\right]\right\}\\ &\times\frac{k^+}{2\pi\xi}\left\langle a_j^{b\dagger}\left(\frac{k^+}{\xi},(1-\xi)v+\xi\bar z\right)a_j^b\left(\frac{k^+}{\xi},(1-\xi)v+\xi z\right)\right\rangle\,. \end{aligned} \tag{51}$$

Throughout the rest of this paper we will continue using the notations u and $\bar u$ to make notations less cumbersome, however it should be understood that they are not independent variables, but rather as shorthand for

$$u\equiv(1-\xi)v+\xi z\,,\qquad \bar u=(1-\xi)v+\xi\bar z\,. \tag{52}$$

To get some intuition about this expression we first consider the soft limit. This corresponds to the situation when the longitudinal momentum of the observed gluon is much smaller than the momentum of the gluons in the valence state $|v\rangle$. Taking $\xi\to 0$ we obtain this limit

$$\begin{aligned} \frac{dN}{d^2kdk^+} &= \frac{\alpha_{\rm s}}{\pi^2}\frac{1}{(2\pi)^2}\frac{1}{N_c^2-1}\int\frac{1}{k^+}e^{ik(z-\bar z)}\frac{(v-\bar z)_i}{(v-\bar z)^2}\frac{(v-z)_i}{(v-z)^2}\\ &\times{\rm tr}\left\{\left[S_v^\dagger T^aS_x-S_v^\dagger T^aS_{\bar z}\right]\left[S_v^\dagger T^aS_v-S_z^\dagger T^aS_v\right]\right\}\left\langle a_j^{\omega\dagger}\left(\frac{k^+}{\xi},v\right)a_j^\omega\left(\frac{k^+}{\xi},v\right)\right\rangle\\ &=\frac{\alpha_{\rm s}}{\pi^2}\frac{1}{(2\pi)^2}\frac{N_c}{N_c^2-1}\int\frac{1}{k^+}e^{ik(z-\bar z)}\frac{(v-\bar z)_i}{(v-\bar z)^2}\frac{(z-v)_i}{(z-v)^2}\\ &\times{\rm tr}\left\{1-S_v^\dagger S_z-S_{\bar z}^\dagger S_v+S_zS_{\bar z}^\dagger\right\}\left\langle a_j^{a\dagger}\left(\frac{k^+}{\xi},v\right)a_j^a\left(\frac{k^+}{\xi},v\right)\right\rangle\,. \end{aligned} \tag{53}$$

The $k_{\rm T}$ factorized form reads ([33] and also [32,17] corrected for typos)

$$\begin{aligned}\frac{dN}{d^2kd\eta} &= \frac{\alpha_{\rm s}}{\pi^2}\frac{1}{(2\pi)^2}\int e^{ik(z-\bar z)} \\ &\times\left\{S_zS^\dagger_{\bar z}+S_vS^\dagger_{\bar v}-S_zS^\dagger_{\bar v}-S_vS^\dagger_{\bar z}\right\}^{ab}\frac{(z-v)_i}{(z-v)^2}\frac{(\bar z-\bar v)_i}{(\bar z-\bar v)^2}\left\langle\rho^a_v\rho^b_{\bar v}\right\rangle \\ &= \frac{\alpha_{\rm s}}{\pi^2}\frac{1}{(2\pi)^2}\int e^{ik(z-\bar z)}\frac{1}{N_c^2-1} \\ &\times{\rm tr}\left\{S_zS^\dagger_{\bar z}+S_vS^\dagger_{\bar v}-S_zS^\dagger_{\bar v}-S_vS^\dagger_{\bar z}\right\}\frac{(z-v)_i}{(z-v)^2}\frac{(\bar z-\bar v)_i}{(\bar z-\bar v)^2}\left\langle\rho^a_v\rho^a_{\bar v}\right\rangle\,,\end{aligned} \tag{54}$$

where the last equality follows from color neutrality of the hadronic state. The color charge density operator here $\rho^a_v=\int\frac{dp^+}{2\pi}a^\dagger_i(p^+,v)T^aa_i(p^+,v)$. In the leading twist approximation the correlator of the charge density operators is local in the transverse space. Physically, this is the case since in this (parton model) approximation there is only a small number of gluons in the hadron, and there are no correlations between different gluons. For a color singlet hadronic state we therefore have

$$\langle\rho^a(v)\rho^a(\bar v)\rangle=\delta^2(v-\bar v)N_c\left\langle\int\frac{dp^+}{2\pi}a^{\dagger a}_i(p^+,v)a^a_i(p^+,v)\right\rangle\,. \tag{55}$$

Thus, in the leading twist approximation Eq. (53) is indeed equivalent to Eq. (54).

It is customary to define the transverse momentum dependent gluon distribution in terms of the gluon distribution function $f_g(x,Q)$ as

$$\begin{aligned}xf_g\left(x,Q=\frac{1}{|u-v|}\right) &\equiv \frac{p^+}{2\pi}\int d^2b\left\langle a^{a\dagger}_i(p^+,u)a^a_i(p^+,v)\right\rangle \\ &= \int d^2b\int\frac{d^2p}{\pi}e^{ip\cdot(u-v)}\phi(p,b;x) \\ &\approx \int d^2b\int_0^{\frac{1}{|u-v|^2}}dp^2\phi(p,b;x)\,,\end{aligned} \tag{56}$$

where $b=\frac{u+v}{2}$. In the soft limit the color charge correlation function and the scattering amplitude are then expressed in terms of the projectile and the target distributions as

$$\langle\rho^a(v)\rho^a(\bar v)\rangle = \frac{1}{8\pi\alpha_{\rm s}}\int d^2pe^{ip\cdot(v-\bar v)}p^2\phi_{\rm P}(p,b)\,, \tag{57}$$

$${\rm tr}[1-S^\dagger(v)S(\bar v)] = 2\pi\alpha_{\rm s}N_c\int d^2pe^{ip\cdot(v-\bar v)}\frac{1}{p^2}\phi_{\rm T}(p,b)\,. \tag{58}$$

In terms of the transverse momentum distribution the single inclusive gluon spectrum in the soft limit has the familiar $k_{\rm T}$ factorized form

$$\begin{aligned}\frac{dN}{d^2kd\eta} &= S\frac{\alpha_{\rm s}N_c}{N_c^2-1}\frac{1}{k^2}\int\limits_l \phi_{\rm T}(l+k,Y-\eta)\phi_{\rm P}(l,\eta)\\ &= S\frac{\alpha_{\rm s}N_c}{N_c^2-1}\int\limits_l\left[\frac{1}{(l+k)^2}+\frac{1}{(l+k)^2}\frac{l^2}{k^2}+2\frac{1}{(l+k)^2}\frac{l\cdot k}{k^2}\right]\phi_{\rm T}(l+k)\phi_{\rm P}(l)\,,\end{aligned} \tag{59}$$

where we have assumed translational invariance in the transverse plane. Here S is the total transverse area of the collision and Y denotes the total rapidity difference between the projectile and the target in the process.

In the limit of large momentum of the produced gluon $k \gg Q_{\rm s}$, $\Lambda_{\rm QCD}$ the momentum integral in Eq. (59) is dominated by two regions of momentum space.

In the first region, $l \ll k$ the dominant term is the first term in Eq. (59) (which corresponds to the first term in Eq. (53)). In this kinematics the incoming projectile gluon has a small transverse momentum (in accordance with the simple parton model picture) and it acquires a large transverse momentum due to elastic scattering from the target field. We will refer to this contribution as elastic

$$\left[\frac{dN}{d^2kd\eta}\right]_{\rm elastic} = \frac{\alpha_{\rm s}N_c}{N_c^2-1}\frac{1}{k^2}\phi_{\rm T}(k)\int\limits_{l<Q\sim k} S\phi_{\rm P}(l)\,. \tag{60}$$

The final states that correspond to this contribution have a single high $p_{\rm T}$ gluon at forward rapidity. The balancing transverse momentum is carried by another gluon kicked out of the target by recoil, and it resides at a very different rapidity, close to the target.

There is, however, another contribution which is equally important at the leading twist. This comes from the momentum range $l = k + q$ with $q \ll k$. Changing variables from l to q the other contribution is clearly just the mirror image of Eq. (60). For reasons explained below we will refer to it as inelastic contribution

$$\left[\frac{dN}{d^2kd\eta}\right]_{\rm inelastic} = \frac{\alpha_{\rm s}N_c}{N_c^2-1}\frac{1}{k^2}\phi_{\rm P}(k)\int\limits_{q<Q\sim k} S\phi_{\rm T}(q)\,. \tag{61}$$

In this kinematics all terms in Eq. (59) are equally important. This contribution corresponds to a projectile gluon coming in with large transverse momentum in the wave function and subsequently scattering with small momentum transfer. The scattering practically does not add to gluons transverse momentum, but decoheres the gluon from the incoming hadronic wave function. One can naturally ask, how do high transverse momentum gluons find themselves in the projectile wave function. The answer is, that they are always there as "unresolved" components of the "parton model" gluons. A low $p_{\rm T}$ gluon can split via a standard DGLAP evolution into a two-gluon state with large relative transverse momentum. The "parton model" gluons are therefore not point like objects, but rather composites, which at first order in $\alpha_{\rm s}$ contain an admixture of a two-gluon state.

We stress that it is not the collinear part of the DGLAP kernel that is responsible for this structure. The collinear emission contributes to multiplication of low momentum gluons in the wave function ("low" momentum here technically means momentum lower than that imparted by the scattering). The splitting however contains also an ultraviolet contribution, which produces gluon pairs with large relative transverse momentum, which sit close to each other in the impact parameter plane. It is the presence of this compact two-gluon configuration that makes a projectile gluon behave as a composite object. When such a composite parton scatters inelastically off a soft target field, its different components can be put on shell, emerging as high $p_{\rm T}$ partons in the final state. Correspondingly, the structure of the final state incidentally is quite different from those that arise from the contribution Eq. (60), as both high $p_{\rm T}$ partons from the projectile wave function end up close to forward rapidity. The inelastic contribution therefore, takes into account production of forward dijets with large $p_{\rm T}$.

At high $p_{\rm T}$, both contributions are of the same order of magnitude. The probability to find a low $p_{\rm T}$ gluon in the projectile is of the order of unity, but then the probability of scattering with large momentum transfer is small, of the order of $\alpha_{\rm s}$. On the other hand, the probability to find a high $p_{\rm T}$ parton in the incoming wave function is of the order of $\alpha_{\rm s}$, however the probability of it scattering softly off the target is of the order of one. If one assumes that the projectile and target wave functions have standard perturbative behavior, $\phi = \frac{\mu^2}{p^2}$, one finds

$$\begin{aligned}\left[\frac{dN}{d^2k d\eta}\right]_{\rm elastic} &= \alpha_{\rm s}\mu_{\rm P}^2\mu_{\rm T}^2 \ln\frac{p^2}{\Lambda_{\rm QCD}^2}\,, \\ \left[\frac{dN}{d^2k d\eta}\right]_{\rm inelastic} &= \alpha_{\rm s}\mu_{\rm P}^2\mu_{\rm T}^2 \ln\frac{p^2}{Q_{\rm s}^2}\,, \end{aligned} \tag{62}$$

where we have assumed that the perturbative behavior for the target kicks in at momenta above Q_s. If the energy of the process is large enough so that the target distributions manifest extended geometric scaling [34], the Q_s^2 in the last equation should be substituted by a higher scale which marks the upper end of the geometric scaling window. At any rate, it is clear that at parametrically large transfer momentum the two contributions are comparable, and both must be kept. In the application we have in mind, the transverse momentum is probably not much higher than Q_s and so the inelastic contribution does not have a logarithmic enhancement. However, the logarithm in the elastic contribution is also not very large, perhaps a factor of 3 or 4. Thus, whether the inelastic contribution can be neglected or not is a numerical question, and one would be well advised not to throw it away prematurely.

We now return to Eq. (51). Our aim is to identify the two contributions described above in this more general formula, and to write the leading twist result in as simple form as possible.

The elastic contribution, as before, corresponds to the region of the phase space where all the transverse momentum of the produced gluon originates from the momentum transfer from the target. This comes from the product of the two last terms in the brackets in Eq. (51). Assuming that the momentum in the rest of the expression is small is equivalent to take $z = \bar{z}$ everywhere else apart from the scattering amplitude $S^\dagger(z)S(\bar{z})$

$$\begin{aligned}\left[\frac{dN}{d^2k d\eta}\right]_{\mathrm{elastic}} &= \frac{\alpha_\mathrm{s} N_c}{2\pi^2}\frac{1}{(2\pi)^2}\int_{x_\mathrm{F}}^1 \frac{d\xi}{\xi}\frac{2}{(1-\xi)}\left[(1-\xi)^2+\xi^2+(1-\xi)^2\xi^2\right] \\ &\times\int_z \frac{1}{(v-z)^2}\frac{k^+}{2\pi\xi}\left\langle a_j^{b\dagger}\left(\frac{k^+}{\xi},\bar{u}\right) a_j^b\left(\frac{k^+}{\xi},u\right)\right\rangle \frac{1}{N_c^2-1}\int_{z-\bar{z}} e^{ik(z-\bar{z})}\mathrm{tr}\left[S^\dagger(z)S(\bar{z})\right] \\ &\simeq \frac{\alpha_\mathrm{s}}{\pi}\frac{1}{(2\pi)^2}\int_{p^2<Q^2}\frac{dp^2}{2p^2}\int_{x_\mathrm{F}}^1\frac{d\xi}{\xi}P_{g/g}(\xi)x_\mathrm{F} f_g\left(\frac{x_\mathrm{F}}{\xi},p^2\right)N_\mathrm{A}(k)\,, \end{aligned} \tag{63}$$

where

$$P_{g/g} = \frac{2N_c}{\xi(1-\xi)}\left[(1-\xi)^2+\xi^2+(1-\xi)^2\xi^2\right] \tag{64}$$

is the standard gluon–gluon splitting function. As explained in [31] the first line is just the DGLAP contribution to the evolution of the gluon distribution. The "parton model" term is not present in our explicit formula because we have for simplicity assumed that the rapidity of the observed gluon is lower than that of any of the valence partons, and thus such gluons can be present in the wave function only as a result of evolution. This assumption is, of course, not necessary. Relaxing it restores the parton model contribution to the elastic gluon production. The elastic term then simply becomes the contribution discussed in [31] (we do not include yet the fragmentation function contribution)

$$\left[\frac{dN}{d^2kd\eta}\right]_{\rm elastic} = \frac{1}{(2\pi)^2} x_{\rm F} f_g\left(x_{\rm F}, Q^2\right) N_{\rm A}(k) \tag{65}$$

with $N_{\rm A}(k) = \frac{1}{N_c^2-1}\int d^2(z-\bar{z})e^{ik\cdot(z-\bar{z})}{\rm tr}[S^\dagger(\bar{z})S(z)]$; and the factorization scale Q must be chosen so that it is of order but smaller than the external momentum k.

To extract the inelastic term in the leading twist approximation we note that it arises as the leading order expansion in powers of $|x-z|$ and $|x-\bar{z}|$, since the separation between the two gluons in the wave function must be much smaller than the typical variation scale of the scattering amplitude $S(x)$. Referring back to Eq. (51) we write

$$\begin{aligned} S_{\bar{u}} &\simeq S_v - \xi(v-\bar{z})_i\partial_i S_v\,, \qquad & S_u \simeq S_v - \xi(v-z)_i\partial_i S_v\,,\\ S_{\bar{z}} &\simeq S_v - (v-\bar{z})_j\partial_j S_v\,, \qquad & S_z \simeq S_v - (v-z)_j\partial_j S_v\,. \end{aligned} \tag{66}$$

Then for the amplitude in Eq. (51) we find

$$\begin{aligned} &{\rm tr}\left\{\left[S^\dagger_{\bar{u}}T^\sigma S_{\bar{u}} - S^\dagger_v T^\sigma S_{\bar{z}}\right]\left[S^\dagger_u T^\sigma S_u - S^\dagger_z T^\sigma S_v\right]\right\}\\ &= N_c(v-\bar{z})_i(v-z)_j\left\{(1-\xi)^2+\xi^2\right\}{\rm tr}\left[\partial_i S_v \partial_j S^\dagger_v\right]\\ &\quad -2\xi(1-\xi)(v-\bar{z})_i(v-z)_j{\rm tr}\left[T^\sigma S_v\partial_i S^\dagger_v T^\sigma S_v \partial_j S^\dagger_v\right]\\ &= \frac{N_c}{2}(v-\bar{z})\cdot(v-z)\left\{(1-\xi)^2+\xi^2\right\}{\rm tr}\left[\partial_i S_v\partial_i S^\dagger_v\right]\\ &\quad -\xi(1-\xi)(v-\bar{z})\cdot(v-z){\rm tr}\left[T^\sigma S_v\partial_i S^\dagger_v T^\sigma S_v\partial_i S^\dagger_v\right]\,, \end{aligned} \tag{67}$$

where the last line strictly speaking holds only after averaging over rotationally invariant target for which $\langle\partial_i S^\dagger\partial_j S\rangle = \frac{1}{2}\delta_{ij}\langle\partial_k S^\dagger\partial_k S\rangle$. One can further simplify this expression noting that $S_v\partial_i S^\dagger_v = \frac{1}{N_c}T^a{\rm tr}[S_v\partial_i S^\dagger_v T^a]$ *i.e.* it is a "pure gauge" vector potential. Then using simple color algebra we find

$${\rm tr}\left[S\partial_i S^\dagger T^a S\partial_i S^\dagger T^a\right] = -\frac{N_c}{2}{\rm tr}\left[\partial_i S^\dagger\partial_i S\right]\,. \tag{68}$$

So that we can write

$$\begin{aligned} &{\rm tr}\left\{\left[S^\dagger_{\bar{u}}T^\sigma S_{\bar{u}} - S^\dagger_v T^\sigma S_{\bar{z}}\right]\left[S^\dagger_u T^\sigma S_u - S^\dagger_z T^\sigma S_v\right]\right\}\\ &= \frac{N_c}{2}\left\{1-\xi+\xi^2\right\}(v-\bar{z})\cdot(v-z){\rm tr}\left[\partial_i S_v\partial_i S^\dagger_v\right]\\ &= N_c^2\left\{1-\xi+\xi^2\right\}(v-\bar{z})\cdot(v-z){\rm tr}\left[\partial_i S_F(v)\partial_i S^\dagger_F(v)\right]\,, \end{aligned} \tag{69}$$

where we have given the final answer in terms of the fundamental representation matrices $S_{\rm F}$. It is clear from our derivation that in the above expressions the target field average should be understood as calculated with resolution Q, where just as in the elastic piece, the factorization scale Q is of order of, but smaller than the transverse momentum of the observed gluon $Q < k$

$$\mathrm{tr}\left[\partial_i S_{\rm F}(v)\partial_i S^\dagger_{\rm F}(v)\right] \to \mathrm{tr}\left[\partial_i S_{\rm F}(v)\partial_i S^\dagger_{\rm F}(v)\right]_Q = N_c \int\limits_{p^2<Q^2} p^2 N_{\rm F}(p)\,. \quad (70)$$

The leading twist part of the inelastic contribution can therefore be written as

$$\begin{aligned}\left[\frac{dN}{d^2k d\eta}\right]_{\rm inelastic} &= \frac{\alpha_{\rm s}}{2\pi^2}\frac{1}{(2\pi)^2}\frac{N_c^2}{N_c^2-1}\\ &\times\int e^{ik(z-\bar z)}\frac{(v-\bar z)_i}{(v-\bar z)^2}\frac{(v-z)_i}{(v-z)^2}(v-\bar z)\cdot(v-z)\mathrm{tr}\left[\partial_i S_{\rm F}(v)\partial_i S^\dagger_{\rm F}(v)\right]_Q\\ &\times\frac{2}{(1-\xi)}\left[(1-\xi)^2+\xi^2+(1-\xi)^2\xi^2\right]\left\{1-\xi+\xi^2\right\}\left\langle a_j^{b\dagger}\left(\frac{k^+}{\xi},\bar u\right)a_j^b\left(\frac{k^+}{\xi},u\right)\right\rangle\,.\end{aligned} \quad (71)$$

This expression can be rewritten in terms of the gluon distribution. To leading twist the dependence of the vacuum average $\langle a_j^{b\dagger}(\frac{k^+}{\xi},\bar u)a_j^b(\frac{k^+}{\xi},u)\rangle$ can be substituted by $\langle a_j^{b\dagger}(\frac{k^+}{\xi},v)a_j^b(\frac{k^+}{\xi},v)\rangle_Q$. Performing the Fourier transform we then obtain

$$\begin{aligned}\left[\frac{dN}{d^2k d\eta}\right]_{\rm inelastic} &= \frac{\alpha_{\rm s}}{\pi^2}\frac{N_c^2}{N_c^2-1}\\ &\times\frac{1}{k^4}\int\limits_{x_{\rm F}}^{1}\frac{d\xi}{\xi}\left\{1-\xi+\xi^2\right\}P_{g/g}(\xi)x_{\rm F}f_g\left(\frac{x_{\rm F}}{\xi},Q\right)\int\limits_{p^2<Q^2}\frac{d^2p}{(2\pi)^2}p^2N_{\rm F}(p)\,.\end{aligned} \quad (72)$$

Of course, in order to calculate the spectrum of produced hadrons, we have to include the gluon fragmentation functions. We assume, as always, that produced gluons fragment into hadrons independently. Taking this into account our result for production is

$$\begin{aligned}\frac{dN}{d^2k d\eta} &= \int_{x_\mathrm{F}}^{1} \frac{dz}{z^2} D_{h/g}(z,Q)\Bigg[x_1 f_g\left(x_1,Q^2\right) N_\mathrm{A}\left(x_2,\frac{k}{z},b=0\right) \\ &\quad + \frac{\alpha_\mathrm{s}}{\pi^2}\frac{N_c^2}{N_c^2-1}\frac{z^4}{k^4}\int_{x_1}^{1}\frac{d\xi}{\xi}\left\{1-\xi+\xi^2\right\} \\ &\quad \times P_{g/g}(\xi) x_1 f_g\left(\frac{x_1}{\xi},Q\right)\int_{p^2<Q^2}\frac{d^2p}{(2\pi)^2}p^2 N_\mathrm{F}(x_2,p,b=0)\Bigg], \end{aligned} \tag{73}$$

where

$$\begin{aligned} N_\mathrm{A}\left(k,b=\frac{\bar z+z}{2}\right) &= \frac{1}{N_c^2-1}\int d^2(z-\bar z)e^{ik\cdot(z-\bar z)}\mathrm{tr}\left[S_\mathrm{A}^\dagger(\bar z)S_\mathrm{A}(z)\right], \\ N_\mathrm{F}\left(k,b=\frac{\bar z+z}{2}\right) &= \frac{1}{N_c}\int d^2(z-\bar z)e^{ik\cdot(z-\bar z)}\mathrm{tr}\left[S_\mathrm{F}^\dagger(\bar z)S_\mathrm{F}(z)\right] \end{aligned} \tag{74}$$

and the longitudinal momentum fractions (neglecting the hadron mass) are

$$x_\mathrm{F} = \frac{k}{\sqrt{s_{NN}}}e^{\eta}, \qquad x_1 = \frac{x_\mathrm{F}}{z}, \qquad x_2 = x_1 e^{-2\eta}. \tag{75}$$

Eq. (73) is our result for hadron production in a toy theory that does not contain quarks. This is obviously not a good approximation to reality especially at forward rapidities, where the quark contribution must be the leading one. We now turn to generalizing the previous discussion by including the quark contribution.

It is straightforward to include the quark contribution to this calculation [12]. Omitting calculational details, the final answer generalizing Eq. (73) is

$$\begin{aligned}\frac{dN_h}{d^2k d\eta} &= \frac{1}{(2\pi)^2}\int_{x_\mathrm{F}}^{1}\frac{dz}{z^2}\Bigg[x_1 f_g\left(x_1,Q^2\right)N_\mathrm{A}\left(x_2,\frac{k}{z}\right)D_{h/g}(z,Q) \\ &+ \Sigma_q x_1 f_q\left(x_1,Q^2\right)N_\mathrm{F}\left(x_2,\frac{k}{z}\right)D_{h/q}(z,Q)\Bigg] + \int_{x_\mathrm{F}}^{1}\frac{dz}{z^2}\frac{\alpha_\mathrm{s}}{2\pi^2}\frac{z^4}{k^4} \\ &\times \int_{p^2<Q^2}\frac{d^2p}{(2\pi)^2}p^2 N_\mathrm{F}(p,x_2)x_1\int_{x_1}^{1}\frac{d\xi}{\xi}\Sigma_{j=q,\bar q,g}w_{i/j}(\xi)P_{i/j}(\xi)f_j\left(\frac{x_1}{\xi},Q\right)D_{h/q}(z,Q), \end{aligned} \tag{76}$$

where the momentum fractions x_1 and x_2 are defined in Eq. (75) and the inelastic weights w_i are

$$\begin{aligned} w_{g/g}(\xi) &= 2\frac{N_c^2}{N_c^2-1}\left(1-\xi+\xi^2\right), && (77)\\ w_{g/q}(\xi) &= w_{g/\bar{q}}(\xi) = \frac{N_c^2}{N_c^2-1}\left[1+(1-\xi)^2-\frac{\xi^2}{N_c^2}\right], && (78)\\ w_{q/q}(\xi) &= w_{\bar{q}/\bar{q}}(\xi) = \frac{N_c^2}{N_c^2-1}\left[1+\xi^2-\frac{(1-\xi)^2}{N_c^2}\right], && (79)\\ w_{q/g}(\xi) &= w_{\bar{q}/g}(\xi) = \frac{1}{2}\left[(1-\xi)^2+\xi^2-\frac{2\xi(1-\xi)}{N_c^2-1}\right] && (80) \end{aligned}$$

in Eq. (77). This is our final result.

In this paper we have derived the complete leading twist expression for inclusive hadron production in the hybrid formalism. We have shown that in addition to elastic scattering terms first derived in [31], there are also terms that correspond to inelastic scattering of the projectile partons on low momentum components of the target field. These terms are given by the second line in Eq. (76). We note that although the inelastic piece has an explicit factor of α_{s} while the elastic contribution does not, the two terms at high k_{T} are in fact of the same order in α_{s}. The reason is that at momenta $k \gg Q_{\mathrm{s}}$ the dipole scattering amplitude $N_{\mathrm{A(F)}}(k)$, which enters the elastic scattering term is itself of the order of α_{s}, while the integral of the amplitude appearing in the inelastic term is of the order of unity.

The final states that correspond to the inelastic process are dihadron pairs where both hadrons are emitted at forward rapidity and have strong back-to-back correlation. Since both produced hadrons have large rapidity, such pairs with large transverse momentum are kinematically allowed only at large collision energy. Thus one might expect this contribution not to be of great importance in RHIC kinematics, however it may be sizable at the LHC.

In this context we believe that including this contribution in calculation *à la* [6] should produce faster approach of nuclear modification factor R_{pA} to unity at large transverse momenta. Here we wish to elaborate on possible role of saturation in the results of [6]. As we have noted above, as long as the transverse momentum is above Q_{s}, saturation should mainly affect the inelastic production piece. This contribution involves the target distribution $\int_{p^2<Q^2}\frac{d^2p}{(2\pi)^2}p^2N_{\mathrm{F}}(p,x_2) \propto f_{\mathrm{target}}(Q,x_2)$ and is thus directly sensitive to saturation effects which suppress the contribution of small momenta $p<Q_{\mathrm{s}}$ to the integral. The elastic production probability (first line of Eq. (76)) depends only on $N(k)$ at large momentum. Naively one expects that this part is unaffected by saturation in the evolution. This does

not necessarily mean that the R_{pA} calculated using only this contribution (as done in [6]) should be equal to unity, but rather that any departure from unity is the effect of a nonscaling initial condition. This should be true if the transverse momentum of the measured particle is in the so-called "geometric scaling" window, where the anomalous dimension is finite, since geometric scaling is not a result of saturation physics but rather of the linear BFKL evolution of the gluon density [34]. To be a little more precise, recall that solution of the BFKL equation above the saturation scale has the form $\phi_{\mathrm{BFKL}}(k,Y) \propto [Q_{\mathrm{s}}(Y)/k]^{2-2\gamma}$ where γ is the anomalous dimension. The anomalous dimension is a slowly varying function of transverse momentum. It is almost constant in a wide window of momenta above Q_{s}, but nevertheless vanishes asymptotically as $k \to \infty$. It also weakly depends on rapidity Y. The saturation momentum Q_{s} is defined within the BFKL solution *per se* as the momentum at which the scattering amplitude is of the order of one. Within leading order BFKL solution $Q_{\mathrm{s}}(Y) = Q_0 \exp\{\lambda Y\}$, where Q_0 is the soft nonperturbative scale which characterizes the initial condition $\phi_0(k, Y=0)$. In the case of calculations of [6] this would be the initial saturation scales, Q_{0p} for the proton target and Q_{0A} for the nuclear target. The nuclear modification factor R_{pA} within a BFKL calculation would then be

$$R_{pA}(Y) = \frac{1}{N_{\mathrm{coll}}}\left[\frac{Q_{\mathrm{s}A}(Y)}{Q_{\mathrm{s}p}(Y)}\right]^{2-2\gamma(Y)} = \left[\frac{Q_{0p}}{Q_{0A}}\right]^{2\gamma(Y)} \tag{81}$$

with the identification $N_{\mathrm{coll}} = Q_{0A}^2/Q_{0p}^2$. Within the running coupling calculation the saturation scale is not a simple exponential of rapidity and thus the explicit expression for the nuclear modification factor and the rapidity dependence is somewhat different. It nevertheless remains the case that as long as the initial conditions for proton and nucleus do not simply scale with $A^{1/3}$ at all momenta, $\phi_p(k, Y=0) \neq A^{1/3}\phi_A(k, Y=0)$, the linear BFKL evolution produces a nuclear modification factor different from unity and slowly varying with rapidity. It is an interesting question whether the numerical results of [6] are consistent with BFKL, or whether saturation effects in the evolution nevertheless give a significant contribution to R_{pA}.

Finally, we note that the final states that contribute to the inelastic production are precisely the states which give the bulk of the contribution to the dihadron correlation function considered in [7]. The calculation of [7] does not address the estimate of large uncorrelated background of produced particles. The small "signal to background" ratio is indeed a very pronounced feature of the data [5, 35]. In this respect it would be interesting to calculate both in a unified framework discussed here. We note that our earlier discussion suggests that the saturation has two distinct effects on the correlated dihadron production. First, as discussed in [17, 36] and [6], the back-to-back correlation is weakened due to independent momentum transfer from the target to each one of the produced hadrons. This does not reduce the number of hadrons produced at forward rapidity, but reduces the correlation between the direction of their transverse momenta. Another distinct effect is that the dihadron production probability is suppressed by the effect of saturation on $\int_{p^2<Q^2} \frac{d^2p}{(2\pi)^2} p^2 N_{\rm F}(p,x_2)$, thus reducing the ratio of the correlated signal to the total number of produced particles.

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