DENSE HADRONIC AND QUARK MATTER AND ITS ASTROPHYSICAL APPLICATIONS* **

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In this paper, we discuss physics of dense matter, which are expected to happen inside compact stars such as neutron stars. After the general introduction to basic properties of dense hadronic and quark matter as well as compact stars, we first focus on QCD phase transitions and its phase structures, based on the Ginzburg–Landau analysis. Then, we show some astrophysical applications into neutron star phenomena such as equation of state, cooling and gravitational wave radiation and some recent developments.

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1. Introduction

Just after the discovery of asymptotic freedom in QCD [1], it had been demonstrated that QCD matter might undergo some phase transitions at high temperature and/or high baryon density [2]. Since then, space-time evolution of hadronic and quark matter in various temperature and/or density regimes has been extensively studied from both theoretical and experimental/observational points of view.

QCD at high temperature is connected to physics of the early universe after Big Bang as well as heavy ion collision experiments such as RHIC and LHC [3]. Experimental results have shown us that created matter, called quark-gluon plasma (QGP) is strongly correlated to each other and behaves like almost the perfect fluid. On the other hand, QCD at finite baryon density is related to physics of compact stars such as neutron star [4]. Neutron

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star is the densest object in the universe. Its mass is around 1.4 $M_{\rm SUN}$ ($M_{\rm SUN}$ is the solar mass), its radius only 10 km. Neutron star is made after the Type-I supernova explodes and cools down via neutrino emissions particularly in the early stage. Some neutron stars are called the magnetar, since they have a gigantic magnetic field. Binary collision of neutron stars is considered to be one of the best candidates to produce the gravitational wave radiation, which might be detected in the future. All the above properties of neutron star are definitely connected to the stellar structure, matter compositions inside it.

In this paper, we demonstrate properties of dense QCD matter, mainly concentrating on the thermodynamic properties and the astrophysical applications. In Sec. 2, we provide our study in dense QCD based on a particular method, *i.e.*, the Ginzburg–Landau analysis. We see the resultant phase structure. In Sec. 3, the astrophysical applications of studies performed in Sec. 2 are shown, with some introduction of recent observational consequences for the neutron star mass and the thermal behavior. Section 4 is devoted to summary of this lecture.

2. Ginzburg–Landau study in dense QCD

2.1. Chiral-super interplay — anomaly-driven critical point

Ginzburg–Landau (GL) analysis for investigating phase transition properties of matter has been utilized in general and universal contexts, from condensed matter to particle physics. Through the analysis, one can reach (1) topological structure of the phase diagram, (2) order of the phase transition and (3) critical properties. The key ingredient for the GL study is to identify the order parameters of the system and construct the effective free energy in terms of them, based on symmetries of underlying theory.

Application of the GL analysis into color superconductivity, which is expected to happen at high density, had been pioneered by the classic work by Bailin and Love [5]. Afterwards, Iida and Baym had developed their work [6]. Let us now construct the GL free energy in dense QCD. At classical level, QCD Lagrangian with 3-flavor massless quarks is invariant under the transformation $\mathcal{G} = {\rm SU}(3)_{\rm L} \times {\rm SU}(3)_{\rm R} \times {\rm U}(1)_{\rm B} \times {\rm U}(1)_{\rm A} \times {\rm SU}(3)_{\rm C}$. Here, ${\rm SU}(3)_{\rm L} \times {\rm SU}(3)_{\rm R}$ denotes chiral transformation, ${\rm U}(1)_{\rm B}$ baryon one, ${\rm U}(1)_{\rm A}$ axial one and ${\rm SU}(3)_{\rm C}$ local color one, respectively. While in the case for massive quarks, chiral symmetry is explicitly broken. At quantum level, on the other hand, ${\rm U}(1)_{\rm A}$ is explicitly broken down to a discrete symmetry group Z_6 by the QCD axial anomaly.

Let us define the order parameter fields in QCD at intermediate baryon density regime, which might be relevant for neutron stars. One knows that at zero or low density, chiral condensate, a signal of chiral symmetry breaking, plays a crucial role whereas at high baryon density, diquark condensate, a signal of color superconductivity, is expected to happen. Thus it is natural to consider both chiral and diquark fields as the order parameters at intermediate region. Let us denote Φ, $d_{\rm L}$ and $d_{\rm R}$ as chiral, left- and right-handed diquark fields, respectively. Their transformation laws under the quantum QCD symmetry group are provided by $\Phi \to e^{-2i\alpha_{\rm A}} V_{\rm L} \Phi V_{\rm R}^\dagger, d_{\rm L} \to e^{2i\alpha_{\rm A}} e^{2i\alpha_{\rm B}} V_{\rm L} d_{\rm L} V_{\rm C}^\dagger$ and $d_{\rm L} \to e^{-2i\alpha_{\rm A}} e^{2i\alpha_{\rm B}} V_{\rm L} d_{\rm L} V_{\rm C}^\dagger$. Here, $V_{\rm L(R)}$ denotes chiral transformation, $V_{\rm C}$ a color rotation, $e^{i\alpha_{\rm B}}$ a transformation associated with $\mathrm{U}(1)_{\rm B}$, and $e^{i\alpha_{\rm A}}$ an axial transformation.

Given these transformation laws and assuming that the order parameters are small enough to write a power series, one can construct the general GL free energy. Below we work at mean field level, *i.e.*, neglecting the quantum fluctuations. Then, the GL free energy $\Omega(\Phi, d_{\rm L}, d_{\rm R})$, measured with respect to that for $\Phi = d_{\rm L} = d_{\rm R} = 0$, is given as follows

$$\Omega\left(\Phi, d_{\rm L}, d_{\rm R}\right) = \Omega_\chi\left(\Phi\right) + \Omega_d\left(d_{\rm L}, d_{\rm R}\right) + \Omega_{\chi d}\left(\Phi, d_{\rm L}, d_{\rm R}\right)\,, \tag{1}$$

where

$$\begin{aligned}
\Omega_\chi &= \frac{a_0}{2}\mathrm{Tr}\,\Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{Tr}\,\Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\mathrm{Tr}\left(\Phi^\dagger\Phi\right)^2 - \frac{c_0}{2}\left(\det\Phi + \det\Phi^\dagger\right)\,,\\
\Omega_d &= \alpha_0\,\mathrm{Tr}\left[d_{\rm L}d_{\rm L}^\dagger + d_{\rm R}d_{\rm R}^\dagger\right] + \beta_1\left(\left[\mathrm{Tr}\left(d_{\rm L}d_{\rm L}^\dagger\right)\right]^2 + \left[\mathrm{Tr}\left(d_{\rm R}d_{\rm R}^\dagger\right)\right]^2\right)\\
&\quad + \beta_2\left(\mathrm{Tr}\left[\left(d_{\rm L}d_{\rm L}^\dagger\right)^2\right] + \mathrm{Tr}\left[\left(d_{\rm R}d_{\rm R}^\dagger\right)^2\right]\right) + \beta_3\,\mathrm{Tr}\left[\left(d_{\rm R}d_{\rm L}^\dagger\right)\left(d_{\rm L}d_{\rm R}^\dagger\right)\right]\\
&\quad + \beta_4\,\mathrm{Tr}\left(d_{\rm L}d_{\rm L}^\dagger\right)\mathrm{Tr}\left(d_{\rm R}d_{\rm R}^\dagger\right)\,,
\end{aligned}$$

$$\begin{aligned}
\Omega_{\chi d} &= \gamma_1\mathrm{Tr}\left[\left(d_{\rm R}d_{\rm L}^\dagger\right)\Phi + \left(d_{\rm L}d_{\rm R}^\dagger\right)\Phi^\dagger\right] + \lambda_1\mathrm{Tr}\left[\left(d_{\rm L}d_{\rm L}^\dagger\right)\Phi\Phi^\dagger + \left(d_{\rm R}d_{\rm R}^\dagger\right)\Phi^\dagger\Phi\right]\\
&\quad + \lambda_2\mathrm{Tr}\left[d_{\rm L}d_{\rm L}^\dagger + d_{\rm R}d_{\rm R}^\dagger\right]\,\mathrm{Tr}\left[\Phi^\dagger\Phi\right] + \lambda_3\left(\det\Phi\,\mathrm{Tr}\left[\left(d_{\rm L}d_{\rm R}^\dagger\right)\Phi^{-1}\right] + \mathrm{h.c.}\right)\,.
\end{aligned} \tag{2}$$

Here, "Tr" and "det" are taken over the flavor indices. The coefficients $a_0, b_1, b_2, \ldots, \lambda_3$ are called the GL parameters, which are functions of temperature and chemical potential. Ω_χ denotes the chiral part, which was originally studied by Pisarski and Wilczek [7]. Ω_d is the diquark part, originally written down by Iida and Baym. What is new is the third part, $\Omega_{\chi d}$, which represents the interactions between chiral and diquark fields [8].

Note here that the terms proportional to c_0 and γ_1 are cubic in order parameter fields and not invariant under $\mathrm{U}(1)_{\rm A}$ transformation, but break it down to the discrete symmetry group Z_6. This is exactly showing the fact that they originate from the axial anomaly. c_0 should be positive so as for the η' mass positive. Since c_0 and γ_1 terms have the same origin, the sign and the magnitude of γ_1 should be related to those of c_0. This observation is crucial for our analysis of the phase structure performed below.

Let us now restrict ourselves to maximally symmetric condensates, namely, a flavor symmetric chiral condensate in which chiral condensate takes place only within the same flavor, and a color-flavor-locked (CFL) diquark condensate [9] which takes place only in the different flavors: $\Phi = \mathrm{diag}(\sigma, \sigma, \sigma)$, $d_{\mathrm{L}} = -d_{\mathrm{R}} = \mathrm{diag}(d, d, d)$. Then, the reduced GL free energy becomes

$$\Omega(\sigma, d) = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) - \gamma d^2\sigma + \lambda d^2\sigma^2 \,. \quad (3)$$

In principle, the system described by Eq. (3) has four possible phases:

$$\begin{aligned} \text{Normal (NOR) phase} &: \ \sigma = 0\,,\ d = 0\,, \\ \text{Color superconducting (CSC) phase} &: \ \sigma = 0\,,\ d \neq 0\,, \\ \text{Nambu–Goldstone (NG) phase} &: \ \sigma \neq 0\,,\ d = 0\,, \\ \text{Coexistence (COE) phase} &: \ \sigma \neq 0\,,\ d \neq 0\,. \end{aligned}$$

The symmetry breaking pattern of each phase can be summarized as follows: in the CSC phase, $\mathcal{G} \to \mathrm{SU}(3)_{\mathrm{C+L+R}} \times \mathrm{Z}(2)_{\mathrm{B}}$; in the NG phase, $\mathcal{G} \to \mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(3)_{\mathrm{L+R}} \times \mathrm{U}(1)_{\mathrm{B}} \times \mathrm{Z}(2)_{\mathrm{A}}$, and in the COE phase, $\mathcal{G} \to \mathrm{SU}(3)_{\mathrm{C+L+R}} \times \mathrm{Z}(2)_{\mathrm{B}}$. Note that the CSC and COE phases have the same symmetry.

The resultant phase structure in this case has been studied in detail [8]. The most essential point is that the cubic coupling γ, which is positive, favors the coexistence of chiral and diquark condensates. This is what we call chiral-super interplay thorough the axial anomaly. A non-zero d^2 acts as external field for σ. As a consequence, the γ term plays a role of quark

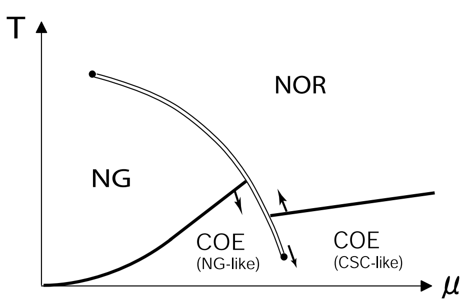

Fig. 1. Schematic phase diagram with two light (up and down) quarks and a medium heavy (strange) quark. The arrows show how the critical point and the phase boundaries move as the strange-quark mass increases.

mass term and then chiral symmetry is all the way down broken. Thus we expect that at very low temperature and intermediate baryon density, a new anomaly-driven critical point might appear in the phase diagram. If we take into account the strange quark mass, another critical point at high temperature and low density will appear [10]. See Fig. 1.

2.2. Excitation spectra — a possible realization of quark–hadron continuity

In this subsection, we examine the mass spectra of mesons in the intermediate density region for the degenerate three-flavor case [11]. We focus on the region near the phase boundaries, where σ and d are sufficiently small in the COE phase, and consider energy scales, p, smaller than the pairing gap d, so that we can neglect excitations of the quarks.

To derive the effective Lagrangian for the light excited states, the pions (π, K and η) in the intermediate density region, we fix the magnitude of the chiral and diquark fields and consider only fluctuations of their phases about their vacuum configurations. We thus parameterize the fields as $\Phi = \sigma\Sigma e^{-2i\theta}$, $d_{\mathrm{L}} = dU_{\mathrm{L}}e^{2i(\tilde{\theta}+\phi)}$, $d_{\mathrm{R}} = -dU_{\mathrm{R}}e^{-2i(\tilde{\theta}-\phi)}$. Here, Σ, U_{L} and U_{R} are SU(3) matrices, and the angles θ and $\tilde{\theta}$ are $\mathrm{U}(1)_{\mathrm{A}}$ phases, and ϕ the $\mathrm{U}(1)_{\mathrm{B}}$ phase. Below we just highlight SU(3) matrix fields U_{L} and U_{R}.

In CFL phase, all eight gluons acquire a mass of the order of $gf_{\tilde{\pi}} \sim \mathcal{O}(g\mu)$ by "eating" the eight colored fluctuations of $U_{\mathrm{L,R}}$, where g is the QCD coupling constant and $f_{\tilde{\pi}}$ is the decay constant associated with $U_{\mathrm{L,R}}$. On the low momentum scales, we consider $p < d \ll gf_{\tilde{\pi}}$, gluons are not low-lying modes. The remaining color-singlet fluctuations correspond to CFL pions, $\tilde{\pi}$ and are parametrized by the field $\tilde{\Delta} = U_{\mathrm{L}}U_{\mathrm{R}}^{\dagger}$, which transforms under $\mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}}$ as $\tilde{\Delta} \to V_{\mathrm{L}}\tilde{\Delta}V_{\mathrm{R}}^{\dagger}$ [12]. Then, the standard pion π^j and the CFL pion $\tilde{\pi}^j$ are defined by $\Sigma = \exp\left(i\lambda^j\pi^j/f_\pi\right)$ and $\tilde{\Delta} = \exp\left(i\lambda^j\tilde{\pi}^j/f_{\tilde{\pi}}\right)$, respectively. Here, the λ^j ($j = 1, \ldots, 8$) are Gell-Mann matrices normalized, so that $\mathrm{Tr}\lambda^i\lambda^j = \frac{1}{2}\delta^{ij}$.

Plugging the expression for the fields Φ, d_{L} and d_{R} into Eq. (2) and expanding in terms of π and $\tilde{\pi}$, one obtains the effective Lagrangian for the ordinary and CFL pions interacting to each other. Due to the interactions, the original pion fields are not the mass eigenstates so that we have to diagonalize the mass matrix, which is expressed as

$$M_{(\pi)} = \begin{pmatrix} \frac{1}{f_\pi^2}\left(\gamma_1 d^2\sigma + \lambda_3 d^2\sigma^2 + A_0 m_q\sigma\right) & -\frac{1}{f_\pi f_{\tilde{\pi}}}\left(\gamma_1 d^2\sigma + \lambda_3 d^2\sigma^2\right) \\ -\frac{1}{f_\pi f_{\tilde{\pi}}}\left(\gamma_1 d^2\sigma + \lambda_3 d^2\sigma^2\right) & \frac{1}{f_{\tilde{\pi}}^2}\left(\gamma_1 d^2\sigma + \lambda_3 d^2\sigma^2 + \Gamma_1 m_q d^2\right) \end{pmatrix}. \quad (4)$$

Here, we are taking into account the non-zero quark mass effect m_q, which is supposed to be equal to all up, down and strange quarks. The eigenstates of the mass matrix $M_{(\pi)}$ can be written as

$$\begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} \pi \\ \tilde{\pi} \end{pmatrix}, \tag{5}$$

with mixing angle ϑ. One can compute the pion masses m_{π_1} and m_{π_2} with and without quark masses. For non-vanishing m_q, one finds

$$m_{\pi_1}^2 = \frac{m_q}{f_\pi^2 + f_{\tilde{\pi}}^2}\left(A_0\sigma + \Gamma_1 d^2\right), \tag{6}$$

$$m_{\pi_2}^2 = m_{\pi_2}^2(m_q = 0) + \frac{m_q}{f_\pi^2 + f_{\tilde{\pi}}^2}\left(\frac{f_{\tilde{\pi}}^2}{f_\pi^2}A_0\sigma + \frac{f_\pi^2}{f_{\tilde{\pi}}^2}\Gamma_1 d^2\right), \tag{7}$$

with the mixing angle

$$\tan\vartheta = \frac{f_{\tilde{\pi}}}{f_\pi} + \frac{f_\pi}{f_{\tilde{\pi}}}\frac{f_\pi^2\Gamma_1 d^2 - f_{\tilde{\pi}}^2 A_0\sigma}{\left(f_\pi^2 + f_{\tilde{\pi}}^2\right)\left(\gamma_1 d^2\sigma + \lambda_3 d^2\sigma^2\right)}m_q\,. \tag{8}$$

Eq. (6) is a generalized form in dense QCD of the Gell-Mann–Oakes–Renner (GOR) relation, connecting the masses of pseudoscalar bosons to the chiral and diquark condensates. The Γ_1 term, which originates from the effect of the axial anomaly in the pion Lagrangian, shows the crucial role of the axial anomaly, not only for the phase structure but also for the excitation spectra in the intermediate density region. When the diquark condensate d decreases as the density becomes low, Eq. (6) reduces to the standard GOR relation $f_\pi^2 m_\pi^2 = A_0 m_q \sigma$. On the other hand, at asymptotically high density, the chiral condensate σ is small and the axial anomaly is highly suppressed as $\Gamma_1 \sim \mu(\Lambda_{\rm QCD}/\mu)^9(1/g)^{14}$ [13]. Then, the linear term in m_q disappears on the right side of Eq. (6) and the leading term becomes $\mathcal{O}(m_q^2)$, with $m_{\pi_1}^2 \propto m_q^2 d^2$. This result is consistent with observations given in [9, 12] at asymptotically high density.

2.3. Meson condensation — CFLK^0 condensate

Up to the previous subsections, we have only considered the cases with massless quarks or quarks of equal mass. In reality, however, it is not the case, *i.e.* the strange quark mass m_s is never small and it is comparable with typical values of the chemical potential in realized neutron stars. Thus it is important to examine the effect of m_s against the phase structure.

To this end, we classify the m_s effects into two parts [14]. One is what we call the direct effect, which means that it gives rise to some modifications of the original GL free energy. The other is the indirect effect, which affects the ansatz for the parametrization of order parameter fields.

We put the quark mass matrix as $M = \mathrm{diag}(m_u, m_d, m_s)$, which transforms under $\mathcal{G}$ as $M \to e^{-2i\alpha_{\mathrm{A}}} V_{\mathrm{L}} M V_{\mathrm{R}}^{\dagger}$. Accordingly, the chiral field Φ is modified from the previous case as $\Phi = \mathrm{diag}(\sigma_u, \sigma_d, \sigma_s)$. On the other hand, for the diquark field, we make the following ansatz

$$d_{\mathrm{L}} = -d_{\mathrm{R}}^{\dagger} = d \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi/2) & i\sin(\phi/2) \\ 0 & i\sin(\phi/2) & \cos(\phi/2) \end{pmatrix} . \tag{9}$$

When $\phi = 0$, Eq. (9) reduces to the previous ansatz for the massless case. So a new field ϕ corresponds to a new condensate by the effect of non-zero m_s. Since ϕ describes a relative phase between $\mathrm{SU}(3)_{\mathrm{L}}$ and $\mathrm{SU}(3)_{\mathrm{R}}$ rotations, we call it a meson condensate. In the current case, the meson to condense is neutral kaon K^0.

Similarly to the previous subsections, one can construct the GL free energy based on symmetries of QCD [14]. The most crucial point is that with the above parameterizations, the Bedaque–Schafer term [15], which is the onset of kaon chemical potential μ_{K^0}, is obtained. As the consequence, in a certain GL parameter region, μ_{K^0} gets bigger than the kaon mass m_{K^0} and then Bose–Einstein condensate (BEC) of neutral kaon occurs. Fig. 2

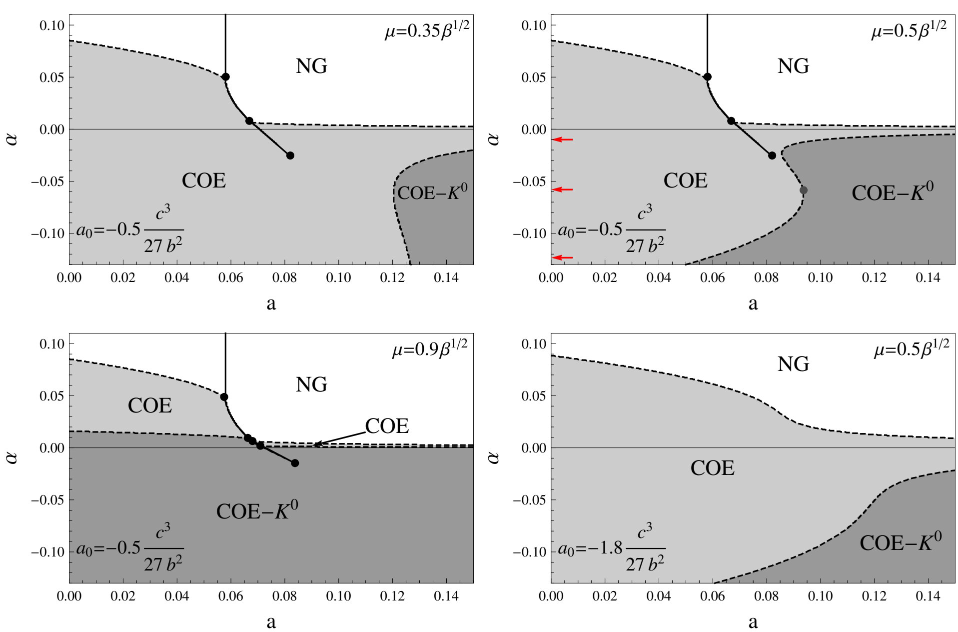

Fig. 2. Ginzburg–Landau phase diagrams with strange quark mass and kaon condensation.

shows the phase diagram in the GL parameter (α, a)-plane, corresponding to four different choices of the parameter set (a_0, μ_{K^0}). The result shows that the critical point vanishes for large quark chemical potential.

3. Astrophysical applications of dense QCD

In Sec. 2, we have shown theoretical aspects for the QCD phase transitions and the excited states at finite baryon density, based on a general ground of the Ginzburg–Landau method. As was already explained in Sec. 1, physics of dense matter is intimately connected to the observational consequences of neutron stars. So far, the experimental effort to investigate properties of dense matter is still a bit far from providing us sufficient data. Also the numerical lattice simulation of dense QCD suffers from the sign problem. Therefore, the application of our study for dense matter into physics of neutron stars potentially constrain theoretical uncertainties. In this section, let us see some recent developments for the observations and theoretical considerations for them.

3.1. Mass-radius relationship of neutron star

Neutron star can be described by the balance between the gravitational force which is attractive and the pressure of the matter inside the star which is repulsive. Assuming spherical symmetry of the star, from the Einstein equation of general relativity, we end up with the following equations to determine the mass (M) and radius (R) of the star

$$\begin{aligned}\frac{dP(r)}{dr} &= -\frac{GM(r)\rho(r)}{r^2}\left(1+\frac{P(r)}{\rho(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{M(r)}\right)\left(1-\frac{2GM(r)}{r}\right)^{-1}, \\ \frac{dM(r)}{dr} &= 4\pi r^2 \rho(r)\,. \end{aligned} \tag{10}$$

This equation is called the Tolman–Oppenheimer–Volkov (TOV) equation [16]. Here, $M(r)$ is the mass of the star up to a radius $r < R$ and $P(r)$ and $\rho(r)$ are the pressure and the energy density at r. To solve the equations, one needs the relation between the pressure and the energy density. This is nothing but the equation of state for the matter inside the star.

Recently, there was a report for the observation of the neutron star mass [17], with which people were very excited. The observed mass is almost 2 M_{SUN}, which should be compared with the standard neutron star mass 1.4 M_{SUN}. Moreover, the observation is quite accurate owing to the use of the Shapiro delay. The authors of [17] have argued that this observation rules out almost all currently proposed hyperon or boson condensate

EOSs. The quark matter can support a star this massive, only if the quarks are strongly interacting and therefore not "free quarks". Here, we show a theoretical study for quark matter on this issue below [18].

In [18], quark matter EOS is parametrized as $P(\mu) = a_4\mu^4 - a_2\mu^2 - B_{\rm eff}$. For the massless and non-interacting quarks, $a_4 = 1$ and $a_2 = 0$. Interaction of quarks and the resultant paring affects both a_4 and a_2, while the effect of non-zero strange quark mass affects a_2. The effective bag constant $B_{\rm eff}$ is fixed so as to provide nuclear to quark matter transition at 1.5 $n_{\rm sat.}$. Then, the authors estimated the mass for various values of a_4 and a_2. As the consequence, they found that $a_4 \leq 0.6$ and $a_2 \leq 10^4$ MeV^2 is consistent with the observed mass. This means that the interaction among quarks is fairly strong and the pairing gap is not too small compared to strange quark mass.

In the analysis of [18], however, possible effects stemming from temperature as well as magnetic field have not been taken into account. So it will be interesting to seek how these effects come out into the current issue.

3.2. Cooling behavior of neutron star

Thermal history of neutron star provides us another important information for the stellar structure. When a neutron star was born just after the supernova explosion, temperature of the star is a few tens of MeV, but afterwards it cools down via neutrino emissions for the first million years. Then, the star is thermalized and the photon emission on the surface gets dominant. The cooling behavior is described by the following equation

$$C_V \frac{dT(t)}{dt} = -L_\nu - L_\gamma \,, \tag{11}$$

where $T(t)$ is the temperature as a function of time t, C_V heat capacity and L_ν, L_γ luminosities of neutrino and photon, respectively. Once the stellar structure and the interaction rates such as mean free path and emissivity of neutrinos are given, one can draw the cooling curve.

Recently, there was a report for rapid cooling of the neutron star in Cassiopeia A [19]. The authors have proposed that this is triggered by neutron $^3P_2$ superfluidity. They studied the issue based on the "minimal cooling" paradigm, which means that the cooling of neutron star via neutrino emissions can be dominated by the so-called modified URCA process and nucleon superfluidity. This is still under investigation.

3.3. The r-mode instability of neutron star

The last interesting property of neutron star to introduce here is associated with its rotation. In fact, the first neutron star was discovered as a rapidly rotating object, namely, pulsar [20]. When the star rotates, the rotation accompanies some flow of matter inside the star. According to the general relativity, such a matter flow causes gravitational wave (GW) radiation. There are various rotational modes. Among them certain non-radial oscillatory modes, in particular, the r(otational)-modes are known to be unstable against the GW radiation [21]. On the other hand, the rotational frequency of neutron stars have been accurately measured. Thus if dissipative phenomena are not strong enough, the oscillations grow exponentially and the star will keep slowing down until some mechanism can damp the r-modes. Therefore, the systematic study of the r-modes is useful in constraining the stellar structure.

So far, some mechanisms have been taken into account for providing enough dissipation to explain the observational data. The simplest solution is to consider the effect of viscosities [22]. But here we show another mechanism, called the mutual friction [23]. The mutual friction is a friction force between normal and superfluid components of matter, provided by the presence of vortices. It manifests in experiment as a certain dissipation present in rotating superfluid state.

We start with considering the so-called Magnus force between superfluid component and a vortex

$$\vec{F}_{\rm M} = \kappa\rho_{\rm s}\,(\vec{v}_{\rm s} - \vec{v}_{\rm L}) \times \vec{\hat{z}}\,, \tag{12}$$

where κ is a circulation, $\rho_{\rm s}$ superfluid density and $\vec{v}_{\rm s}, \vec{v}_{\rm L}$ superfluid and vortex velocities, respectively. On the other hand, there is a force produced by the normal component as

$$\vec{F}_{\rm N} = -D(\vec{v}_{\rm n} - \vec{v}_{\rm L}) - D'\vec{\hat{z}} \times (\vec{v}_{\rm n} - \vec{v}_{\rm L}) \tag{13}$$

with the coefficients D and D' which are computed microscopically. Then, the force valance condition for the vortex is given by $\vec{F}_{\rm M} + \vec{F}_{\rm N} = 0$ and then the vortex velocity is obtained as $\vec{v}_{\rm L} = \vec{v}_{\rm s} + \alpha'(\vec{v}_{\rm n} - \vec{v}_{\rm s}) + \alpha\vec{\hat{z}} \times (\vec{v}_{\rm n} - \vec{v}_{\rm s})$. Here, the coefficients α and α' are called the Hall–Vinen parameters [23]. If a perturbation of the superfluid velocity $\delta\vec{v}_{\rm L}$ is introduced, there is no more guarantee that two forces $\vec{F}_{\rm M}$ and $\vec{F}_{\rm N}$ are balanced. That is the origin of the energy dissipation, *i.e.*, $(dE/dt)_{\rm MF} = \delta\vec{F} \cdot \delta\vec{v}_{\rm L}$. In [24], the mutual friction of CFL baryon superfluid has been estimated in terms of a rotational (Nambu–Goldstone) mode around the vortex. This can be extended so as to include the mode perpendicular to the vortex (the Higgs mode) [25].

4. Summary

In this paper, we have emphasized an intimate connection between physics of dense matter and that of astrophysics. Dense hadronic and quark matter can be described by QCD with finite chemical potential, but since QCD is strongly coupled in density regime of our interest and the lattice simulation does not work in the regime so far, one needs some effective field theory approaches to describe QCD phase transitions and the phase diagram. As such an example, we have present the Ginzburg–Landau analysis. On the other hand, in order to comprehend physical properties of neutron stars such as stellar structure, cooling behavior, origin of strong magnetic field and stability for gravitational wave radiation, we have shown some applications of theoretical study for dense matter, *i.e.*, equation of state (EOS), neutrino interaction rates and the r-mode instability. In the future, much more progress in both theoretical and observational/experimental directions is expected.

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