NUCLEAR COLLECTIVE MODELS AND PARTIAL SYMMETRIES*

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It is shown that a mathematical modelling of the collective vibrations in the presence of the tetrahedral symmetry, *in contrast to the previous simplistic predictions*, may lead to large quadrupole moments Q_0 in the tetrahedral symmetry nuclear bands. Their tetrahedral character originates from the fact that the vibrations take place around a tetrahedral minimum, however, a large amplitude vibrations collect large contributions to Q_0.

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Following microscopic calculations, *cf.* *e.g.* [1], an idea of an existence of point-group symmetries generating four-fold degeneracy of the nucleonic levels (tetrahedral and/or octahedral ones) has been advocated. Theoretical description of the corresponding collective nuclear states requires constructing Hamiltonians with appropriate symmetries. For example, one can expect that the collective wave-functions of the ground-state band built on a quadrupole-triaxial state belong to an irreducible representation of the D_{2h} group. In the case of a possible coexistence of *e.g.* the D_{2h}-quadrupole

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with the tetrahedral- or octahedral-symmetry bands the adapted Hamiltonians need to be constructed having a low symmetry, but containing terms ("sub-Hamiltonians") capable of describing bands with higher symmetries.

In this paper, we examine symmetries of collective Hamiltonians using from now on the intrinsic nuclear frames. A possibility of decomposing an arbitrary Hamiltonian $\hat{\mathcal{H}}$ into a sum of distinct-symmetry, orthogonal[1] sub-Hamiltonians was discussed in Ref. [2]. Accordingly, one writes $\hat{\mathcal{H}} = \sum_{\rm G} \hat{\mathcal{H}}_{\rm G}$, where every sub-Hamiltonian $\hat{\mathcal{H}}_{\rm G}$ is invariant under its own symmetry group G. Such a decomposition introduces several coexisting families of distinct-symmetry rotational bands whose states are mutually orthogonal but they can couple through (possibly strong) electromagnetic transitions. Every sub-Hamiltonian defines its own sub-set of rotational bands and governs the symmetry-related physical properties.

Alternative constructions can be envisaged. Consider a deformed nucleus in which rotations and vibrations are separated

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{\rm vib} + \hat{\mathcal{H}}_{\rm rot}\,. \tag{1}$$

In the following, symbols $\rm G_{vib}$ and $\rm G_{rot}$ denote symmetry groups of the sub-Hamiltonians $\hat{\mathcal{H}}_{\rm vib}$ and $\hat{\mathcal{H}}_{\rm rot}$, respectively, and $\rm G_H$ is the symmetry group of the resulting collective Hamiltonian $\hat{\mathcal{H}}$. Equation (1) represents the decomposition of $\hat{\mathcal{H}}$ into two non-orthogonal sub-Hamiltonians having their own symmetries. This decomposition is less powerful since it involves vector spaces that are not orthogonal, however, it also allows to analyse spectra and eigen-states in terms of symmetries.

The assumption about no (or weak) coupling between rotations and vibrations splits the eigen-value equations associated with $\hat{\mathcal{H}}$ into two *viz.*

$$\hat{\mathcal{H}}_{\rm vib}\, \phi_{\nu_{\rm v} \mathit{\Gamma}_{\rm v} a_{\rm v}}(\alpha) \;=\; \epsilon^{\rm (vib)}_{\nu_{\rm v} \mathit{\Gamma}_{\rm v}}\, \phi_{\nu_{\rm v} \mathit{\Gamma}_{\rm v} a_{\rm v}}(\alpha)\,, \tag{2}$$

$$\hat{\mathcal{H}}_{\rm rot}\, \mathcal{R}^{JM}_{\nu_{\rm r} \mathit{\Gamma}_{\rm r} a_{\rm r}}(\mathit{\Omega}) \;=\; \epsilon^{\rm (rot)}_{J;\nu_{\rm r} \mathit{\Gamma}_{\rm r}}\, \mathcal{R}^{JM}_{\nu_{\rm r} \mathit{\Gamma}_{\rm r} a_{\rm r}}(\mathit{\Omega})\,. \tag{3}$$

The eigen-values $\epsilon^{J}_{\nu_{\rm v} \mathit{\Gamma}_v;\nu_{\rm r} \mathit{\Gamma}_{\rm r}}$ and eigenfunctions $\mathit{\Psi}^{JM}_{\nu_{\rm v} \mathit{\Gamma}_{\rm v} a_{\rm v};\nu_{\rm r} \mathit{\Gamma}_{\rm r} a_{\rm r}}(\alpha, \mathit{\Omega})$ of the full Hamiltonian are now sums of eigen-values and products of eigenfunctions obtained from the solutions of the equations (2) and (3), respectively. Labels J and M (angular momentum projection in the laboratory frame) denote angular momentum quantum numbers whereas $\{\nu_{\rm v}, \nu_{\rm r}\}$ are additional indices labelling equivalent irreducible representations of groups $\rm G_{vib}$ and $\rm G_{rot}$, respectively. The labels $a_{\rm v}$ and $a_{\rm r}$ distinguish basis vectors within the irreducible representations $\mathit{\Gamma}_{\rm v}$ and $\mathit{\Gamma}_{\rm r}$.

[1] Two terms of the Hamiltonian, *e.g.* $\hat{\mathcal{H}}_{\rm G'}$ and $\hat{\mathcal{H}}_{\rm G''}$ are called orthogonal if they act on the mutually orthogonal vector spaces.

A typical example illustrating the construction discussed so far is Bohr Hamiltonian with neglected coupling between vibrations and rotations

$$\hat{\mathcal{H}}_{\rm B} = \hat{\mathcal{H}}_{{\rm vib};2}(\beta,\gamma) + \hat{\mathcal{H}}_{\rm rot}(\Omega)\,. \tag{4}$$

Using the generators R_1, R_2 and R_3 of the symmetrisation group $\overline{\rm O}_h$, derived in the Ref. [4], it is easy to check that the vibrational part, $\hat{\mathcal{H}}_{{\rm vib};2}$, is invariant under the octahedral group ${\rm G}_{\rm vib} = \overline{\rm O}_h$ acting *only* in the deformation subspace. On the other hand, the rotational part $\hat{\mathcal{H}}_{\rm rot}$ is invariant under the group ${\rm G}_{\rm rot} = \overline{\rm D}_{2h}$ acting *only* in the rotor space.

The most important message so far is that these are the symmetries of commuting sub-Hamiltonians rather than the formal symmetry group of the total Hamiltonian $\hat{\mathcal{H}}$ that determine the effective symmetry structures of vibrational and rotational states.

In order to be able to illustrate the preceding discussion, we have used a simplified model Hamiltonian, $\hat{\mathcal{H}} = \hat{\mathcal{H}}_{{\rm vib};\lambda=2} + \hat{\mathcal{H}}_{{\rm vib};\lambda=3} + \hat{\mathcal{H}}_{\rm rot}$, discussed in detail in Ref. [6], *cf.* in particular Eqs. (10)–(12) and the surrounding text. There, we have postulated a separable form of the collective Hamiltonian with the effective potential energy treated within a harmonic approximation — an approach which allows to obtain an order-of-magnitude estimates. In the analysis quoted, the lowest negative parity in ^{156}Gd band was interpreted as tetrahedral-symmetry band using the symmetry of the assumed vibrational sub-Hamiltonian.

Recently, a new measurement on ^{156}Gd has been published in Ref. [7] suggesting that the tetrahedral-symmetry candidate negative-parity band has the quadrupole moment comparable with the one in the ground-state band — the result in an apparent contradiction to the original prediction of small quadrupole moments for tetrahedral bands, *cf.* Ref. [1]. To illustrate the formalism introduced so far we have used the matrix elements of transition-operators as introduced in Ref. [6] and the separable Hamiltonian of Eq. (1). Calculations show that the quadrupole moment of the members of the tetrahedral band, for small values of the octupole oscillator parameter $\eta_3 = \sqrt{B_3\omega_3/\hbar} = \sqrt{C_3/\hbar\omega_3}$ introduced in Ref. [6], is growing rapidly in function of the decreasing stiffness coefficient C and allows to reproduce the experimental values of the quadrupole moment of the tetrahedral-candidate negative parity band in ^{156}Gd, close to the result of [7]. All the parameters entering our estimates have been obtained from fitting to the microscopic universal Woods–Saxon total-energy calculation results that predict a coexistence between the quadrupole and the tetrahedral configurations.

Essential parameters entering the calculations are: (a) for the quadrupole part of the Hamiltonian, the quadrupole deformation parameters $\beta = 0.23$, $\gamma = 10^\circ$ and the quadrupole oscillator parameter $\eta_2 = 12$; (b) for the octupole part of the Hamiltonian the tetrahedral deformation parameter

$\xi = \mathrm{Im}\,\alpha_{32} = 0.12$ and a much smaller, compared to the ^{156}Dy nucleus, an octupole harmonic oscillator parameter $\eta_3 = 0.515$; parameters η contain the information about the stiffness coefficients C of the microscopically calculated total energy surfaces. Using these parameters we obtain the quadrupole moment of the ground-state band equal to 788 $e^2\,\mathrm{fm}^4$ compared to the experimental value of 683 $e^2\,\mathrm{fm}^4$.

The dipole transitions from the tetrahedral band to the ground state band are dependent on irreducible representation to which the octupole state belongs. The tetrahedral, octupole states may belong to one of the three irreducible representations of the tetrahedral group $\overline{\mathrm{T}}_d$. One can show that the scalar representations, $A1$, does not allow for dipole transitions to quadrupole band and it is not interesting in the present context. The 3-dimensional representation $T2$ contains the axial-symmetric vector corresponding to axial octupole one phonon excitation of the nucleus whereas the other 3-dimensional representation, $T1$, represents the tetrahedral excitation. Both representations may contribute to the non-zero static tetrahedral deformation ξ and both reproduce the experimental reduced $E2$ transition probabilities within the negative-parity band $B(E2; T1 \to T1) = B(E2; T2 \to T2) \sim 298$ W.u. However, representation $T2$, corresponding to the "axial octupole excitation", does not reproduce the experimental ratio $B(E2; T2 \to T2)/B(E1; T2 \to q) \sim 3.6 \times 10^3$ fm^2, which should be of the order of 10^6. On the other hand, the representation $T1$, related to the "tetrahedral excitation", gives the correct ratio $B(E2; T1 \to T1)/B(E1; T1 \to q) \sim 1 \times 10^6$ fm^2.

Moreover, the reduced transition probabilities from the octupole states $T1$ to the ground-state band are $B(E1; T1 \to q) \sim 7 \times 10^{-3}$ W.u., which is in agreement with the measured values, whereas the reduced transition probabilities from $T2$ to the ground-state band are not reproduced by calculated $B(E1; T2 \to q) \sim 0.2$ W.u. which is 2 orders of magnitude too large.

Summarising, the collective-model calculations suggest possibly large contribution to the quadrupole moment of the negative parity tetrahedral-candidate bands coming from large octupole vibrations in case of soft octupole collective potential. Further investigations are in progress.

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