THE EFFECT OF VALENCE NEUTRONS ON SPIN–ORBIT SPLITTING*

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We study the splitting of single-particle energies between spin–orbit partners (*ls* splitting) for the isotopic chains of some even–even closed shell nuclei in the Hartree–Fock–Bogolyubov framework. $Z = 8$, 20, 28 and 50 magic proton nuclei of stability valley and their even–even neighbours are selected and a limited systematic investigation is performed for the isotopic chains of C, O, Si, Ca, Ni, Sr, Sn, Te, and Ce isotopes. They are compared with existing data. The modification of proton spin–orbit partners energy splitting of those isotopes is investigated with valence neutron numbers extending to the neutron drip-line. Our calculation shows that the proton *ls* splitting in chosen nuclei becomes smaller with the increasing neutron number. This reduction is accounted for two different effects, namely, the effect of neutron diffuseness and tensor interaction. The neutron diffuseness values are also calculated for the nuclei under investigation. The deduced *ls* splitting reduction is compared for the nuclei with the same isospin number. The larger reduction is attributed to the tensor interaction assuming that the *ls* reduction due to the neutron surface thickness will be the same for the nuclei having the same neutron diffuseness parameter. The contribution from tensor interaction is mainly accounted for the interaction between the spin–orbit partner proton orbital and the neutron orbital having different angular momentum. Briefly, the effect of increased valence neutron number on proton effective single particle levels is discussed.

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1. Introduction

Spin–orbit splitting (*ls* splitting) is important in the structure of nuclei. A large *ls* splitting between single particle orbits with the same orbital angular momentum is responsible for the shell structure of nuclei [1,2]. Exotic

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nuclei far from the line of beta-stability have gained considerable interest in recent years both on the experimental as on the theoretical side. New accelerators with radioactive beams allow the experimental study of nuclei far from stability. The shell structure in neutron-rich nuclei changes from that in stable nuclei. We can obtain the change of shell structure information from single-particle orbits around closed-shell or closed-subshell nuclei. The case is investigated by following the evolution of the spin–orbit splittings of $\Delta E_{ls} = E_{n,l,j=l-1/2} - E_{n,l,j=l+1/2}$ proton pair of states along the above mentioned isotopic chains [3].

In this paper, we derive the spin–orbit splittings from calculated single-particle energies in nonrelativistic Hartree–Fock–Bogolyubov with Skyrme-type forces. The code HFBTHO (v1.66p) is used to obtain the single-particle energies of even–even nuclei which utilizes an axially symmetric deformed HO potential [4]. The core polarization effects should affect essentially in the same way the two members of a spin–orbit doublet if they are both below, or both above the Fermi level [5]. Some studies show that *ls* splitting in neutron-rich nuclei become small because the diffuseness of neutron density distribution reduces the spin–orbit coupling in neutron-rich nuclei [6,7]. Moreover, the monopole interaction produced by tensor force between proton and neutron makes *ls* splitting smaller [8].

2. Hartree–Fock–Bogolyubov method

Hartree–Fock–Bogolyubov calculations can be performed in the whole mass region over the nuclear chart. Such mean-field calculations can reproduce binding energies and radii of nuclei including unstable ones using effective forces with relatively simple forms like the Skyrme force [9,10]. A two-body Hamiltonian of a system of fermions can be expressed in terms of a set of annihilation and creation operators $(c, c^\dagger)$

$$H = \sum_{n_1 n_2} e_{n_1 n_2} c^\dagger_{n_1} + \tfrac{1}{4} \sum_{n_1 n_2 n_3 n_4} \overline{v}_{n_1 n_2 n_3 n_4} c^\dagger_{n_1} c^\dagger_{n_2} c_{n_4} c_{n_3} \,, \tag{1}$$

where $\overline{v}_{n_1 n_2 n_3 n_4} = \langle n_1 n_2 | V | n_3 n_4 - n_4 n_3 \rangle$ are anti-symmetrized two-body interaction matrix-elements. For Skyrme forces, this two-body Hamiltonian appears as the HFB energy in the form of a local energy density functional. The total Hamiltonian is the sum of the mean-field and pairing energy densities. In this work, a parametric form of total HFB energy is used as in Ref. [4] utilizing the SkP and SLy4 interactions of Skyrme force [11,12]. The user-defined parameters set of the Skyrme force is modified in order to include the tensor effect.

3. Results

The modification of the neutron density due to the increasing valence neutron number induces a reduction of the SO splitting. This reduction appears in a self-consistent manner in the mean field description, as soon as the neutron density is modified with the increasing valence neutron number. Tensor interaction is another process which modifies the SO splitting acting on the spin–orbit partners level in opposite way. Therefore, the neutron radii and the single particle level energies are simultaneously calculated along the isotopic chain. The so-called neutron diffuseness parameter $t = r_n - r_p$ and the reduction of spin–orbit partners energy splitting is deduced for the selected isotopes. In Fig. 1, the calculated reduction of spin–orbit energy splitting for C and O isotopes between $1p_{1/2}$ and $1p_{3/2}$ shells are found as 6.4% and 7.8% respectively.

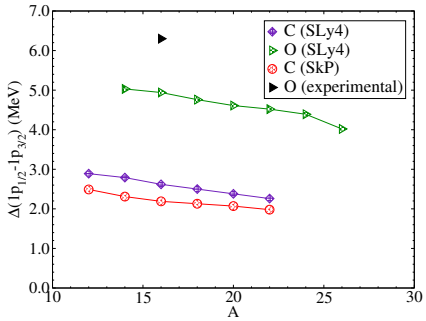

Fig. 1. Energy splittings between spin–orbit partners for proton levels in C and O isotopes, as functions of mass number. Experimental value in $A = 16$ is shown [13].

On the other hand, the neutron diffuseness parameter $t = r_n - r_p$ is obtained as 0.68 for C isotopes and 0.45 for O isotopes for the same valence neutron number. The larger neutron diffuseness parameter for the C isotopes is an expected result from the properties of the lighter nuclei. Therefore, one expects a larger spin–orbit splitting reduction for lighter C isotopes than O isotopes if the reduction would only be raised from the increasing neutron thickness. A slightly greater reduction for oxygen isotopes can be attributed to the effect of monopole tensor interaction. For oxygen isotopes $1p_{1/2}$ and $1p_{3/2}$ proton shells are both fully occupied and the tensor interaction of these proton shells between $d_{5/2}$ neutrons works on opposite way. The tensor interaction between $1d_{5/2}$ neutrons and $1p_{1/2}$ protons is attractive. As a result, $1p_{1/2}$ proton level lies deeper in potential well as $1p_{3/2}$ is pushed up. So the energy gap between $1p_{1/2}$ and $1p_{3/2}$ proton spin–orbit partner is reduced. In the case of carbon isotopes $1p_{1/2}$ the proton shell is empty so there is no tensor interaction between $1d_{5/2}$ neutrons and $1p_{1/2}$ protons.

The neutron diffuseness parameter t is equal and 0.45 fm for both isotopes of ^{18}C and ^{24}O which have approximately the same valence neutron number. The contribution of monopole tensor interaction to the shell gap reduction can be deduced assuming that the valence neutrons give the same neutron diffuseness for both isotopes. Spin–orbit energy gap reduction is around 0.39 MeV from ^{12}C to ^{18}C isotopes while it is 0.55 MeV between ^{18}O and ^{24}O isotopes. The difference of spin–orbit energy reduction between carbon and oxygen isotopes is around 0.16 MeV. This value can be inferred as the tensor interaction between p and d shells in carbon and oxygen region which is in agreement with the value obtained from two body matrix elements for $A = 20$ region [14].

In Fig. 2, d shell reduction for Si isotopes is slightly greater than that of Ca isotopes. In Si isotopes, while protons occupy $1d_{5/2}$ level of spin–orbit splitting, $1d_{3/2}$ is empty. Whenever proton Fermi energy is taken into account, one of the spin–orbit doublets is above and the other one is below the Fermi level. A slightly greater reduction of spin–orbit splitting makes us think of core polarization effect. Lying above the Fermi energy level $1d_{3/2}$ proton single particle level is pulled down with increasing neutron number but $1d_{5/2}$ proton level is not influenced. In the case of Ca isotopes both levels are occupied *i.e.* they lie under the Fermi level thus both of them are affected in the same way by the core polarization effect.

The reduction of spin–orbit energy splitting between $1p_{1/2} - 1p_{3/2}$ and, $1d_{3/2} - 1d_{5/2}$ proton orbits is approximately equal and around 10% for Si isotopes.

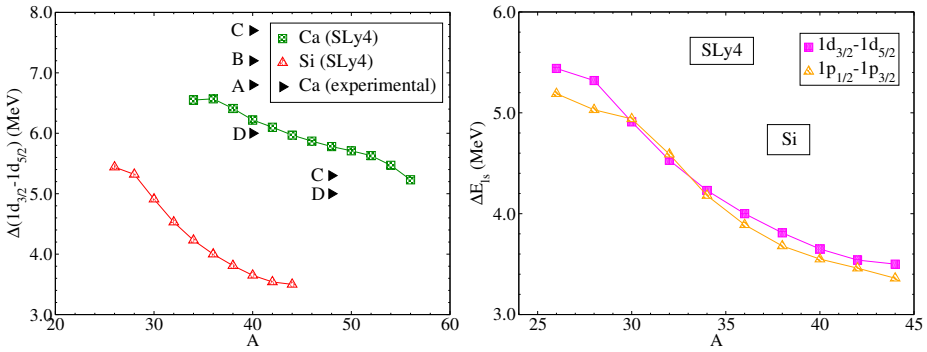

Fig. 2. Energy splittings between spin–orbit partners for proton levels in Ca and Si isotopes, as functions of mass number. Experimental values in $A = 40$ and $A = 48$ are shown A [15], B [16], C [17], and D [18].

In Fig. 3 for the Ni isotopes, f shell spin–orbit reduction is slightly greater than that of g shell reduction. The spin–orbit reduction value is calculated as 10% for f shell while it is 8% for g shell. Spin–orbit reduction

is 10% for Ni isotopes for which the $1f_{5/2}$ proton level is above the Fermi level while the $1f_{7/2}$ spin–orbit partner lies under the Fermi level. The spin–orbit reduction for Sr isotopes is around 3%. The both spin–orbit partners are occupied and lie under the Fermi level. The both spin–orbit partners feel core polarization effect in the same way so extra energy decreasing is out of question. The only mechanism for the reduction is the increasing neutron surface diffuseness.

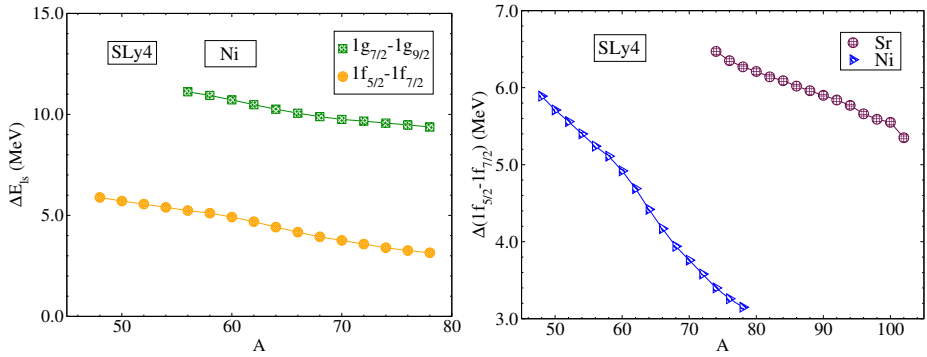

Fig. 3. Energy splittings between spin–orbit partners for proton levels in Ni and Sr isotopes, as functions of mass number.

f, g and h shell reductions are approximately equal and around 5%. There is no effect of increasing angular momentum on spin–orbit partners energy splitting reduction for Sn isotopes. SO energy reduction is around 5% for Sn isotopes while this value is around 2.5% for Te and Ce isotopes. Among these three isotope chains the Sn isotope chain is the only one on which the core polarization affects (see in Fig. 4).

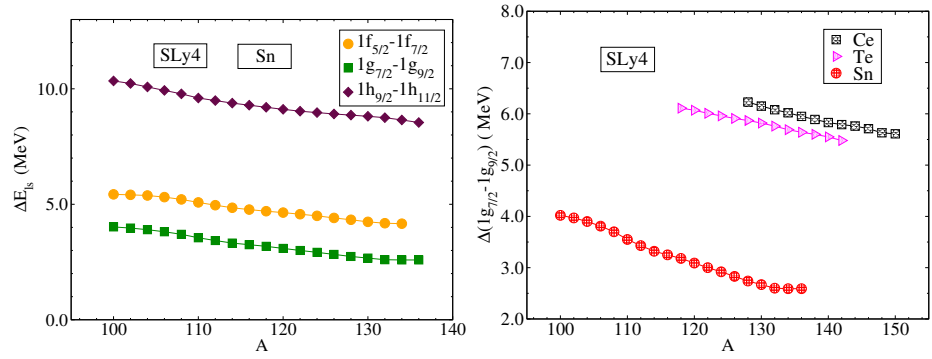

Fig. 4. Energy splittings between spin–orbit partners for proton levels in Sn, Ce and Te isotopes, as functions of mass number.

4. Conclusions

As is shown through figures 1–4, the spin–orbit splitting for the proton orbits between spin–orbit partners is generally reduced with increasing valence neutron number. This reduction is remarkable for lighter nuclei and lower angular momentum orbital. The reduction for lighter nuclei such as C, O, Si and Ca isotopes is as twice as the reduction for heavier nuclei such as Sn, Te and Ce isotopes. Experimentally available spin–orbit splitting values are very scarce and they differ up to 1 MeV for the same isotopes as shown for ^{40}Ca isotope in Fig. 2. The experimental values given in Fig. 1 and Fig. 2 for ^{16}O and ^{40}Ca, respectively, seem a bit higher than the calculated values. This is partly due to the reason that spin–orbit force does not include explicitly in HFB calculations, using an LS strength parameter as in the shell-model calculations. Instead, the LS force and therefore (ls) splitting is obtained self-consistently in the mean field calculations. As it is expected the tensor force does not affect the LS occupied doubly magic and stable nuclei such as ^{12}C, ^{16}O and ^{40}Ca, but there is remarkable reduction in ls splitting for the isotopes of these nuclei having large number of valence neutrons. Another remarkable conclusion is that the spin–orbit splitting reduction becomes slightly smaller for the higher orbital *i.e.* the reduction slightly decreases with increasing orbital angular momentum contrary to the fact that spin–orbit splitting increases with increasing orbital angular momentum.

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