OVERVIEW ON THE CLUSTER STRUCTURE AND THE ALPHA CONDENSATION*

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Recent trends in nuclear cluster physics are discussed. Three subjects of the discussion are (1) clustering in neutron-rich nuclei, (2) alpha-condensate-like states, and (3) coexistence of cluster and mean-field-type states. The subject (1) includes molecular-orbital and atomic-orbital structures in Be and F isotopes and also dineutron correlation in halo nucleus ^{11}Li. The subject (2) is discussed by using the investigations of ^{12}C and ^{16}O. The discussion of the subject (3) is based on two points: One is the AMD studies of cluster and mean-field-type states in many nuclei and the other is the monopole transitions between cluster states and the ground state with mean-field-type structure.

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1. Introduction

Recent trends in nuclear cluster physics are discussed. Discussions are on three subjects. The first subject is on clustering in neutron-rich nuclei. We discuss on molecular-orbital and atomic-orbital structures in the case of neutron-rich isotopes of Be and F, and also on dineutron correlation in halo nucleus ^{11}Li. The second subject is on the alpha-condensate-like states and we discuss the investigations in ^{12}C and ^{16}O, namely 3α and 4α condensate-like states. The third subject is on the coexistence of cluster and mean-field-type states in stable nuclei, where degrees of freedom due to valence neutrons do not come in. Our discussion is about two points: One is the actual features of coexistence of cluster and mean-field-type states which are clarified by AMD studies in many nuclei. On the basis of the actual features of coexistence, we propose a mechanism of coexistence which is based on the dual character of the ground state wave function. The other point of

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our discussion is the monopole transitions between cluster states and the ground state with mean-field-type structure, which serves as a verification of our proposed mechanism of coexistence.

2. Clustering in neutron-rich nuclei

2.1. Molecular and atomic orbital structures

Clustering in neutron-rich nuclei is governed mainly by two kinds of dynamics. One is the inter-cluster dynamics and the other is the dynamics of excess neutrons around clusters. The inter-cluster dynamics is basically common to that in stable nuclei. The dynamics of excess neutrons is of molecular-orbital type in low excitation energy region and in the higher excitation energy region there show up atomic-orbital type and mixed type dynamics.

Neutron-rich Be isotopes are now known to be well described by molecular orbits around two α clusters in low excitation energy region. Most states of Be isotopes belong to respective rotational bands, and their intrinsic states are well explained in terms of particle-hole configurations of molecular orbit model. Fig. 1 shows good reproduction of the observed energy spectra of ^{10}Be by the AMD calculation of Ref. [1]. In the ground $K^\pi = 0^+_1$ band and the $K^\pi = 2^+$ band, valence neutrons around 2α core are in the p-shell molecular orbits, while in the $K^\pi = 1^-$ band one valence neutron is raised into the intruder sd-shell molecular orbit called σ-orbit and in the $K^\pi = 0^+_2$ band two valence neutrons are raised into the intruder σ-orbit. When the σ-orbit is occupied by neutron(s), the inter-α distance is enlarged in order to reduce the kinetic energy of the σ-orbit, which means that the spatial clustering of the $K^\pi = 0^+_2$ band is more prominent than that of the $K^\pi = 1^-$ band which in turn is more prominent than that of the ground band.

The nucleus ^{12}Be presents interesting features of coexistence of various kinds of nuclear structure. In spite of the neutron magic number $N = 8$, the ground band structure is dominantly composed of the configuration with two neutrons in σ-orbit. The normal configurations of valence neutrons contained all in p-shell appear as dominant components of the excited 0^+_2 states. Furthermore, it is argued that the molecular rotational band of Ref. [2] has a structure which contains a large amount of ^{6}He+^{6}He clustering component in which valence neutrons move not over the whole system but around either of two α clusters. Such neutron orbits in ^{6}He +^{6} He structure may be called 'atomic' orbits. Recently Ito and his co-workers proposed a unified theoretical framework to treat both molecular and atomic orbital configurations and applied it to ^{12}Be [3]. They could reproduce the ground band, the excited 0^+_2 band, and the molecular rotational band of Ref. [2]. They predict the existence of the excited states with mixed configurations of molecular and atomic orbitals.

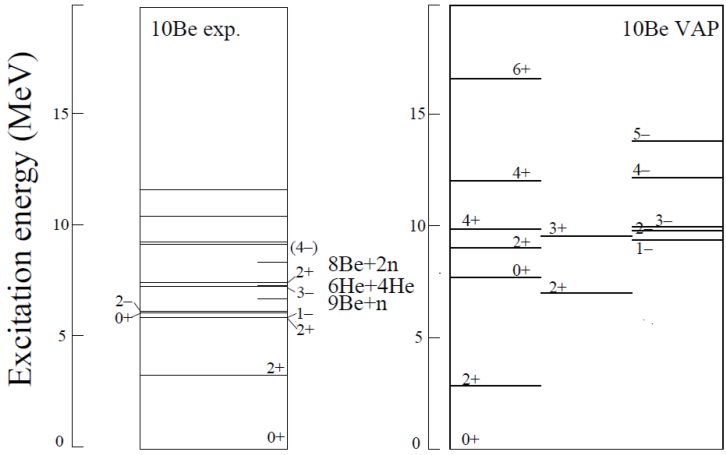

Fig. 1. Energy spectra of ^{10}Be by AMD denoted as VAP.

In a recent AMD calculation of ^{22}Ne of Ref. [4], it is argued that there are states with $^{18}\mathrm{O}+\alpha$ cluster structure in addition to the molecular-orbital states with two valence neutrons around $^{16}\mathrm{O}+\alpha$ core and the ground band states with deformed mean-field structure. The two valence neutrons of ^{18}O of the $^{18}\mathrm{O}+\alpha$ cluster structure may be regarded as atomic-orbit neutrons. The coexistence of mean-field-type states and cluster states with molecular and atomic orbits is quite interesting.

It is well known that the low-lying excited band with $K^\pi = 1/2^-_1$ of ^{19}F upon $1/2^-_1$ state at 0.11 MeV has a cluster structure of $^{15}\mathrm{N}+\alpha$ [5,6]. AMD calculations have confirmed this knowledge [7]. Ref. [7] reports that the AMD calculation predicts the systematic existence of excited states with cluster structure composed of $^{15}\mathrm{N}+\alpha$ core and surrounding valence neutrons in F isotopes. Fig. 2 shows the existence of three types of excited cluster bands in F isotopes all of which have a $^{15}\mathrm{N}+\alpha$ core. All three types of bands have proton configuration with one hole in p-shell just like the $K^\pi = 1/2^-_1$ band of ^{19}F. The first type band has valence neutrons in which two neutrons occupy σ' orbital while the second and third bands have one and no neutrons occupying σ' orbital, respectively. In this figure, the first, second, and third bands are denoted as "proton hole with σ'^2", "proton hole with $\pi\sigma'$", and "proton hole with π^2", respectively. The σ' orbital is analogous to the σ orbital in Be isotopes and is the lowest intruder orbital coming down from the pf-shell. In the left-hand side of this figure, the density distributions of core (solid contour lines) and valence neutron orbital (colour plots) are shown for these three bands and the ground band in the case of ^{21}F. It is to be noted here that, in this figure of density distribution, the core of the

ground band is not of $^{15}\mathrm{N}+\alpha$ type but of $^{16}\mathrm{O}+t$ type. A very interesting prediction of Fig. 2 is that the excited states with molecular orbital structure of first and second type come down rapidly in low excitation energy region as neutron number increases.

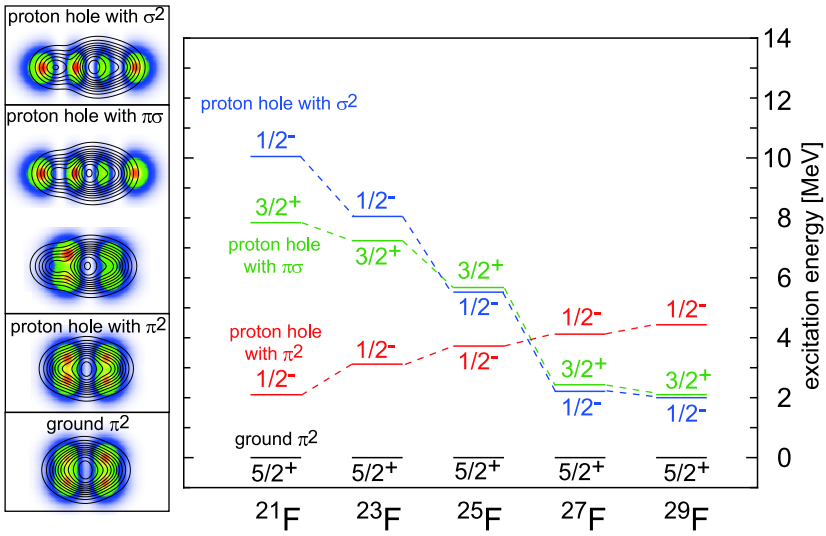

Fig. 2. Excitation energies of band-head states of three types of excited cluster bands in F isotopes obtained by AMD. Three bands all have a $^{15}\mathrm{N}+\alpha$ core. Left-side figure shows density distributions of core (solid contour lines) and valence neutron orbital (colour plots) for these three bands and the ground band in the case of ^{21}F.

2.2. Dineutron clustering

^{11}Li is a representative two-neutron-halo nucleus. In this nucleus the neutron magicity $N = 8$ is regarded as broken and recently the dineutron clustering discussed by Hansen [8] seems to have revived. These three characteristics, two-neutron halo, broken magicity, and dineutron clustering, are considered to be related to each other. In Ref. [9], the Pauli blocking effect of the neutron pairing correlation was attributed to the reason of the magicity breaking: When two valence neutrons occupy the $0p_{1/2}$-orbit, the pairing excitation of $(0p_{3/2})^2_{J=0}$ neutrons to $(0p_{1/2})^2_{J=0}$ is Pauli-blocked, while when two valence neutrons occupy the $1s_{1/2}$-orbit, the pairing excitation of $(0p_{3/2})^2_{J=0}$ neutrons to $(0p_{1/2})^2_{J=0}$ is no more Pauli-blocked. This situation implies that the configuration with the last valence neutron in the $1s_{1/2}$-orbit is energetically favourable over the ordinary configuration with the last valence neutron in the $0p_{1/2}$-orbit, which is equivalent to the effective lowering of the $1s_{1/2}$-orbit, namely the breaking of $N = 8$ magicity.

Ikeda and his collaborators [10] discussed that the Pauli-blocking effect is not sufficient for reproducing about 50% mixing of the $(1s_{1/2})^2_{J=0}$ component in ^{11}Li observed experimentally [11]. They proposed that the Pauli blocking effect of the neutron–proton tensor-force correlation causes similar amount of the effective lowering of the $1s_{1/2}$-orbit and the combined effect of pairing and tensor correlations can reproduce about 50% mixing of the $(1s_{1/2})^2_{J=0}$ component in ^{11}Li. The mechanism of the narrowing the $1s$–$0p$ spacing by the tensor-force correlation is very similar to that by the pairing correlation. In the case of tensor-force correlation, an important excitation of the ^{9}Li core is the excitation of the $(0s_{1/2})^2_{(1,0)}$ neutron–proton pair to the $(0p_{1/2})^2_{(10)}$ neutron–proton pair. The suffix (10) stands for $(J = 1, T = 0)$ which is the quantum number of a deuteron. It is clear that the $(0p_{1/2})^2$ configuration for the last two valence neutrons Pauli-blocks the tensor correlation of the ^{9}Li core while the $(1s_{1/2})^2$ configuration for the last two valence neutrons does not Pauli-block. In Ref. [10], good reproduction of the Coulomb breakup reaction cross-section [12] is reported and the calculated two-neutron correlation density distribution is shown to indicate clear dineutron clustering and to have long dineutron tail implying prominent two-neutron halo.

3. Alpha-condensate-like states

3.1. 3α condensate-like states

The second 0^+ state of ^{12}C which is called the Hoyle state is located near the 3α breakup threshold. Its small excitation energy of 7.66 MeV is regarded as difficult to explain by the shell model. The no-core shell model has by far not succeeded in its reproducing it [13]. In 1970s it was shown that microscopic 3α cluster model studies [14, 15, 16] give gas-like structure to the Hoyle state which well explains the observed data. The 3α gas-like structure means a structure with very large radius (about 1/3 of the ground state density) composed of weakly coupled 3α-clusters with relative S-wave. The weak coupling of 3α-clusters means the small energy of relative motion and hence locates the Hoyle state near the 3α breakup threshold at 7.27 MeV and hence explains naturally the low excitation energy of 7.66 MeV.

The results by the cluster model studies have been supported by theoretical approaches which do not assume alpha clustering. They are calculations by AMD [17, 18] and those by FMD with UCOM technique [19]. Since both AMD and FMD+UCOM build up wave functions totally on nucleonic degrees of freedom, the obtained wave functions contain non-clustering components with broken spatial symmetry. The non-clustering components give good reproduction of the observed β decay strengths from ^{12}N [18]. In

Ref. [19], it is reported that the magnitude of the non-clustering components of the second 2^+ state is very small ($\sim 1\%$), which is consistent with the non observation of β decay to 2^+_2 state.

The Hoyle state was reconsidered in a new light in Ref. [20] where the following wave function $\Phi^{3\alpha\mathrm{THSR}}(B)$ (3α THSR wave function) was proposed for the Hoyle state

$$\Phi^{3\alpha\mathrm{THSR}}(B) = \mathcal{A}\left\{\exp\left[-\frac{2}{B^2}\left(\vec{X}_1^2+\vec{X}_2^2+\vec{X}_3^2\right)\right]\ \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\}, \tag{1}$$

where $\vec{X}_j$ stands for the centre of mass coordinate of the j-th α cluster. The THSR wave function represents the state where three α clusters occupy the same single $0S$-orbit $\exp(-(2/B^2)\vec{X}^2)$, namely a 3α condensate-like state which is the finite size counterpart of the macroscopic α-particle BEC (Bose–Einstein condensation) in infinite nuclear matter at low density [21]. It is to be noted that $\Phi^{3\alpha\mathrm{THSR}}(B)$ expresses a state composed of weakly coupled 3α-clusters with relative S-wave. As a very striking fact, it was soon discovered [22] that the microscopic 3α wave functions of Refs. [15] and [16] have overlaps of more than 95% with a single 3α THSR wave function with a large size parameter B.

3.2. 4α condensate-like states

The existence of the 3α condensate-like state in ^{12}C strongly suggests the general existence of the $n\alpha$ condensate-like states. Recently a full four-body calculation in the framework of 4α OCM [23] was reported. The calculated lowest six 0^+ states were claimed to correspond well to the observed six 0^+ states up to the 0^+_6 state at 15.1 MeV as shown in Fig. 3. The calculated 0^+_1 has, as its dominant component, the double closed shell wave function. The calculated 0^+_2 and 0^+_3 states proved to have, as their dominant components, $^{12}\mathrm{C}(0^+_1)+\alpha$ and $^{12}\mathrm{C}(2^+_1)+\alpha$ cluster structures, respectively, which well agree with established knowledge about 0^+_2 and 0^+_3 states. One of the most important results of Ref. [23] is that the calculated 0^+_6 state has a structure of 4α condensate-like state. The radius of the calculated 0^+_6 state has a very large value of about 5 fm. This large size suggests that the calculated 0^+_6 state is composed of a weakly interacting gas of α particles. The reduced width amplitudes of the calculated 0^+_6 state proved to have a large amplitude only in the $^{12}\mathrm{C}(0^+_2)+\alpha$ channel, where the $^{12}\mathrm{C}(0^+_2)$ (Hoyle state) wave function is given by the 3α OCM calculation.

We should note an important fact that the $n\alpha$ condensate-like state $\Phi^{n\alpha}$ has, in general, narrow width in spite of its high excitation energy. The reason is explained as follows in the typical case of ^{16}O: When $\Phi^{4\alpha}$ decays to ordinary ^{12}C states, large difference of structure between parent

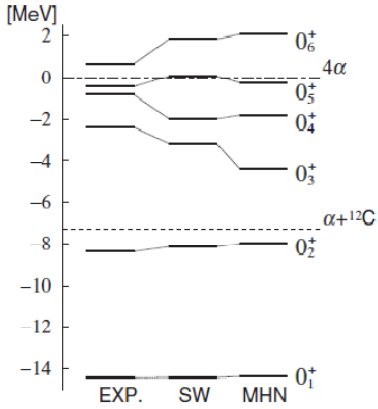

Fig. 3. Energy spectra by 4α OCM [23] with two kinds of effective nuclear force, SW and MHN.

and daughter states makes reduced widths very small, while when $\Phi^{4\alpha}$ decays to gas-like states of ^{12}C, small Q values of decay make barrier penetrability very small. The calculated widths of 0_4^+, 0_5^+ and 0_6^+ states are ~ 150, ~ 50 and $\sim$ 50 keV, respectively, while the observed values are 600, 185 and 166 keV, respectively.

4. Coexistence of cluster and mean-field-type states

Cluster states and mean-field-type states coexist widely in nuclei. Study of the coexistence phenomena has recently much developed. A review of actual features of the coexistence ranging from p-shell to pf-shell is given in Ref. [24]. A good example can be seen in the case of ^{44}Ti. Fig. 4 shows a good reproduction of the observed energy spectra of ^{44}Ti by the AMD calculation of Ref. [25]. The observed and calculated energy levels expressed by bold lines are the states with $^{40}\mathrm{Ca} + \alpha$ structure. The number $N(= 13, 14, 15)$ attached to the calculated rotational bands with bold lines stands for the quantum number $N = 2n + L$ of the ^{40}Ca-α relative wave function with n denoting the number of nodes.

The structure of the calculated $K^\pi = 0_2^+$ band which corresponds to the observed $K^\pi = 0_2^+$ band proved to have actually superdeformation of $\beta \approx 0.5$ and furthermore, to have triaxial shape which yields the side band with $K^\pi = 2^+$ corresponding to the observed $K^\pi = 2^+$ band. The observed $E2$ transitions, namely intra-band transitions inside the ground band and two superdeformed bands with $K^\pi = 0_2^+$ and $K^\pi = 2$ and also the inter-band transitions between $K^\pi = 0_2^+$ and $K^\pi = 2$ bands, are well reproduced by Ref. [25] without use of any effective charge.

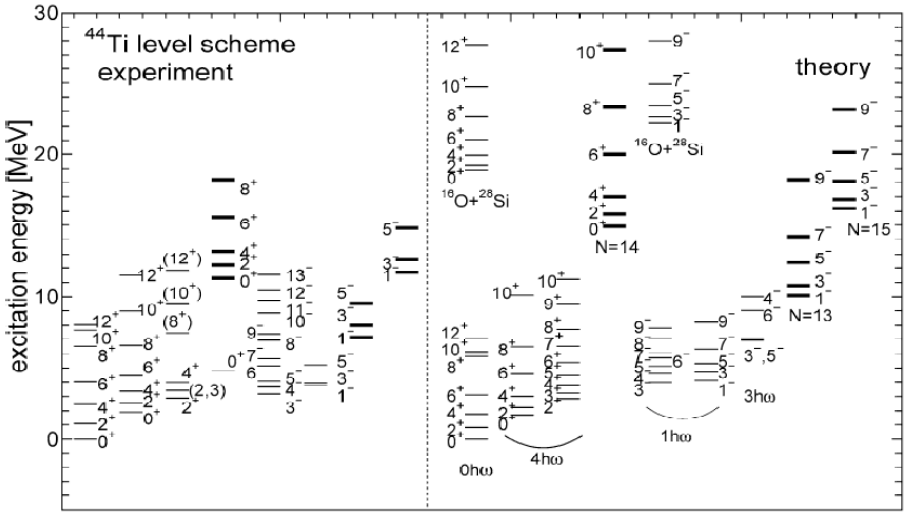

Fig. 4. Energy spectra of ^{44}Ti by AMD [25]. $(E_{\rm th})_{\rm exp}$ for ^{40}Ca + α breakup is 5.14 MeV.

The density distribution of the intrinsic state of the calculated ground band does not show clustering feature. However, the percentage of the ^{40}Ca + α component contained in the ground state is about 40% which is of sizable magnitude in view of the density distribution with no clustering feature. This seeming puzzle can be resolved by noticing that the SU(3) shell model wave function $|^{40}\mathrm{Ca}, (0f,1p)^4; (12,0)L\rangle$ which is the dominant component of the ground state wave function can be rewritten in the form of ^{40}Ca + α cluster model wave function by the Bayman–Bohr theorem [26]

$$
\begin{aligned}
&\left|{}^{40}\mathrm{Ca}, (0f,1p)^4; (12,0)L\right\rangle \\
&= D_L \mathcal{A}\left\{R_{12,L}(r_{\mathrm{Ca}-\alpha}, (40/11)\nu) Y_L(\hat{r}_{\mathrm{Ca}-\alpha}) \phi\left({}^{40}\mathrm{Ca}\right) \phi(\alpha)\right\} \times g\left(\vec{X}_G, 40\nu\right) , \\
&g(\vec{r},\gamma) = (2\gamma/\pi)^{3/4} \exp\left(-\gamma r^2\right) ,
\end{aligned}
$$

where $R_{N,L}(r,\gamma)$ and $\vec{X}_G$ stand for the radial harmonic oscillator function and the total centre-of-mass coordinate, respectively. This relation allows us to consider that the formation of the ^{40}Ca + α cluster states with the wave function $\mathcal{A}\{\chi(\vec{r})\phi(^{40}\mathrm{Ca})\phi(\alpha)\}$ is made by exciting the ^{40}Ca − α relative motion from $R_{12,0}(\vec{r}_{\mathrm{Ca}-\alpha}, (40/11)\nu)$ of the ground state to $\chi(\vec{r})$.

We can consider that the formation of cluster states is made generally in the same way as the formation of the ^{40}Ca + α cluster states in ^{44}Ti mentioned above. For example, the formation of the ^{12}C + α cluster states in ^{16}O with the wave function $\mathcal{A}\{\chi(r)[Y_\ell(\hat{r})\phi_L(^{12}\mathrm{C})]_J\phi(\alpha)\}$ is described as

follows: The ground state is dominantly described by the double-closed-shell wave function $\Phi_{\rm DCS} = \det|(0s)^4(0p)^{12}|$ and this $\Phi_{\rm DCS}$ can be equivalently rewritten in the form of the $^{12}{\rm C}+\alpha$ cluster wave function by the Bayman–Bohr theorem

$$\begin{aligned}&\det\left|(0s)^4(0p)^{12}\right| \\ &= c_L\mathcal{A}\left[R_{4,L}(r_{{\rm C}-\alpha},3\nu)\left[Y_L(\widehat{r}_{{\rm C}-\alpha})\phi_L\left(^{12}{\rm C}\right)\right]_{J=0}\phi(\alpha)\right] g\left(\vec{X}_G,16\nu\right), \quad (2)\end{aligned}$$

where L can be any of $L=0,2,4$. The formation of the $^{12}{\rm C}+\alpha$ cluster states is made by exciting the $^{12}{\rm C}-\alpha$ relative motion from $R_{4,L}(r_{{\rm C}-\alpha},3\nu)$ of the ground state to $\chi(r)$.

Usually, cluster states can be considered to be formed by the activation of the clustering degrees of freedom embedded in the ground state. The ground state wave function contains, as a dominant component, SU(3) model wave function which has dual character of shell model wave function and cluster model wave function. Sometimes, however, cluster states are formed by the activation of the clustering degrees of freedom embedded in some excited state which is formed by mean-field dynamics from the ground state and has a dual character of shell and cluster model wave functions. AMD calculation of Ref. [27] shows that $^{16}{\rm O}+^{16}{\rm O}$ molecular states are formed by the activation of $^{16}{\rm O}-^{16}{\rm O}$ degree of freedom embedded in the superdeformed band states.

Recently it was pointed out in Ref. [28] that an important observable quantity which directly verifies the dual nature of the nuclear wave function expressed by the Bayman–Bohr theorem is the large strength of the monopole transition between shell model type ground state and the excited cluster states. $E0$ transition strengths between ground state and cluster states in light nuclei are, in general, actually strong with their order of magnitude comparable with single-nucleon strength [29,28]. This fact looks strange because, in the shell model description, cluster states are many-particle many-hole states implying that $E0$ strengths from the ground state are much smaller than the single-nucleon strength. The explanation by Ref. [28] is as follows in the case of $E0$ transition in $^{16}{\rm O}$ between the ground state and the 0_2^+ state described by the wave function $\mathcal{A}\{\chi_0(r)[Y_0(\hat{r})\phi_0(^{12}{\rm C})]_0\phi(\alpha)\}$: From Eq. (2), we see that the $E0$ transition looks to be the $E0$ transition of relative motion between $R_{4,0}(r,3\nu)$ and $\chi_0(r)$. Actually, the exact analytical calculation gives

$$M\left(E0,0_2^+\to 0_1^+\right)=\frac{1}{2}\sqrt{\frac{\tau_{0,4}}{\tau_{0,6}}}\eta_6\left\langle R_{40}(r,\nu)\left|r^2\right|R_{60}(r,\nu)\right\rangle, \quad (3)$$

where $\tau_{0,N} = \langle \Psi_{0,N} | \mathcal{A}\{\Psi_{0,N}\} \rangle$ with $\Psi_{0,N} = R_{N,0}(r, 3\nu)[Y_0(\widehat{r})\phi_0(^{12}\mathrm{C})]_0\phi(\alpha)$. Although, due to antisymmetrization, $\tau_{0,N}$ is fairly smaller than unity for non large N, it shows up only in the form of ratio $\tau_{0,4}/\tau_{0,6}$ which is close to unity. The quantity η_6 is the coefficients of the $2\hbar\omega$-jump component contained in $|0_2^+\rangle$, and its magnitude is around 0.4. Eq. (3) shows clearly that the $E0$ transition is just the $E0$ transition with respect to one degree of freedom, namely the inter-cluster relative motion, which explains why $M(E0, 0_2^+ \to 0_1^+)$ has a large magnitude comparable with the single-nucleon strength.

5. Summary

Recent trends in nuclear cluster physics were discussed. Discussions were on three subjects. (1) Clustering in neutron-rich nuclei. For this subject we discussed (a) molecular-orbital and atomic-orbital structures in the cases of Be, Ne, and F isotopes, and (b) dineutron correlation in ^{11}Li. (2) Alpha-condensate-like states. For this subject we discussed two systems, ^{12}C and ^{16}O. (3) Coexistence of cluster and mean-field-type states. For this subject we discussed (a) AMD study of the coexistence in ^{44}Ti, (b) Mechanism of the coexistence and dual character of the ground state wave function, and (c) $E0$ transition between cluster states and ground state, which gives us a verification of our proposed mechanism of coexistence.

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