# RECENT STUDIES ON TROJAN HORSE METHOD[*]

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The study of nuclear reactions that are important for the understanding of astrophysical problems received an increasing attention over the last decades. The Trojan Horse Method was proposed as a tool to overcome some of the problems connected with the measurement of cross-sections between charged particles at astrophysical energies. Here we present some recent studies on this method.

## 1. Introduction

Nuclear reactions between charged particle at the very low energies typical of astrophysical environment are severely hindered by the presence of the Coulomb barrier between the colliding nuclei. This barrier has values of the order of some MeV while the energies in the astrophysical systems typically range from tens to hundreds of keV. This implies that the nuclear reactions proceed via tunnel effect with a tunnelling probability that is governed by the Gamow penetration factor. This has an exponentially decreasing behaviour with energy. Hence the tunnelling probability drops off very quickly

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when the relative energies of the colliding particles get below the Coulomb barrier. As a consequence the cross-sections of the processes also diminish and their typical values in the relevant energy region for astrophysics (the Gamow window) are often of the order of micro- and even nanobarns: the cross-section of the most important reaction for the helium burning stage [1], $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$, lies in the abyss of $10^{-17}$ barn at its Gamow energy, 300 keV!

Until the first underground laboratory for nuclear astrophysics (LUNA) was set up in Italy, this situation has prevented from measuring the cross-sections of nuclear reactions of astrophysical interest directly in the Gamow window region. The pieces of information needed in astrophysical models were then derived from the extrapolation of measurements performed at higher energies complemented by theoretical model calculations.

After the work of LUNA, it became clear that even the electron screening effect, *i.e.* the effect of the electron cloud surrounding the colliding ions in a laboratory experiment, that was often disregarded before the 80s, can actually play an important role in enhancing the measured value of the cross-sections at such low energies, thus giving a bias that was transferred to the astrophysical models. This situation obliged to be very careful even in evaluating cross-sections directly measured in the Gamow window region. As a consequence, extrapolations had to be brought back in the data analysis, though, of course, the global level of accuracy increased.

A complementary way of getting information on the cross-section of nuclear reactions relevant for astrophysics is by using indirect methods. A common feature of these methods is that, instead of studying the reaction of interest for astrophysics, a surrogate one is considered. The cross-section of this latter can then be related with that of the orginal one by thoretical models. Among these methods, the Asymptotic Normalization Coefficient (ANC), the Coulomb dissociation (CD) and the Trojan Horse (THM) ones got much attention over the last two decades. Here we will review recent work aiming at bringing the THM to an even higher level of reliability and precision.

## 2. The Trojan Horse Method

The Trojan Horse Method (THM) [2, 3, 4, 5, 6] has been proposed and exploited to study the cross-sections of nuclear reactions between charged particles at astrophysical energies. It will be only shortly recalled here. Details of the method are discussed in the bibliographic references cited in this paper. According to this method, a reaction of the type

$$x + b \rightarrow z + w \tag{1}$$

can be studied at energies relevant for nuclear astrophysics by using a three-body reaction of the type

$$a + b \rightarrow z + w + s \, . \tag{2}$$

In reaction (2) $a$ is a nucleus with a strong cluster structure of the type $x \otimes s$ and it is often called the *trojan horse* nucleus. The relative energy between $a$ and $b$ is assumed to be higher than their Coulomb barrier and also than the energy required to break-up particles $a$ into its components $x$ and $s$. Given the relative energy of the colliding particles $a$ and $b$, reaction (2) can proceed without suffering from the strong exponential decrease of the cross-section that affects reaction (1) at low energies.

If reaction (2) proceeds through a quasi-free reaction mechanism, where particle $s$ behaves as a spectator to the sub-process $x + b \rightarrow z + w$, then it can be described by the diagram shown in figure 1. As particle $x$ that initiates the $x + b \rightarrow z + w$ reaction with the real particle $b$ is virtual, then this process is said to be half off energy shell (HOES).
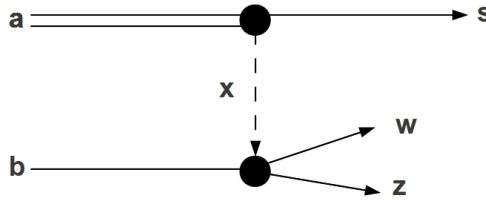


Fig. 1. The THM basic diagram. The upper vertex describes the virtual decays of particle $a$ into $x \otimes s$ while the lower one refers to the process of interest, namely $b + x \rightarrow w + z$.

The cross-section of the three-body reaction can be factorized in a kinematical term, a term describing the virtual decay of $a$ into $x$ and $s$ and one giving the interaction of $x$ with $b$ to give $z$ and $w$. This can be written in PWIA as

$$\frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} \propto (\text{KF}) |\Phi(p_s)|^2 \left( \frac{d\sigma}{d\Omega} \right)_{\text{cm}} . \tag{3}$$

Hence, by measuring the cross-section of the three body process (2), calculating the kinematical factor (KF) and the impulse distribution of $s$ inside $a$ (which corresponds to the virtual decay mentioned above) represented by $|\Phi(p_s)|^2$, one can derive the cross-section $\left( \frac{d\sigma}{d\Omega} \right)_{\text{cm}}$ for the process of astrophysical interest (1).

The THM experimental conditions are defined by the identification of the phase space region where the quasifree mechanism is expected to be dominant, a certain degree of freedom being left by the three body kinematics. The c.m. interaction energy of the HOES entrance channel is connected

to that of outgoing particles by the relation $E_{b-x} = E_{w-z} - Q_2$, where $E_{w-z}$ is the relative energy of the outgoing particles $w$ and $z$ and $Q_2$ is the $q$-value of the (on shell) two body process $x + b \rightarrow z + w$. This is known as post collision prescription.

A typical features of the THM is that it is, in general, sufficient to study the three-body reaction in a co-planar geometry. In this case, by energy and momentum conservation, it turns out that it is sufficient to measure three out of the four (two energies and two angles) kinematical variables of the outgoing particles to completely determine the kinematics of the whole process. In general, this also allows for the use of a simple detection setup in a THM experiment.

The THM has been used a number of times [4,7,8,9,10,11,12,13,14,15] to measure cross-sections of astrophysical interest and it is presently regarded as a very powerful tool by the nuclear astrophysics community. It must be stressed here that the simple theoretical approach discussed above, while being basically correct, has been improved along the years introducing more sophisticated techniques that allow to treat complicated situations as in the cases, where the cross-section is dominated by the presence of narrow, close or interfering resonances.

## 3. Recent studies on impulse distributions for Trojan Horse

In order to evaluate the influence of the details of the ingredients used in the formula shown in the previous paragraph, studies were recently performed. From the simple formula shown above, it is clear that the results will be sensitive to the details of the impulse distribution functions used in deriving the two body cross-section. In particular, the dependence of the width of the impulse distribution on the value of the transferred momentum has recently been studied [16]. This dependence is well known and many papers have studied it for decades [17]. The new study carefully considered the implications of such dependence in connection with the use of Impulse Approximation in Trojan Horse measurements of reactions of astrophysical interest.

The transferred momentum is defined as

$$\vec{q_t} = \left( \frac{m_B}{m_b} \right)^{1/2} \vec{p_b} - \left( \frac{m_b}{m_B} \right)^{1/2} \vec{p_B} \, , \tag{4}$$

where $B$ represents the $z + w$ system. The dependance of the width of the momentum distribution of the proton inside dueteron with $q_t$ is shown in figure 2. In this figure the experimental behaviour is fitted using an empirical formula

$$W(q_t) = f_0 \left[ 1 - e^{(-q_t/q_0)} \right] \, . \tag{5}$$

In this formula the values of the parameters are $q_0 = 60 \pm 12$ MeV/$c$ and $f_0 = 58$ MeV/$c$. This piece of information is used in order to use the proper momentum distribution width in the given experimental conditions hence minimizing the uncertainty related to this parameter in the results of THM applications.
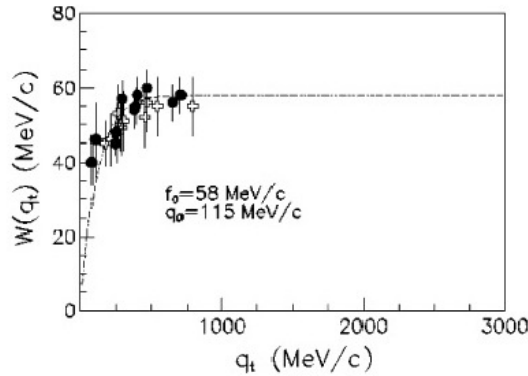


Fig. 2. FWHM of the momentum distribution of the proton inside dueteron as a function of the transferred momentum $q_t$. See [16] for details.
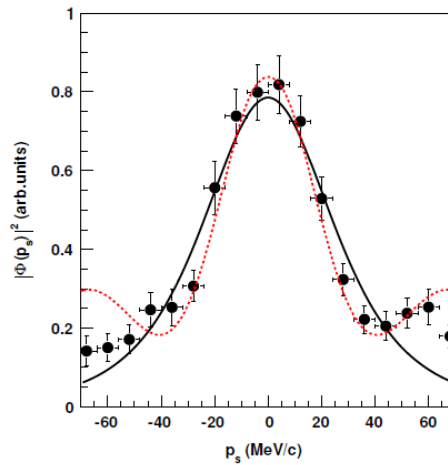


Fig. 3. Momentum distribution of the spectator inside the nucleus a (in this case neutron inside deuteron). The solid line represents the simple Hultén function approximation, the dashed line is the result of a FRESCO calculation. See [15] for details.

The use of a simple PWIA in obtaining THM cross-sections for nuclear astrophysics has been often questioned in the past. The key point is that in THM applications PWIA was and is not used in a wide range of the

spectator momentum values. On the contrary, this range is limited to values of the spectator momentum of the order of very few tens of MeV/$c$. Within these limits, not only distorsion effects that heavily affect the wave funcions of the particles taking part in the reaction at high momentum values are negligeable, as clearly shown in figure 3, but also PWIA, with its limited number of parameters, allows for the extraction of the two body cross-section with good reliability and a weak dependence of the result on the set of parameters used in the analysis.

## 4. Conclusion

The THM has proven over the year to be a powerful method for measuring cross-sections of astrophysical interest in a variety of situations. Much attention is deserved to the reliability of each single element entering the derivation of the two-body indirectly measured cross-section from the three-body one. This has proven the reliability of the results obtained so far and gives confidence for the use of the method in more difficult cases as those involving the use of radioactive ion beams.

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