NEAR-THRESHOLD CONFIGURATION MIXING*

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The near-threshold mixing of Shell Model (SM) states is studied using the real-energy Continuum Shell Model (CSM). It is shown that the salient features of the configuration mixing can be traced back to the energy dependence of the nearby double-pole of the S-matrix of the complex-extended effective Hamiltonian of the CSM.

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1. Introduction

How does the continuum coupling work close to the decay threshold? Are the SM eigenstates stable against the continuum coupling at the channel threshold? These questions, which have been at the centre of the scientific debate when the paradigm of the nuclear SM was born [1, 2], are of great importance nowadays in the context of rare isotope studies. They can be addressed in the Shell Model for open quantum systems which has been already postulated by Fano [2] but its realization took nearly 40 years and required solving many conceptual and technical problems (for recent reviews see [3, 4]). Below, we shall use the modern version of the CSM, the Shell Model Embedded in the Continuum (SMEC) [5, 3, 6] to illustrate certain universal features of many-body wave functions in the vicinity of the dissociation threshold.

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2. The SMEC syllabus

The SMEC is formulated in the Hilbert space using the projection technique. Its detailed description can be found in recent reviews [3, 7]. The Hilbert space is divided into orthogonal subspaces $\mathcal{Q}_i$ with a different number i of particles in the scattering continuum ($i = 0, 1, \ldots$). An open quantum system (OQS) description of "internal" dynamics, *i.e.* in $\mathcal{Q}_0$, includes couplings to the "environment" of decay channels and is given by the energy-dependent effective Hamiltonian [3]

$$H^{\mathrm{eff}}(E) = H_0 + H_1(E) \simeq H_0 + V_0^2 h(E)\,, \tag{1}$$

where H_0 is the closed quantum system (CQS) Hamiltonian, V_0 is the continuum-coupling constant, E is the scattering energy and $h(E)$ is the coupling term between localized states ($\mathcal{Q}_0$) and the environment of decay channels ($\mathcal{Q}_1$, $\mathcal{Q}_2$,…). For $E < 0$ (bound system), eigenvalues $\mathcal{E}_\alpha(E)$ of H^{eff} are real. In the scattering continuum, $\mathcal{E}_\alpha(E)$ correspond to poles of the S-matrix [3] and H^{eff} is complex-symmetric (non-hermitian). The competition between hermitian (H_0) and non-hermitian ($V_0^2 h(E)$) parts of the effective Hamiltonian (1) may lead to the appearance of coalescing eigenvalues (exceptional points (EP)), *i.e.* to the formation of double-poles of the S-matrix [8]. Signatures of EPs in the continuum part of the spectrum have been discussed in Ref. [8].

The wave function mixing in the CQS is closely related to features of EPs of the complex-extended CQS Hamiltonian. Similarly, to understand the configuration mixing in the OQS it is compulsory to analyse the spectrum EPs of the complex-extended OQS Hamiltonian. Since the OQS Hamiltonian (1) is energy dependent, the essential information about the configuration mixing in the scattering continuum is contained in the lines of coalescing eigenvalues $\mathcal{E}_{\alpha_1}(E) = \mathcal{E}_{\alpha_2}(E)$ (exceptional threads (ETs)) of the effective Hamiltonian for *complex* coupling V_0. In the next section, we shall study the mixing of CQS eigenvalues (SM eigenvalues) close to the proton decay threshold in terms of the threads of EPs (ETs) of the complex-extended SMEC Hamiltonian.

3. Exceptional threads close to the proton decay threshold

EPs have been studied mostly in schematic systems. Here we present the analysis of a near-threshold configuration mixing and the role of ETs therein on the example of spectrum of ^{16}Ne. Information about the model space and the many-body Hamiltonian in this calculation can be found in [8].

The upper part of Fig. 1 shows the real and imaginary parts of the off-diagonal mixing coefficient b_{12} between two $J^\pi = 0^+$ CQS eigenstates ($J^\pi = 0^+$ SM eigenstates) of ^{16}Ne, due to their coupling to the same (elastic) decay channel $\left[{}^{15}\mathrm{F}(1/2^+) \oplus \pi s_{1/2}\right]^{0^+}$ which corresponds to the emission of a proton in the relative s state ($\ell = 0$). Calculations have been performed using the SMEC Hamiltonian for the *real* continuum-coupling constant $V_0 = -1000$ MeV fm^3. The mixing coefficient is plotted here as a function of the proton scattering energy E. The threshold corresponds to $E = 0$. The maximum of b_{12} is shifted to positive energies due to the Coulomb barrier.

In the lower part of Fig. 1, we show real and imaginary parts of the continuum-coupling constant for which the lowest $J^\pi = 0^+$ ETs appear for decaying (d) and capturing (c) resonances of the complex-extended SMEC Hamiltonian. For $E < 0$, ETs for c- and d-resonances have the same imaginary parts of V_0 and their real parts have the opposite sign. This symmetry is lost for $E > 0$.

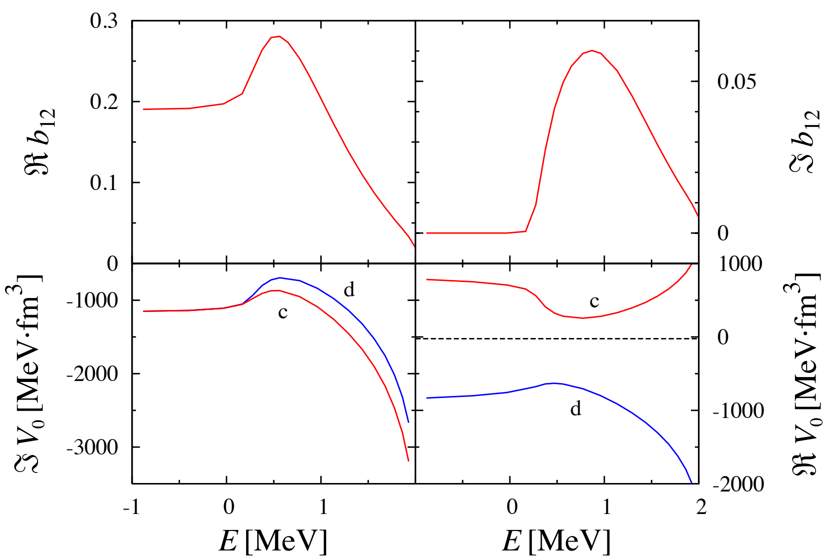

Fig. 1. Upper part: the real and imaginary parts of the off-diagonal mixing coefficient b_{12} between two $J^\pi = 0^+$ SM eigenstates of ^{16}Ne, due to their coupling to the same (elastic) decay channel $\left[{}^{15}\mathrm{F}(1/2^+) \oplus \pi s_{1/2}\right]^{0^+}$. The continuum-coupling constant equals: $V_0 = -1000$ MeV fm^3. Lower part: the real and imaginary parts of the continuum-coupling constant $V_0(E)$ corresponding to the lowest ETs for decaying and capturing $J^\pi = 0^+$ resonances. For more details, see the text.

It can be clearly seen that the point of the closest approach between the ETs and the ReV_0-axis corresponds to the region of rapid variation of $b_{12}(E)$. For $E > 0$, the ET for d-resonances is closer to the ReV_0-axis than the corresponding ET for c-resonances and, therefore, its influence on the eigenfunction mixing is stronger.

It should be stressed that a complete understanding of the configuration mixing phenomena in OQS is directly related to the properties of ETs in the complex-extended OQS Hamiltonian. ETs in close proximity to a *physical* SMEC Hamiltonian (Im$V_0 \simeq 0$), and not EPs of the *physical* SMEC Hamiltonian (Im$V_0 = 0$), provide insight into a configuration mixing in the OQSs. Further developments, in particular, the formulation of a unified theory of structure and reactions based on the Gamow Shell Model [9], are mandatory to describe the formation of correlated domains of states in weakly-bound neighbouring nuclei, and the cluster dominance in the neighbourhood of the corresponding cluster decay threshold [10].

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REFERENCES

[1] J.B. Ehrman, *Phys. Rev.* **81**, 412 (1951); R.G. Thomas, *Phys. Rev.* **88**, 1109 (1952).

[2] U. Fano, *Phys. Rev.* **124**, 1866 (1961).

[3] J. Okołowicz, M. Płoszajczak, I. Rotter, *Phys. Rep.* **374**, 271 (2003).

[4] N. Michel *et al.*, *J. Phys. G* **37**, 064042 (2010).

[5] K. Benneaceur *et al.*, *Nucl. Phys.* **A651**, 289 (1999).

[6] J. Rotureau, J. Okołowicz, M. Płoszajczak, *Phys. Rev. Lett.* **95**, 042503 (2005).

[7] B. Blank, M. Płoszajczak, *Rep. Prog. Phys.* **71**, 046301 (2008).

[8] J. Okołowicz, M. Płoszajczak, *Phys. Rev.* **C80**, 034619 (2009).

[9] N. Michel *et al.*, *Phys. Rev. Lett.* **89**, 042502 (2002); R. Id Betan *et al.*, *Phys. Rev. Lett.* **89**, 042501 (2002).

[10] J. Dobaczewski *et al.*, *Prog. Part. Nucl. Phys.* **59**, 432 (2007).