ON OPTIMAL SHAPES OF FISSIONING AND ROTATING NUCLEI* **

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Following the idea of Strutinsky we have evaluated in a liquid-drop type of approach the shapes of fissioning nuclei along the fission valley in a model independent way. These optimal shapes, which correspond to the minimum of the LD energy for a given elongation, are compared with the shapes obtained in some often used shape parametrisations. The effect of rotation on the optimal shapes of nuclei is discussed within an optimal-shape theory generalised to non-axial forms.

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A parameter-free description of nuclear shapes is one of the fundamental problems in nuclear-structure theory. Following the idea of Strutinsky *et al.* [1] we have found parametrisation-independent shapes of rotating and fissioning nuclei. These so-called *optimal shapes* correspond to the minimum of the liquid drop (LD) energy and fulfil constrains for *e.g.* volume, elongation, quadrupole moment or mass asymmetry and centre of mass position. Let us recall here the main equations of the optimal-shape theory [1, 2].

The binding energy of a charged and rotating liquid drop is the sum of the volume, surface, Coulomb and rotational terms

$$E = E_{\rm V} + E_{\rm S}^0\, B_{\rm S}({\rm def}) + {\rm E}_{\rm C}^0\, {\rm B}_{\rm C}({\rm def}) + {\rm E}_{\rm R}^0\, {\rm B}_{\rm R}({\rm def}) + {\rm const.} \tag{1}$$

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Here $E_{\rm S}^0$, $E_{\rm C}^0$ and $E_{\rm R}^0$ are the surface, Coulomb and rotational energies of the spherical drop, and $B_i = E_i(\mathrm{def})/E_i^0$ are the corresponding shape functions characterising the deformation. These functions can be written as integrals

$$B_i(\mathrm{def}) = \int\limits_{z_{\rm min}}^{z_{\rm max}} dz \int\limits_0^{2\pi} d\varphi \; b_i\left(\rho, \rho_z, \rho_\varphi\right) \tag{2}$$

over the nuclear surface $\rho(z,\varphi)$ expressed in the cylindrical coordinates. Here, we have used the shorthand notation $\rho_z = \partial\rho/\partial z$ and $\rho_\varphi = \partial\rho/\partial\varphi$. The integration limits $z_{\rm min}$ and $z_{\rm max}$ correspond to the z-coordinates of the tips of the nucleus.

Following the notation of Ref. [3] one can write the LD deformation energy (in units of $E_{\rm S}^0$) as

$$E_{\rm def} = (B_{\rm S} - 1) + 2{\rm x}_{\rm LD}(B_{\rm C} - 1) + {\rm y}_{\rm LD}(B_{\rm R} - 1)\,, \tag{3}$$

where ${\rm x}_{\rm LD} = E_{\rm C}^0/2E_{\rm S}^0$ and ${\rm y}_{\rm LD} = E_{\rm R}^0/E_{\rm S}^0$ are the fissility and rotational parameters, respectively. For a charged drop one has ${\rm x}_{\rm LD} > 0$, while ${\rm x}_{\rm LD} < 0$ corresponds to gravitating objects.

The variation principle (with respect to the different shapes of nucleus), similar to that developed by Strutinsky *et al.* in Ref. [1], applied to the deformation energy (3)

$$\delta_\rho E_{\rm def} = \delta_\rho \int\limits_{z_{\rm min}}^{z_{\rm max}} dz \int\limits_0^{2\pi} d\varphi \; \xi(\rho, \rho_z, \rho_\varphi; z, \varphi) = 0\,, \tag{4}$$

leads to a *generalised Euler–Langevin* partial differential equation

$$\frac{\partial}{\partial z}\frac{\partial \xi}{\partial \rho_z} + \frac{\partial}{\partial \varphi}\frac{\partial \xi}{\partial \rho_\varphi} - \frac{\partial \xi}{\partial \rho} = 0 \tag{5}$$

which should be solved numerically. Here $\xi(\rho, \partial\rho/\partial z, \partial\rho/\partial\varphi; z, \varphi)$ is the "surface-energy density" of the deformation energy. The second term in the above equation disappears when one deals with axially symmetric shapes, only. The volume conservation and the deformation of the nucleus are imposed through appropriate constrains for fixed volume V and *e.g.* fixed quadrupole moment Q

$$\xi \;\longrightarrow\; \widetilde{\xi}\left(\rho, \rho', z\right) = \xi\left(\rho, \rho', z\right) - \lambda_1 V(\rho, z) - \lambda_2 Q(\rho, z)\,, \tag{6}$$

where λ_i are the corresponding Lagrange multipliers.

The optimal fission barriers and corresponding nuclear shapes obtained by solving Eq. (5) for different values of the fissility parameter ${\rm x}_{\rm LD}$ are displayed in Fig. 1 as function of the distance between the mass centers of symmetric fission fragments.

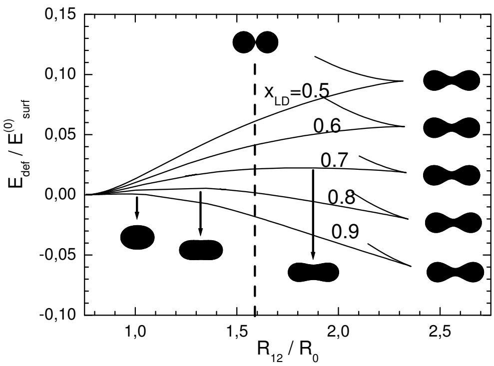

Fig. 1. Optimal fission barriers and corresponding nuclear shapes for a few values of the fissility parameter [2].

For a fixed value $x_{LD} = 0.75$ of the fissility parameter, the form of the fission barrier obtained with optimal shape prescription is compared in Fig. 2 with the barriers obtained using the Funny-Hills (FH) shape parametrisation [4] and the modified Funny-Hills (MFH) shapes [5].

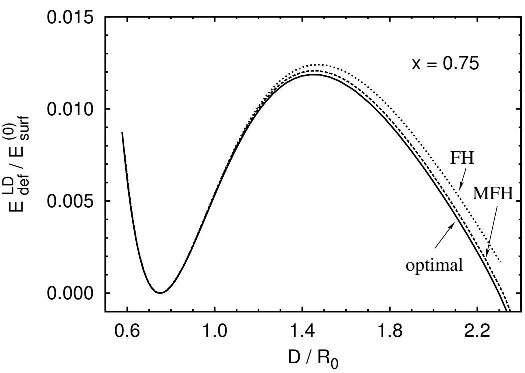

Fig. 2. Comparison of the optimal fission barrier with the estimates obtained with two-parameter shape parametrisations, the Funny-Hills [4] and the Modified Funny-Hills [5] shapes.

It is seen that the MFH estimate of the barrier is very close to the optimal one. In the expansion into spherical harmonics, a similar quality can only be obtained when including six terms corresponding to the first even multipolarities, which demonstrates how powerful the FH or MFH parametrisation are, which contain only two deformation parameters in the case of axially symmetric shapes. The evolution of the shapes and the non-axiality in the uncharged drop for some values of the rotational parameter y_{LD} is shown in Fig. 3, where the optimal shapes (l.h.s. plot) and the non-axiality parameter η see Eq. (7)) are drawn as function of z-coordinate.

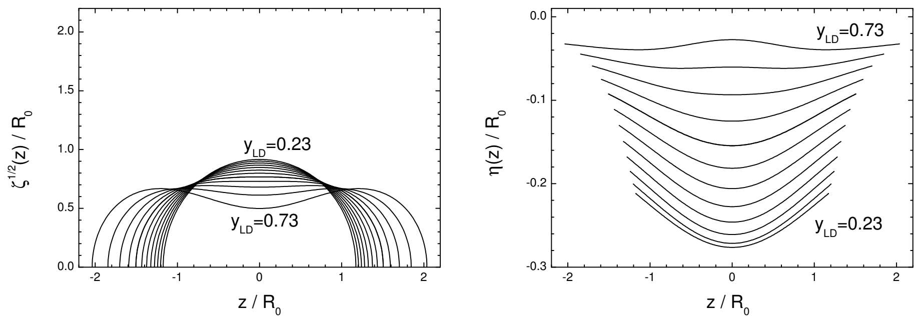

Fig. 3. The optimal shapes (l.h.s. plot) and the non-axiality parameter η (r.h.s. plot) of the rotating uncharged drop are as function of z-coordinate

These data are obtained by solving the two-dimensional Euler–Lagrange equation (5) using the following simplified Ansatz for the form of the shape profile

$$\rho^2(z,\varphi) = \zeta(z)\,[1 + \eta(z)\cos(2\varphi)] \tag{7}$$

which simplifies significantly the numerical calculations. It came as a surprise to us to notice that the non-axiality parameter varies along the symmetry axis (z-axis) when rotation is present and should therefore not be taken as constant. This important result makes questionable shape parametrisations with constant non-axiality, like in an ellipsoid.

Summarising, we conclude that Strutinsky's theory of optimal shapes offers an useful tool to investigate the potential energy surface in liquid-drop type models. Different types of nuclear deformations like elongation or mass asymmetry can be achieved in the optimal-shape theory by adding appropriate constrains *e.g.* for quadrupole or octupole moment. The optimal shapes allow to test different shape parametrisations of the surface of fissioning and rotating nuclei. Our new two dimensional version of the optimal-shape theory allows to study in details the Jacobi transitions and Poincare instabilities. Further calculations are in progress.

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