# RELATIVE EVEN AND ODD PARITY LEVELS WITHIN THE NUCLEI IN THE IRON REGION\*

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In the current study, the ratio of the nuclear level densities with odd and even parities for <sup>58</sup>Fe were calculated and compared with those of <sup>60</sup>Ni, using a microscopic–macroscopic computational method. It was shown that the level densities of odd and even parities for even–even nuclei are not equal at low energies. However, increasing excitation energy reduces the difference between odd and even parities and equality is gained gradually.

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## 1. Introduction

Knowledge of the nuclear level density is important in nuclear astrophysics, in reactor physics, and in nuclear medicine. Furthermore, it is essential for the understanding of the nucleosynthesis in stars and reliable estimation of nuclear abundances often requires accurate nuclear level densities [1, 2, 3]. The study on nuclear level density as a function of excited energy was initiated by Bethe about sixty years ago [4]. Later, this quantity received increased attention by taking into account mass number, spin, and isospin. The recent findings proving the parity dependency of nuclear level density have further stimulated the studies on this quantity [5]. Theoretical calculations of astrophysical interactions, consider an equal distribution for parity states for all the energy. However, it is obvious that this assumption is not valid at low energies. In particular for even–even nuclei, paring between

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identical nucleons leads to only even parity states in the low energy region. In the present study, we investigated the parity dependence of nuclear level density of  $^{58}$ Fe and compared it with that of  $^{60}$ Ni. The reason for choosing these two nuclei is that the nuclei in the iron region play an essential role in nucleosynthesis.

### 2. Summary of theory

The starting point of this work is the assumption of statistically independent particles at finite temperature. The single particle (s.p.) levels clustered into two groups according to their parity and the group which has smaller probability of occupying is denoted by  $\pi$ . If the particles occupy the s.p. states randomly and independently, it is expected that the probability of occupying *n* levels of group with the parity of  $\pi$  is given by Poisson distribution [6]

$$p(n) = \frac{f^n}{n!} e^{-f} \,. \tag{1}$$

Here f indicates the average occupancy of levels with parity  $\pi$  which in turn depends on temperature. Since the probabilities  $p\mp$  can be related to the partition functions  $Z\mp$  of the even/odd parity and the total partition function  $Z = Z_- + Z_+$ , then the ratio of the odd-parity to the even-parity probabilities is given by

$$\frac{p_-}{p_+} = \frac{Z_-/Z}{Z_+/Z} = \frac{\Sigma_n^{\text{odd}} p(n)}{\Sigma_n^{\text{even}} p(n)} = \frac{e^{-f} \sinh f}{e^{-f} \cosh f} = \tanh f \,. \tag{2}$$

In the independent particle model, f has to be treated separately for neutrons and protons and will be evaluated from the Fermi–Dirac distribution

$$f = \Sigma_{k \in \pi} f_k^{\text{FD}} = \Sigma_{k \in \pi} \frac{1}{1 + e^{\beta(\epsilon_k - \bar{\lambda})}}, \qquad (3)$$

where  $\bar{\lambda}$  and  $\epsilon_k$  are the average chemical potential and s.p. energy, respectively.

# 3. Results and discussion

The <sup>58</sup>Fe nucleus was evaluated with the above model. The nuclei of iron region are generally located in the middle of fp shell, and it is very unlikely that the even parity  $g_{9/2}$  shell is occupied. Therefore, n can be considered as the occupation number of  $g_{9/2}$  shell. To obtain f as a function of  $\beta$  from Eq. (3), the shells  $g_{9/2}$  were summed up. As is expected by increasing  $\beta$ , the occupancy probability of  $g_{9/2}$  shell was decreased. This is because at low temperatures the nucleons cannot gain the sufficient excitation energy to rise to upper levels. The deformation parameter needed for obtaining the s.p. energy,  $\epsilon_k$  is inserted from the mentioned values in [7]. Also  $\bar{\lambda}$  was equaled with Fermi energy. After obtaining f as a function of  $\beta$  by using the mentioned computational method, the parity ratio  $\frac{Z_-}{Z_+}$  is calculated by Eq. (2) and is shown by solid line in Fig. 1. We aimed to investigate the parity dependency of nuclear level densities. In order to calculate the ratio of odd to even level densities, the absolute values of and  $Z_-$  and  $Z_+$  are needed. Using equations (2) and  $Z = Z_- + Z_+$ , these values can be individually calculated. Where  $f(\beta)$  is known and therefore by calculating  $Z(\beta)$  both  $Z_-$  and  $Z_+$ values are obtained.  $Z(\beta)$  is given by  $Z(\beta) = e^{-\beta E_{\rm G}} \int \omega(E_x) e^{-\beta E_x} dE_x$ .



Fig. 1. Ratio of  $\frac{Z_-}{Z_+}$  versus inverse temperature  $\beta$  for <sup>60</sup>Ni (solid line) by using Eq. (2) and (squares) by Monte Carlo calculations [7].

To do this  $\omega(E_x)$  is obtained by backshifted Bethe formula (BBF) ( $E_G$  is the ground state energy). Applying the well-known Laplace transform of the partition function and employing the saddle point approximation  $\beta = \beta_0$ , then

$$\frac{\omega_{-}(E_x)}{\omega_{+}(E_x)} = \left[ \sqrt{\frac{\frac{\partial^2 \ln Z_{+}}{\partial \beta^2}}{\frac{\partial^2 \ln Z_{-}}{\partial \beta^2}}} e^{\left(\frac{\partial \ln Z_{+}}{\partial \beta} - \frac{\partial \ln Z_{-}}{\partial \beta}\right)} \tanh f \right]_{\beta = \beta_0}, \quad (4)$$

 $\beta_0$  can be obtained as  $\beta_0 = \sqrt{\frac{\alpha}{E_x - \Delta}}$ ,  $\alpha$  is level density parameter and  $\Delta$  is the backshifted parameter. The calculations were performed in the current study for <sup>58</sup>Fe and <sup>60</sup>Ni using 6.254 MeV<sup>-1</sup> and 6.48 MeV<sup>-1</sup> for their  $\alpha$  and 1.136 MeV and 1.87 MeV for their  $\Delta$  parameters, respectively [8].

#### 4. Conclusion

Similar to other even–even nuclei,  $^{60}$ Ni and  $^{58}$ Fe nuclei possess positive parity in the ground state. This positive parity remains even at low excitation energies where nucleons pairing is strong. As it can be seen from

Fig. 2 increasing the excitation energy is concomitant with higher probability of odd parity or increased  $\omega_{-}/\omega_{+}$  ratio. This is due to the lower pairing strength of nucleons and higher nucleon energy which can be up to the energy gap between two single particle levels with the opposite parity. Therefore,  $\omega_{-}$  and  $\omega_{+}$  would be equal if nucleons become dissociated and obtain the required energy to reach the levels with the opposite parity to the Fermi level parity.  $\omega_{-}/\omega_{+}$  ratio is expected to approach unity at high temperatures. Thus, the assumption of independency of nuclear level density to parity holds true only at high temperatures. Also the equality of level densities with opposite parity occurs faster for <sup>60</sup>Ni than for <sup>58</sup>Fe. This is because  $g_{9/2}$  is the first level with opposite parity at ground state and the gap between the last occupied nucleon level and even parity  $g_{9/2}$  in <sup>60</sup>Ni nucleon is less than that for the  ${}^{58}$ Fe nucleon. Therefore, it can be concluded that besides temperature and energy of nucleons, the nucleon structure and pairing interactions can significantly affect the parity dependency of nuclear level densities.



Fig. 2. (Left) The ratio of  $\frac{\omega_{-}}{\omega_{+}}$  versus  $E_x$  for <sup>60</sup>Ni nucleus (solid line) by using Eq. (2) and (squares) by Monte Carlo calculations. (Right) The ratio of  $\frac{\omega_{-}}{\omega_{+}}$  versus  $E_x$  for <sup>58</sup>Fe nucleus (solid line) by using Eq. (2).

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