# FISSION RATE AND TIME OF HIGHLY EXCITED NUCLEI IN MULTI-DIMENSIONAL STOCHASTIC CALCULATIONS\*

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(Received December 22, 2010)

A four-dimensional stochastic approach to dynamics of nuclear fission induced by heavy ions was applied to calculations of the fission rate and time of highly excited compound nuclei. The research took into account not only three shape collective coordinates introduced on the basis of the  $\{c, h, \alpha\}$ -parametrization but also orientation degree of freedom (K-state) spin about the symmetry axis. Overdamped Langevin equation was used to describe the evolution of the K-state. Impact of orientation degree of freedom on the fission rate and time of the compound nuclei was studied in detail for the reactions with high-energy projectiles <sup>14</sup>N and <sup>16</sup>O on target nuclei <sup>19</sup>Au, <sup>208</sup>Pb, <sup>232</sup>Th and <sup>238</sup>U. It was revealed that inclusion of the K-state in the dynamical model produces considerable increase in the mean fission time and decrease in the stationary fission rate. The K-state impact on the fission rate and time almost fully canceled the opposite effect produced by inclusion of nuclear neck and mass-asymmetry coordinates in the 1D Langevin calculations. The difference of 5-25% between 4D and 1D calculations was found as the result of this research.

DOI:10.5506/APhysPolB.42.493 PACS numbers: 25.85.-w, 05.10.Gg, 21.10.Tg

## 1. Introduction

The problem of calculating the fission rate and mean fission time have traditionally been thought to be of importance in collective nuclear dynamics. The mean fission time is directly related to the mean multiplicity of prescission neutrons and is frequently used as a source of information about nuclear viscosity. The fission rate is in direct proportion to the fission width, which in turn, determines the probability for the decay of a nucleus.

<sup>\*</sup> Presented at the Zakopane Conference on Nuclear Physics "Extremes of the Nuclear Landscape", August 30–September 5, 2010, Zakopane, Poland.

For the first time, the fission width of an excited nucleus was calculated by Bohr and Wheeler within statistical transition-state theory [1]. In that classic study, the partial fission width was defined as a ratio of the number of phase-space states accessible to the nucleus being considered at the saddle point to level-density at the ground state. As is well known, this formula yields strongly underestimated values of the number of prescission neutrons [2]. A year later, Kramers considered [3] a solution to the problem of the passage of a Brownian particle through a potential barriers on the basis of the Fokker–Plank equation.

It should be recalled that the calculations in the studies of Bohr and Wheeler and Kramers were performed for the case of one collective coordinate. In collective nuclear dynamics, one-dimensional problems can be considered to obtain analytic estimates. However, at least three collective coordinates are required for realistical description of the fission process [4].

Kramers's and Bohr–Wheeler's expressions for the fission width are valid only for compound nuclei with zero spin about a symmetry axis, because it was always assumed that angular momentum is not only perpendicular to the reaction plane, but also to the fission direction. As first pointed out by Lestone [5] this assumption is not consistent with statistical model as well as with dynamical treatment of the orientation degree of freedom. He suggested to describe evolution of the K mode using the overdamped Langevin equation [5] and stressed that the large volume of heavy-ion-induced fission data needs to be reanalyzed with dynamical treatment of the orientation degree of freedom.

Therefore, in the present research we have studied the impact of orientation degree of freedom on the fission rate and time of highly excited compound nuclei in multi-dimensional stochastic calculations.

# 2. Model

Within a stochastic approach [4] the evolution of the shape collective degrees of freedom of a nucleus undergoing fission is described by analogy with the movement of a Brownian particle placed in a heat bath formed by all other intrinsic degrees of freedom of the nucleus. Relevant calculations usually rely on the set of Langevin stochastic differential equations. The potential energy of the nucleus was calculated within the framework of liquid-drop model with finite range of nuclear forces. We have used onebody dissipation based on the "wall" and "wall-plus-window" formula with a reduction coefficient from the "wall" formula  $k_s = 0.25$ . Evaporation of light prescission particles along Langevin trajectories were taken into account using Monte Carlo simulation technique. Scission configurations were determined from the scission condition of the finite neck radius, on average equal to  $0.3R_0$ . Technical details of the calculations can be found in [4, 6]. Following Lestone [7], we choose to treat K as an overdamped collective coordinate. Thus its evolution is defined by a reduced Langevin equation. The difference form of this equation reads as follows

$$K^{(n+1)} = K^{(n)} - \frac{\gamma_K^2 I^2}{2} \frac{\partial E_{\rm rot}}{\partial K} \tau + \Gamma_K^{(n)} \gamma_K I \sqrt{T\tau} \,, \tag{1}$$

where  $\gamma_K = 0.077$  (MeV  $10^{-21}$  s)<sup> $-\frac{1}{2}$ </sup> is a parameter that controls the coupling between K and the thermal degrees of freedom; I is the total spin of the system;  $\Gamma_K$  is a random number from a normal distribution with unit variance. The rotational part of potential energy has the form

$$E_{\rm rot}(\boldsymbol{q}, I, K) = \frac{\hbar^2 K^2}{2J_{\parallel}(\boldsymbol{q})} + \frac{\hbar^2 \left[ I(I+1) - K^2 \right]}{2J_{\perp}(\boldsymbol{q})} = \frac{\hbar^2 I(I+1)}{2J_{\perp}(\boldsymbol{q})} + \frac{\hbar^2 K^2}{2J_{\rm eff}(\boldsymbol{q})}, \quad (2)$$

where  $J_{\text{eff}}(\boldsymbol{q}) = \left[J_{\parallel}^{-1}(\boldsymbol{q}) - J_{\perp}^{-1}(\boldsymbol{q})\right]^{-1}$  is the effective moment of inertia;  $J_{\parallel}(\boldsymbol{q})$  and  $J_{\perp}(\boldsymbol{q})$  are the rigid body moments of inertia about and perpendicular to the symmetry axis.

## 3. Results and discussions

In Langevin calculations the time-dependent fission rate is defined as follows

$$R_{\rm f}(t) = \frac{1}{N - N_{\rm f}(t)} \frac{\Delta N_{\rm f}(t)}{\Delta t} , \qquad (3)$$

where N is the total number of simulated particles (trajectories);  $N_{\rm f}(t)$  is the number of particles that reach the scission configurations during the time t;  $\Delta N_{\rm f}(t)$  is the number of particles that reach the scission point during the time interval  $t \to t + \Delta t$ .

The fission rate as a function of time for various system dimensions is displayed in Fig. 1(a). In Fig. 1(a) particle evaporation is not considered to show the difference between stationary fission rate levels. It is important to note that fission rate will not reach stationary level when particle evaporation is taken into account, since the nuclear temperature decreases during the fission process. The mean fission time was calculated using concept of the mean first passage time [8]. As can be seen from the Fig. 1 orientation degree of freedom tends to increase mean fission time and decrease fission rate. This effect can be explained based on the expression for the rotational energy. From (2) follows that inclusion of the K raises the fission barrier. So the system with non-zero K needs to overcome a higher barrier resulting in a longer mean fission time and lower fission rate.



Fig. 1. (a) Time dependence of the fission rate for the reaction  ${}^{16}\text{O} + {}^{208}\text{Pb} \longrightarrow {}^{224}$  Th with  $E_{\text{lab}} = 90$  MeV. (b) Calculated mean fission time  $\langle t_f \rangle$  as a function of  $E_{\text{lab}}$  with particle evaporation taken into account.

In the present work the influence of the dimensionality of the dynamical model on the fission rate and mean fission time was investigated. Four-dimensional Langevin equation was employed to calculate the characteristics of fissioning nucleus. Overdamped Langevin equation was used to describe the evolution of the K-state. The orientation degree of freedom impact on the fission rate and time almost fully canceled the opposite effect produced by inclusion of nuclear neck and mass-asymmetry coordinates in the 1D Langevin calculations [9]. The difference of 5-25% between 4D and 1D calculations was found as the result of this research. This difference is determined mainly by the temperature of the compound nuclei. The results obtained for other compound nuclei mentioned in the abstract will be published in [10].

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