# A SYMMETRY OF THE CPHC MODEL OF ODD–ODD NUCLEI AND ITS CONSEQUENCES FOR PROPERTIES OF $M1$ AND $E2$ TRANSITIONS*

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A new symmetry of the Core-Particle-Hole Coupling model of odd–odd nuclei is discussed. This symmetry is used to explain a staggering pattern of the $M1$ and $E2$ transitions within and between partner bands.

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## 1. Introduction

For several years the concept of chiral symmetry [1] has been applied to explain some properties of medium heavy odd–odd nuclei, see [2] for a recent review. These properties are: the existence of nearly degenerate partner

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bands with the same electromagnetic features and a characteristic staggering pattern of $M1$ and $E2$ transitions with $\Delta I = 1$. In [3] we studied such properties within the Core-Particle-Hole Coupling model [4] using both $\gamma$-soft and rigid cores. It turns out that such staggering, obtained by numerical calculations in [3], can be explained by a kind of selections rules following a new, not discussed as yet, symmetry of the model. This symmetry is a combination of the parity operation in the five dimensional space of deformation of the core (we stress that it is not the parity in the ordinary space) and an exchange of states of the unpaired particles. The considered symmetry is not fully preserved in realistic cases, but small deviation from this symmetry does not destroy the staggering patterns in the $M1$ and $E2$ transition probabilities. A detailed presentation will be published elsewhere [5].

## 2. Symmetry of the CPHC model

### *2.1. Hamiltonian*

We assume the Hamiltonian of the odd–odd nucleus of the form

$$H_{\mathrm{o-o}} = H_{\mathrm{core}} - \chi Q \cdot q_\pi + \chi Q \cdot q_\nu + \chi q_\pi \cdot q_\nu \,, \tag{1}$$

where $Q$, $q_\pi$, $q_\nu$ are mass quadrupole operators of the core, proton–particle and neutron–hole respectively. The sign of the two last terms of Eq. (1) reflects the fact that neutron is a hole. The core is described by the general Bohr Hamiltonian

$$H_{\mathrm{core}} = H_{\mathrm{GBH}}(\beta,\gamma,\Omega) = T_{\mathrm{vib}}(\beta,\gamma) + T_{\mathrm{rot}}(\beta,\gamma,\Omega) + V(\beta,\gamma)\,, \tag{2}$$

where $\beta,\gamma$ are the deformation variables in the intrinsic frame and $\Omega$ stands for the Euler angles, for details see [6]. The kinetic energy is determined by the six inertial functions $B_{\beta\beta}$, $B_{\beta\gamma}$, $B_{\gamma\gamma}$ and $B_k, k = 1,2,3$, depending, in general, on $\beta,\gamma$ variables. The moments of inertia are given by the relation $J_k(\beta,\gamma) = 4B_k\beta^2\sin^2(\gamma - 2\pi k/3)$.

The symmetry operation $S$ is a combination of the $\alpha$-parity of the core, $P_\alpha$, and the exchange of the proton and neutron states, $C_{\pi\nu}$

$$S = P_\alpha C_{\pi\nu}\,. \tag{3}$$

The $P_\alpha$ operator is defined simply in terms of the laboratory variables

$$P_\alpha \alpha_\mu = -\alpha_\mu\,, \qquad \mu = -2,\ldots,2\,. \tag{4}$$

It should be mentioned that $P_\alpha$ has no relation to the parity in the ordinary space ($\alpha_\mu$ are invariant against the space parity). In the axial case $P_\alpha$ transforms prolate into oblate shapes and *vice versa*. In the intrinsic frame the $\alpha$-parity can be implemented in several equivalent ways. The form given below is convenient if we consider the only one sextant $0 \le \gamma \le \pi/3$ in the deformation plane

$$P_\alpha(\beta, \gamma, \Omega) = (\beta, \pi/3 - \gamma, R_x(\pi/2)\Omega)\,, \tag{5}$$

here $R_x$ is a rotation around the intrinsic $x$-axis.

The exchange operator $C_{\pi\nu}$ in the proton–neutron space is defined as

$$C_{\pi\nu}|(\pi, j_\pi m_\pi)(\nu, j_\nu m_\nu)\rangle = |(\pi, j_\nu m_\nu)(\nu, j_\pi m_\pi)\rangle\,. \tag{6}$$

Such a definition makes sense for the same single-particle spaces for both kinds of particles. Below we restrict ourselves to only one $j$-shell $j_\pi = j_\nu = j$ for the proton and neutron. Obviously, squares of all three operators $P_\alpha$, $C_{\pi\nu}$, $S$ are equal to identity.

An important result of the present paper is that if $H_{\rm core}$ is invariant against $P_\alpha$ then the odd–odd Hamiltonian $H_{\rm o-o}$ (Eq. 1) is invariant against $S$ and its eigenstates can be labeled by the additional number $s = \pm 1$. This can be easily proved taking into account that $Q$ is odd against $P_\alpha$ as being proportional to $\alpha$.

The $P_\alpha$ invariance of $H_{\rm core}$ leads to several consequences for the properties of the even–even core which are not discussed here. We mention only the conditions that must be fulfilled in order to $H_{\rm GBH}$ be $P_\alpha$-invariant. Again, we consider the case of the first sextant of the deformation plane:

$$\begin{aligned} f(\beta, \pi/3 - \gamma) &= f(\beta, \gamma) \qquad \text{for} \quad f = B_{\beta\beta}, B_{\gamma\gamma}, B_x\,, \\ B_{\beta\gamma}(\beta, \pi/3 - \gamma) &= -B_{\beta\gamma}(\beta, \gamma)\,, \\ B_y(\beta, \pi/3 - \gamma) &= B_z(\beta, \gamma)\,, \\ V(\beta, \pi/3 - \gamma) &= V(\beta, \gamma)\,. \end{aligned} \tag{7}$$

### *2.2. Electromagnetic transition operators*

We show below that the $M1$ and $E2$ transitions between states with the same $s$ number are much smaller than those between the states with different $s$. In the $E2$ case it is readily seen because the single-particle parts of the transition operator are almost negligible compared to the core part, which is odd against the $S$ symmetry.

For a single-$j$ orbital the $M1$ transition operator can be effectively written as

$$T(M1) = \sqrt{\frac{3}{4\pi}}\mu_N(g_{\rm R} R_{\rm core} + g_\pi j_\pi + g_\nu j_\nu)\,, \tag{8}$$

where $R_{\rm core}$, $j_\pi$, $j_\nu$ are angular momenta operators. $R_{\rm core}$ is even against $S$ and the one-particle part, which does not have definite symmetry, cannot be neglected, but it can be checked by direct calculation that for $g_{\rm R} - (g_\pi + g_\nu)/2 = 0$ the matrix elements of $T(M1)$ obey

$$\langle I, s|T(M1)|I', s\rangle = 0 \qquad \text{for} \qquad I \neq I' \,. \tag{9}$$

In the case of the $A \sim 130$ nuclei considered in [3], the values of the gyromagnetic factors are $g_{\rm R} = 0.44$, $g_\pi = 1.22$, $g_\nu = -0.21$ which gives nonzero but very small values of $M1$ transition probabilities between states with the same $s$ number.

Presently we apply the obtained results to the odd–odd Hamiltonian with $\gamma$-soft core (Wilets–Jean type) studied in [3]. This Hamiltonian is obviously $S$-invariant. The new quantum number $s$ which labels the states and discussed properties of the $M1$ and $E2$ operators provide a clear interpretation of the pattern of strong transitions shown in Fig. 1. Such a pattern was obtained in [3] by numerical calculations.

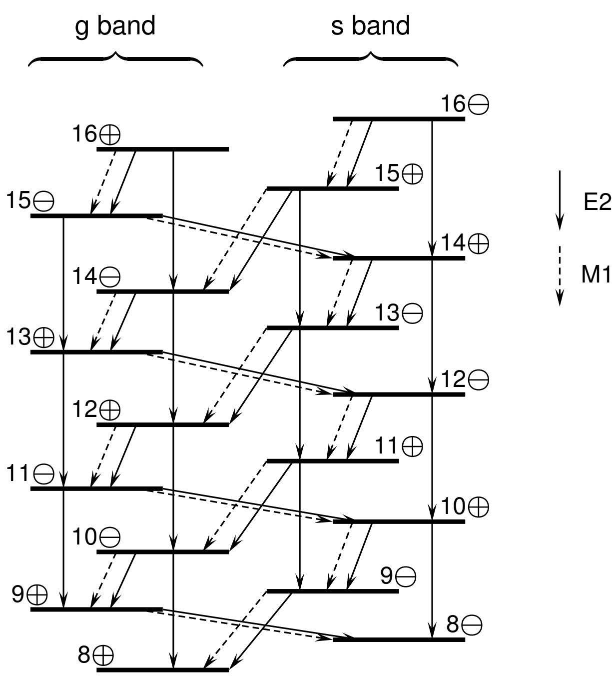


Fig. 1. Schematic picture of the partner bands and strong $E2$ and $M1$ transitions in an odd–odd nucleus. Symbols ⊕ and ⊖ correspond to $s = \pm 1$. Compare Fig. 3 in [3].

Some remarks should be added. Firstly, the symmetry $P_\alpha$ of the core is not quite new and was mentioned in various papers, *e.g.* [7,8]. Secondly, for the rigid core (Davydov–Filippov) the rotation $R_x(\pi/2)$ plays an analogous role to $P_\alpha$. An invariance with respect to this operation requires 'maximal' triaxiality, that is fixed deformation $\gamma = \pi/6$ of the core. Thirdly, one can see some similarities with the symmetry operation discussed in [9] in the context of the particle-rotor model.

## 3. Conclusions

The discussed symmetry of the CPHC model gives a valuable insight into the background of staggering of the $M1$ and $E2$ transitions in some odd–odd nuclei. To be preserved, the symmetry $S$ needs rather stringent conditions so it is very interesting to what extent it can be broken without complete disappearing of the staggering pattern. We discuss this topic extensively in [5]. Another interesting question, on which the work is in progress, is to what extent the even–even cores calculated in the microscopic theory obey the $P_\alpha$ symmetry.

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