HAWKING RADIATION OF ROTATING D-BRANES FROM ANOMALY CANCELLATION

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We apply the method of anomaly cancellation initiated by Robinson and Wilczek *et al.* to the Hawking radiation of rotating D-brane solutions of superstring theories. We obtain that their reduced field theories near their horizons are two-dimensional chiral field theories in a set of curved backgrounds. Therefore, we can calculate their angular momentum fluxes and energy-momentum fluxes from the method of anomaly cancellation. We obtain that the energy-momentum fluxes of the rotating D-branes compose thermal radiations, their thermal temperatures match with their Hawking temperatures obtained from black brane thermodynamics.

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1. Introduction

Hawking radiation is one of the most interesting problems of a black hole. Since its primal discovery by Hawking more than three decades ago [1], people have given many different explanations to this phenomenon [2,3,4,5,6], but the same result has been obtained. Different derivation methods of Hawking radiation have shown that Hawking radiation is related with the quantum field effect in the spacetime of a black hole.

Recently, a new method for the derivation of Hawking radiation has been set up by Robinson and Wilczek *et al.* which is called anomaly cancellation [7,8]. Because a black hole's horizon is a one-way membrane, the effective field theory of quantum fields near the horizon is a two-dimensional chiral field theory. Hence there are gauge and gravitational anomalies for the currents outside the horizon. However, the effective action of quantum fields of a black hole is still gauge invariant and general covariant. Then, to combine certain boundary conditions for the covariantly anomalous currents on the horizon, gauge and energy-momentum fluxes of a black hole can be derived. Such a method for the derivation of Hawking radiation has revealed the connections between Hawking radiation and anomalies of quantum fields of a black hole. But one can see that the original mind of such a method has occurred in [5] many years ago.

In [9, 10, 11], the method of anomaly cancellation for the derivation of Hawking radiation has been generalized to higher-dimensional rotating black holes. Then, people have applied such a method for the study of Hawking radiation of many different kinds of black holes [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. In [32], to combine the techniques of conformal field theory, the anomaly cancellation method for the radiation of a black hole has been generalized to the radiation of higher-spin currents. Some further developments and applications on the method of anomaly cancellation have been carried out in [33, 34, 35].

There are many rotating D-brane solutions existing in superstring theories [36,37,38,39]. Rotating D-branes have many interesting thermodynamical properties [40, 41, 42]. Because a rotating D-brane has an event horizon like that of a rotating black hole, it also has Hawking radiation like that of a rotating black hole. Because it is now shown that the method of anomaly cancellation is universal to the Hawking radiation of many different kinds of black holes, while the Hawking radiation of rotating D-branes from the method of anomaly cancellation has not been investigated in the literature yet, it is necessary for us to give an analysis of the Hawking radiation of rotating D-branes from the method of anomaly cancellation. In addition, some technical details in the anomaly cancellation method for a rotating object with multiple rotating parameters are still necessary to be investigated further.

This paper is organized as follows. In Sec. 2, we study the effective field theories of quantum fields near the horizons of rotating D-branes. We obtain that they are equal to two-dimensional chiral field theories in a set of curved backgrounds. Therefore, we can apply the anomaly cancellation method to these black hole like objects. In Sec. 3, we calculate the angular momentum fluxes and energy-momentum fluxes of the rotating D-branes from the method of anomaly cancellation. We obtain that the energy-momentum fluxes of the rotating D-branes compose thermal radiations, their thermal temperatures match with their Hawking temperatures obtained from black brane thermodynamics. In Sec. 4, we give the expressions of the Hawking temperatures of all of the rotating D-branes in 10-dimensional space-time. In the Appendix A, we supply with the fluxes of two-dimensional black body radiation with multiple chemical potentials for the purpose of comparison.

2. Quantum fields near the horizons of rotating *D*-branes

Rotating D-branes of supergravity and superstring theories in 10-dimensional space-time have been constructed in [36,37,38,39]. They are higher dimensional rotating objects carrying Ramond–Ramond charges of the Ramond–Ramond $(p+1)$-form fields $A_{p+1}(x)$. Their actions are given by

$$S=\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left[R-\frac{1}{2}(\partial\Phi)^2-\frac{1}{2(p+2)!}e^{-a\Phi}|F_{p+2}|^2\right], \tag{1}$$

where Φ is the dilaton field and $F_{p+2}(x)=dA_{p+1}(x)$ is the Ramond–Ramond field strength. For even dimensional world volume case, the metrics of the rotating D-branes in Einstein-frame are given by

$$\begin{aligned} ds_D^2 \;=\;& H^{-\frac{\tilde{d}}{D-2}}\left(-(1-\frac{2m}{r^{\tilde{d}}\Delta})dt^2+dx_1^2+\ldots+dx_p^2\right) \\ &+H^{\frac{d}{D-2}}\left[\frac{\Delta dr^2}{H_1\ldots H_N-2mr^{-\tilde{d}}}+r^2\sum_{i=1}^{N}H_i\left(d\mu_i^2+\mu_i^2d\phi_i^2\right)\right. \\ &\left.-\frac{4m\cosh\alpha}{r^{\tilde{d}}H\Delta}dt\left(\sum_{i=1}^{N}l_i\mu_i^2d\phi_i\right)+\frac{2m}{r^{\tilde{d}}H\Delta}\left(\sum_{i=1}^{N}l_i\mu_i^2d\phi_i\right)^2\right], \end{aligned} \tag{2}$$

where $D=10$ is the total space-time dimension, $d=p+1$ is the world volume dimension, $\tilde{d}=D-d-2$, $2N=\tilde{d}+2$, N is the total number of the rotating parameters. In metric (2), the functions Δ, H and H_i are given by

$$\begin{aligned} \Delta \;&=\; H_1\ldots H_N\sum_{i=1}^{N}\frac{\mu_i^2}{H_i}, \qquad H=1+\frac{2m\sinh^2\alpha}{r^{\tilde{d}}\Delta}, \\ H_i \;&=\; 1+\frac{l_i^2}{r^2}, \qquad\qquad i=1,2,\ldots,N. \end{aligned} \tag{3}$$

The N quantities μ_i satisfy the constraint $\sum\limits_{i=1}^{N}\mu_i^2=1$. They can be parametrized by $(N-1)$ unconstrained angles as

$$\begin{aligned} \mu_i \;&=\; \sin\varphi_i\prod_{j=1}^{i-1}\cos\varphi_j, \qquad i\le N-1, \\ \mu_N \;&=\; \prod_{j=1}^{N-1}\cos\varphi_j. \end{aligned} \tag{4}$$

In (4), $\prod_{j=1}^{n} \cos\varphi_j$ is defined to be equal to 1 when $n \leq 0$. The dilaton field Φ and the Ramond–Ramond $(p+1)$-form field $A_{p+1}(x)$ are given by

$$e^{2\Phi/a} = H\,, \qquad A_{(p+1)} = \frac{1-H^{-1}}{\sinh\alpha}\left(\cosh\alpha dt + \sum_{i=1}^{N} l_i \mu_i^2 d\phi_i\right) \wedge d^p x\,. \qquad (5)$$

In (1) and (5), the constant a is determined by $a^2 = 4 - 2d\tilde{d}/(d+\tilde{d})$. For the odd dimensional world volume case, we have $2N = \tilde{d}+1$, the metrics of the rotating D-branes have the same form as metric (2), but with the range of the index i including 0, *i.e.*, $i = 0, 1, \ldots, N$, so there are N unconstrained angles. Equations (3) to (5) are simply generalized to this case, and we need to define $l_0 = 0$, $\phi_0 = 0$ and $H_0 = 1$.

The horizon $r_{\rm H}$ of a rotating D-brane is determined by the largest real root of the equation

$$\prod_{i=1}^{N} H_i - \frac{2m}{r^{\tilde{d}}} = 0\,. \qquad (6)$$

The rotating D-brane extends in the $x_1, x_2, \ldots, x_p$ directions and rotates in the perpendicular $\phi_1, \phi_2, \ldots, \phi_N$ directions. The angular velocities $\Omega_1, \Omega_2, \ldots, \Omega_N$ of the event horizon in $\phi_1, \phi_2, \ldots, \phi_N$ directions can be obtained by requiring $\eta^2 = 0$ on the horizon, where $\eta = \frac{\partial}{\partial x_0} + \Omega_i \frac{\partial}{\partial \phi_i}$ is the Killing vector. They are evaluated to be

$$\Omega_i = \frac{2l_i}{\cosh\alpha(r_{\rm H}^2 + l_i^2)}\,, \qquad i = 1, 2, \ldots, N\,. \qquad (7)$$

The Hawking temperature of a rotating D-brane can be evaluated through the formula of black hole thermodynamics

$$T_{\rm H}^2 = \lim_{r\to r_{\rm H}} \frac{1}{16\pi^2(-\eta^2)} \nabla_\mu \eta^2 \nabla^\mu \eta^2\,. \qquad (8)$$

Being a constant, it can be calculated at any special point on the horizon. Through defining two functions $A(r)$ and $B(r)$ as

$$\begin{aligned} A(r) &= 1 - \frac{2m}{r^{\tilde{d}}\left(1+\frac{l_1^2}{r^2}\right)\left(1+\frac{l_2^2}{r^2}\right)\ldots\left(1+\frac{l_N^2}{r^2}\right)}\,, \\ B(r) &= 1 + \frac{2m\sinh^2\alpha}{r^{\tilde{d}}\left(1+\frac{l_1^2}{r^2}\right)\left(1+\frac{l_2^2}{r^2}\right)\ldots\left(1+\frac{l_N^2}{r^2}\right)}\,, \end{aligned} \qquad (9)$$

the result of the Hawking temperature of a rotating D-brane can be given by

$$T_{\rm H} = \frac{1}{4\pi} \frac{A'(r)}{B^{1/2}(r)}\bigg|_{r=r_{\rm H}}\,. \qquad (10)$$

The anomaly cancellation method for the derivation of Hawking radiation has shown the connections between the Hawking radiation and the anomalies of quantum fields of a black hole. Because classically the particles cannot exit though a black hole horizon, the effective field theory of quantum fields near a black hole horizon should be a two-dimensional chiral field theory. Thus when we reduce field theories to two-dimensional space-time near the horizon of a black hole, there should be gauge and gravitational anomalies for the currents outside the horizon.

But for a four-dimensional or a higher-dimensional black hole, to observe from four-dimensional or higher-dimensional space-time, people do not consider that field theories near the horizon are chiral; at the same time, people can observe that Hawking radiation is existing outside the horizon. This means that when we reduce field theories to two-dimensional space-time near the horizon of a black hole, the effective field theories still satisfy the gauge invariance and general covariance conditions, and there are some other currents which have counteracted the anomalies of the two-dimensional chiral field theories.

These currents can be determined by certain boundary conditions which are physically acceptable. As we can see in Sec. 3 the existence of these currents results in the correct Hawking radiation of a black hole in four-dimensional and higher-dimensional space-time. Thus, in order to investigate the Hawking radiation of the rotating D-branes from the method of anomaly cancellation, we first need to study the effective field theories of quantum fields near the horizons of rotating D-branes.

We consider a free scalar field φ in the background of metric (2). We suppose that the scalar field is zero-mass and does not couple with the Ramond–Ramond gauge field. Then, the action of the system is given by

$$\begin{aligned} S_{\text{free}}(\varphi) &= \int d^D x\sqrt{-g}\; g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \\ &= \int d^D x\sqrt{-g}\left[\partial_\mu(\varphi\partial^\mu\varphi) - \varphi\partial_\mu\partial^\mu\varphi\right] , \end{aligned} \tag{11}$$

where g is the determinant of metric (2). The scalar field $\varphi(x)$ can be expanded in terms of the harmonic functions of a rotating D-brane space-time. To consider the odd dimensional world volume case, we can write

$$\begin{aligned} \varphi(x) = \sum_{k_1,\dots,k_p}\sum_{l_1,\dots,l_N}\sum_{m_1,\dots,m_N} &\varphi_{l_1\dots l_N m_1\dots m_N k_1\dots k_p}(r,t) \\ &\times Y_{l_1,\dots,l_N,m_1,\dots,m_N}(\theta,\varphi_1,\dots,\varphi_{N-1},\phi_1,\dots,\phi_N)X(k_1,\dots,k_p)\,. \end{aligned} \tag{12}$$

In (12), $Y_{l_1,\ldots,l_N,m_1,\ldots,m_N}(\theta,\varphi_1,\ldots,\varphi_{N-1},\phi_1,\ldots,\phi_N)$ are the normalized spherical harmonics on a $2N$-dimensional unit sphere. $Y_{l_1,\ldots,l_N,m_1,\ldots,m_N}(\theta,\varphi_1,\ldots,\varphi_{N-1},\phi_1,\ldots,\phi_N)$ can be written in the form

$$\begin{aligned}&Y_{l_1,\ldots,l_N,m_1,\ldots,m_N}(\theta,\varphi_1,\ldots,\varphi_{N-1},\phi_1,\ldots,\phi_N)\\&= P^{m_1\ldots m_N}_{l_1\ldots l_N}(\cos\theta,\cos\varphi_1,\ldots,\cos\varphi_{N-1})e^{i\sum\limits_{i=1}^{N} m_i\phi_i}\,,\end{aligned} \tag{13}$$

where $P^{m_1,\ldots,m_N}_{l_1,\ldots,l_N}(\cos\theta,\cos\varphi_1,\ldots,\cos\varphi_{N-1})$ are the generalized associated Legendre functions on a $2N$-dimensional unit sphere. The functions $X(k_1,\ldots,k_p)$ are plane waves in the transverse space. They are given by

$$X(k_1,\ldots,k_p) = e^{i(k_1x^1+\ldots+k_px^p)}\,. \tag{14}$$

The number set $(l_1,\ldots,l_N,m_1,\ldots,m_N,k_1,\ldots,k_p)$ are the quantum numbers of a partial wave. We suppose that the functions $Y_{l_1,\ldots,l_N,m_1,\ldots,m_N}(\theta,\varphi_1,\ldots,\varphi_{N-1},\phi_1,\ldots,\phi_N)$ and $X(k_1,\ldots,k_p)$ satisfy the normalization condition

$$\begin{aligned}&\int d\Omega_{2N}d^px Y_{l_1,\ldots,l_N,m_1,\ldots,m_N}(\theta,\varphi_1,\ldots,\varphi_{N-1},\phi_1,\ldots,\phi_N)X(k_1,\ldots,k_p)\\&\times Y^*_{u_1,\ldots,u_N,v_1,\ldots,v_N}(\theta,\varphi_1,\ldots,\varphi_{N-1},\phi_1,\ldots,\phi_N)X^*(q_1,\ldots,q_p)\\&= \delta_{l_1u_1}\ldots\delta_{l_Nu_N}\delta_{m_1v_1}\ldots\delta_{m_Nv_N}\delta_{k_1q_1}\ldots\delta_{k_pq_p}\,,\end{aligned} \tag{15}$$

where the integration of the coordinates x^i takes a unit area on the p-dimensional transverse space of a rotating D-brane.

To substitute (12) into (11) and complexify the first φ inside the integral, through some calculation, to take the limit near horizon and leave the dominant terms only [8, 9, 10, 24], we obtain finally

$$\begin{aligned}S_{\text{free}}(\varphi) &= -\sqrt{-g}|_{r_{\mathrm{H}}}\sum_{k_1,\ldots,k_p,l_1\ldots,l_N,m_1,\ldots,m_N}\int dtdr\varphi^*_{l_1,\ldots,l_N,m_1,\ldots,m_N,k_1,\ldots,k_p}(r,t)\\&\quad\times\left[-\frac{B^{\tilde d/(D-2)}(r)}{A(r)}\left(\partial_t+\frac{2im_jl_j}{\cosh\alpha\left(r^2+l_j^2\right)}\right)^2+\frac{A(r)}{B^{d/(D-2)}(r)}\partial_r^2\right]\\&\quad\times\,\varphi_{l_1,\ldots,l_N,m_1,\ldots,m_N,k_1,\ldots,k_p}(r,t)\,.\end{aligned} \tag{16}$$

From (16), we obtain that, near the horizon of a rotating D-brane, the effective field theory of a scalar field reduces to an infinite set of two-dimensional complex scalar fields in a curved background with metric

$$ds^2 = -\frac{A(r)}{B^{\tilde d/(D-2)}(r)}dt^2+\frac{B^{d/(D-2)}(r)}{A(r)}dr^2\,, \tag{17}$$

where the functions $A(r)$ and $B(r)$ have been defined in (9). The horizon of the two-dimensional metric (17) is determined by equation $A(r_{\rm H}) = 0$, just the same equation as (6). In addition, each of the two-dimensional complex field carries N U(1) charges m_j, which are just the N angular momentum quantum numbers of the original partial waves. The corresponding N U(1) gauge fields are given by

$$\mathcal{A}_t^j(r) = -\frac{2l_j}{\cosh\alpha\left(r^2 + l_j^2\right)}, \qquad j = 1, 2, \ldots, N. \tag{18}$$

The N axial isometries along each ϕ_j directions of metric (2) have turned into the N U(1) gauge symmetries. Corresponding to the N U(1) gauge symmetries, there are N U(1) currents $J_i^\mu(r)$, $i = 1, 2, \ldots, N$. For the rotating D-branes of even dimensional world volume, the calculation is similar as above, and the results of (16)–(18) are also similar. Like that of the rotating black holes [8, 9, 10], in the reduced two-dimensional metric (17), the relevance with the azimuthal angles in the original metric (2) related with the rotation of the brane has been dismissed.

If the scalar field carries Ramond–Ramond charges, then it will couple with the Ramond–Ramond gauge fields of the rotating D-branes. However, because now the explicit forms of the covariant couplings of quantum fields with Ramond–Ramond gauge fields are not clear, we do not consider such a coupling and the related Ramond–Ramond charge fluxes in this paper. However, we can expect that if we include the couplings of scalar field with Ramond–Ramond gauge fields in the action, the curved backgrounds of the reduced two-dimensional field theories of the rotating D-branes are still given by metric (17).

3. Hawking radiation of rotating *D*-brane from anomaly cancellation

For the reduced near horizon metric (17), to consider the area outside its horizon, we divide it into two parts: $[r_{\rm H}, r_{\rm H}+\epsilon]$ and $[r_{\rm H}+\epsilon, \infty]$. $[r_{\rm H}, r_{\rm H}+\epsilon]$ is the area near the horizon, where the physics has certain exotic properties. $[r_{\rm H}+\epsilon, \infty]$ is the area far away from the horizon, where the physics has the normal properties. The parameter ϵ can take to be arbitrarily small, thus, for observable quantities, they can be taken in the area $[r_{\rm H}+\epsilon, \infty]$. In the near horizon area $[r_{\rm H}, r_{\rm H}+\epsilon]$, to consider that a black hole's horizon is a one-way membrane, for the $(1+1)$-dimensional field theory, the ingoing (left moving) modes will move towards the centre singularity, therefore, they will not affect the physics of the area $[r_{\rm H}, r_{\rm H}+\epsilon]$. That is to say, in the area $[r_{\rm H}, r_{\rm H}+\epsilon]$, only the outgoing (right moving) modes are responsible for the observable physics. This fact makes the effective field theory there be a two-dimensional chiral field theory. Thus, in the area $[r_{\rm H}, r_{\rm H}+\epsilon]$, there are gauge and gravitational anomalies for the currents. In the area $[r_{\rm H}+\epsilon, \infty]$, the ingoing and outgoing modes are both existing, field theory there is a normal one and there are no anomalies for the gauge and gravitational currents.

For a rotating D-brane, it is a thermodynamical equilibrium system, all currents in the space-time outside the horizon are static. Outside the horizon, the N U(1) gauge currents can be decomposed into the form

$$\sqrt{-g_2}J_i^{\mu}(r) = \sqrt{-g_2}J^{\mu}_{i({\rm H})}(r)H(r) + \sqrt{-g_2}J^{\mu}_{i({\rm o})}(r)\Theta_+(r)\,, \tag{19}$$

where $\Theta_+(r) = \Theta(r - r_+ - \epsilon)$ (here we use r_+ to represent the radius of the event horizon), $H(r) = 1 - \Theta_+(r)$, and $g_2 = -B^{(d-\tilde{d})/(D-2)}(r)$ is the determinant of metric (17). From (19), $\sqrt{-g_2}J^{\mu}_{i({\rm o})}(r)$ are the currents in the region $[r_{\rm H}+\epsilon, \infty]$, they satisfy the ordinary conservation equation. Thus we have

$$\partial_r\left[\sqrt{-g_2}J^r_{i({\rm o})}(r)\right] = 0\,. \tag{20}$$

On the other hand, $\sqrt{-g_2}J^{\mu}_{i({\rm H})}(r)$ are the currents in the region $[r_{\rm H}, r_{\rm H}+\epsilon]$. As mentioned above, they are anomalous and thus obey the anomalous conservation equation [43, 44]

$$\partial_r\left[\sqrt{-g_2}J^r_{i({\rm H})}(r)\right] = \frac{m_i}{4\pi}\partial_r A_t(r)\,, \tag{21}$$

where $A_t(r) = m_i\mathcal{A}^i_t(r)$ is the sum of N U(1) fields. The solutions of (20) and (21) are given by

$$\begin{aligned}\sqrt{-g_2}J^r_{i({\rm o})} &= c_{i({\rm o})}\,,\\ \sqrt{-g_2}J^r_{i({\rm H})} &= c_{i({\rm H})} + \frac{m_i}{4\pi}\left(A_t(r) - A_t(r_{\rm H})\right),\end{aligned} \tag{22}$$

where $c_{i({\rm o})}$ and $c_{i({\rm H})}$ are two sets of integration constants. In fact, $c_{i({\rm o})}$ are just the currents for an observer to observe at infinity, $c_{i({\rm H})}$ are the values of the consistent currents of the outgoing modes at the horizon.

On the other hand, current anomaly is a pure quantum field effect, its existence does not change the gauge invariance of the effective action. A gauge transformation for the effective action of the two-dimensional field theory yields [7, 8, 16]

$$\begin{aligned}\delta W &= -\int d^2x\sqrt{-g_2}\lambda_i\nabla_\mu J_i^{\mu}\\ &= -\int d^2x\lambda_i\Big[\partial_r\left(\frac{m_i}{4\pi}A_tH(r)\right) + \left(\sqrt{-g_2}J^r_{i({\rm o})} - \sqrt{-g_2}J^r_{i({\rm H})} + \frac{m_i}{4\pi}A_t\right)\\ &\quad\delta\left(r - r_+ - \epsilon\right)\Big]\,,\end{aligned} \tag{23}$$

where λ_i are N U(1) gauge parameters. The first term in the second line of (23) can be cancelled by the quantum effect of the ingoing modes near horizon. Thus gauge invariance of the effective action leads to the vanishing of the coefficients of the δ-function. The combination with (22) yields the relation

$$c_{i(\mathrm{o})} = c_{i(\mathrm{H})} - \frac{m_i}{4\pi} A_t(r_{\mathrm{H}})\,. \tag{24}$$

The constants $c_{i(\mathrm{H})}$ in (24) can be determined through introducing the covariantly anomalous currents

$$\sqrt{-g_2}\widetilde{J}^r_i(r) = \sqrt{-g_2}J^r_i(r) + \frac{m_i}{4\pi}A_t(r)H(r)\,, \tag{25}$$

together with the boundary conditions

$$\sqrt{-g_2}\widetilde{J}^r_i(r_{\mathrm{H}}) = 0\,, \tag{26}$$

i.e., the covariant currents vanish away on the horizon [7,8,9]. Such a boundary condition makes physical quantities regular on the future horizon [7,8,9]. The combination of (19), (25) and (26) yields

$$\sqrt{-g_2}J^r_{i(\mathrm{H})}(r_{\mathrm{H}}) = -\frac{m_i}{4\pi}A_t(r_{\mathrm{H}})\,. \tag{27}$$

To combine (22) and (27), we obtain

$$c_{i(\mathrm{H})} = \sqrt{-g_2}J^r_{i(\mathrm{H})}(r_{\mathrm{H}}) = -\frac{m_i}{4\pi}A_t(r_{\mathrm{H}})\,. \tag{28}$$

To substitute (28) into (24), we obtain

$$c_{i(\mathrm{o})} = -\frac{m_i}{2\pi}A_t(r_{\mathrm{H}}) = \frac{m_i}{2\pi}\sum_{j=1}^{N}\frac{2m_j l_j}{\cosh\alpha\left(r_{\mathrm{H}}^2 + l_j^2\right)}\,. \tag{29}$$

Thus we have obtained the N U(1) gauge currents of the two-dimensional black hole (17). In (29), because m_i are just the N angular momentum quantum numbers of the partial waves of the original rotating D-brane, (29) are just the angular momentum fluxes of the rotating D-brane.

Next, we need to calculate the energy-momentum flux of the two-dimensional black hole (17). The energy-momentum tensor outside the horizon can be decomposed into the form

$$\sqrt{-g_2}T^{\mu}_{\nu}(r) = \sqrt{-g_2}T^{\mu}_{\nu(\mathrm{H})}(r)H(r) + \sqrt{-g_2}T^{\mu}_{\nu(\mathrm{o})}(r)\Theta_+(r)\,, \tag{30}$$

where $\Theta_+(r) = \Theta(r - r_+ - \epsilon)$ (here we use r_+ to represent the radius of the horizon), $H(r) = 1 - \Theta_+(r)$. Thus, $\sqrt{-g_2}T^{\mu}_{\nu(\mathrm{H})}(r)$ is the energy-momentum tensor in the area $[r_{\mathrm{H}}, r_{\mathrm{H}} + \epsilon]$, $\sqrt{-g_2}T^{\mu}_{\nu(\mathrm{o})}(r)$ is the energy-momentum tensor in the area $[r_{\mathrm{H}} + \epsilon, \infty]$.

In a two-dimensional curved space-time, $\sqrt{-g_2}T_t^r(r)$ is just the energy-momentum flux in the radial direction. For $\sqrt{-g_2}T^\mu_{\nu(\mathrm{o})}(r)$, as mentioned above, it has no gauge and gravitational anomalies, therefore, it satisfies the normal conservation equation

$$\partial_r\left[\sqrt{-g_2}T^r_{t(\mathrm{o})}(r)\right] = J^r_{(\mathrm{o})}\partial_r A_t(r)\,, \tag{31}$$

where the right-hand side of (31) comes from the U(1) gauge currents. $J^r_{(\mathrm{o})}$ is a constant determined by $J^r_{(\mathrm{o})} = -\frac{1}{2\pi}A_t(r_{\mathrm{H}})$. From (29), we have

$$J^r_{(\mathrm{o})} = \frac{1}{2\pi}\sum_{j=1}^{N}\frac{2m_j l_j}{\cosh\alpha\left(r_{\mathrm{H}}^2+l_j^2\right)} \equiv c_{\mathrm{o}}\,. \tag{32}$$

The integration of (31) gives

$$\sqrt{-g_2}T^r_{t(\mathrm{o})}(r) = a_{\mathrm{o}} + c_{\mathrm{o}}A_t(r)\,, \tag{33}$$

where a_{o} is an integration constant. Because $A_t(r_\infty) = 0$, a_{o} is just the energy-momentum flux for an observer to measure at spatial infinity.

For $\sqrt{-g_2}T^\mu_{\nu(\mathrm{H})}(r)$, as mentioned above, it has gauge and gravitational anomalies, therefore, it satisfies the anomalous conservation equation [7, 8, 45, 46]

$$\begin{aligned}\partial_r\left[\sqrt{-g_2}T^r_{t(\mathrm{H})}(r)\right] &= \sqrt{-g_2}J^r_{\mathrm{H}}(r)\partial_r A_t(r) + A_t(r)\partial_r \\ &\quad\times\left[\sqrt{-g_2}J^r_{(\mathrm{H})}(r)\right] + \partial_r N^r_t(r)\,,\end{aligned} \tag{34}$$

where the first term comes from the U(1) gauge currents, the second term comes from the U(1) gauge anomaly, the third term comes from the gravitational anomaly of the consistent energy-momentum tensor. For a general two-dimensional non-Schwarzschild-type spherically symmetric metric

$$ds^2 = -f(r)dt^2 + g^{-1}(r)dr^2\,, \tag{35}$$

the quantity $N^r_t(r)$ is given by [7, 8]

$$N^r_t(r) = \frac{1}{192\pi}\left[f''(r)g(r) + f'(r)g'(r)\right]\,; \tag{36}$$

therefore, for metric (17), we have

$$\begin{aligned}N^r_t(r) &= \frac{1}{192\pi}\left[\left(\frac{A(r)}{B^{d/(D-2)}(r)}\right)\left(\frac{A(r)}{B^{\tilde{d}/(D-2)}(r)}\right)''\right. \\ &\quad\left.+\left(\frac{A(r)}{B^{d/(D-2)}(r)}\right)'\left(\frac{A(r)}{B^{\tilde{d}/(D-2)}(r)}\right)'\right]\,.\end{aligned} \tag{37}$$

From (22) and (24), we have

$$J^r_{\mathrm{H}}(r) = c_{\mathrm{o}} + \frac{1}{4\pi} A_t(r)\,. \tag{38}$$

The integration of (34) gives

$$\sqrt{-g_2}\, T^r_{t(\mathrm{H})}(r) = a_{\mathrm{H}} + \int_{r_{\mathrm{H}}}^{r} dr \partial_r \left(c_{\mathrm{o}} A_t(r) + \frac{1}{2\pi} A_t^2(r) + N^r_t(r) \right), \tag{39}$$

where a_{H} is an integration constant.

On the other hand, because the anomaly of energy-momentum tensor is a pure quantum field effect, it does not affect the general covariance of the effective action. To perform an infinitesimal coordinate transformation for the two-dimensional field theory along the time direction with parameter ξ^t, we have [7,8,16]

$$\begin{aligned}
\delta W &= -\int d^2x \sqrt{-g_2}\, \xi^t \nabla_\mu T^\mu_t \\
&= -\int d^2x\, \xi^t \Bigg\{ c_{\mathrm{o}} \partial_r A_t(r) + \partial_r \left[\frac{1}{4\pi} A_t^2(r) H(r) + N^r_t(r) H(r) \right] \\
&\qquad + \left[\sqrt{-g_2} \left(T^r_{t(\mathrm{o})}(r) - T^r_{t(\mathrm{H})}(r) \right) + \frac{1}{4\pi} A_t^2(r) + N^r_t(r) \right] \delta(r - r_+ - \epsilon) \Bigg\}\,.
\end{aligned} \tag{40}$$

In (40), the first term comes from the classical current. The second term can be cancelled by the quantum effect of the ingoing modes near horizon. Thus, general covariance of the effective action leads to the vanishing of the coefficient of the δ-function. To combine with (33) and (39), we obtain

$$a_{\mathrm{o}} = a_{\mathrm{H}} + \frac{1}{4\pi} A_t^2(r_{\mathrm{H}}) - N^r_t(r_{\mathrm{H}})\,. \tag{41}$$

According to (37), and because on the horizon $A(r_{\mathrm{H}}) = 0$, we have

$$N^r_t(r_{\mathrm{H}}) = \frac{1}{192\pi} \left[\left(\frac{A(r)}{B^{d/(D-2)}(r)} \right)' \left(\frac{A(r)}{B^{\tilde{d}/(D-2)}(r)} \right)' \right] \Bigg|_{r=r_{\mathrm{H}}}\,. \tag{42}$$

In order to determine the constant a_{H} of (41), we need to introduce the covariantly anomalous energy-momentum tensor $\widetilde{T}_{\mu\nu}$ which satisfies the covariant anomaly equation [46]

$$\nabla^\mu \widetilde{T}_{\mu\nu} = \frac{1}{96\pi\sqrt{-g_2}} \epsilon_{\mu\nu} \partial^\mu R\,, \tag{43}$$

where R is the Ricci scalar. For the component $\widetilde{T}^r_t(r)$ which is needed in the following calculation, for two-dimensional metric (35), to solve (43), we obtain

$$\sqrt{-g_2}\widetilde{T}^r_t(r) = \sqrt{-g_2}T^r_t(r) + \frac{1}{192\pi}\left[f''(r)g(r) - 2f'(r)g'(r)\right] . \tag{44}$$

Thus, for two-dimensional metric (17), we have

$$\begin{aligned}\sqrt{-g_2}\widetilde{T}^r_t(r) &= \sqrt{-g_2}T^r_t(r) + \frac{1}{192\pi}\left[\left(\frac{A(r)}{B^{d/(D-2)}(r)}\right)\left(\frac{A(r)}{B^{\tilde{d}/(D-2)}(r)}\right)''\right. \\ &\quad \left. -2\left(\frac{A(r)}{B^{d/(D-2)}(r)}\right)'\left(\frac{A(r)}{B^{\tilde{d}/(D-2)}(r)}\right)'\right] .\end{aligned} \tag{45}$$

Like that in [7,8,9], we add the boundary condition

$$\sqrt{-g_2}\widetilde{T}^r_t(r_{\rm H}) = 0 \tag{46}$$

to $\sqrt{-g_2}\widetilde{T}^r_t(r)$, because such a boundary condition makes physical quantities regular on the future horizon for a free falling observer. The combination of (45), (46) and (30) yields

$$\begin{aligned}\sqrt{-g_2}T^r_{t(\rm H)}(r_{\rm H}) &= \frac{1}{192\pi}\left[2\left(\frac{A(r)}{B^{d/(D-2)}(r)}\right)'\left(\frac{A(r)}{B^{\tilde{d}/(D-2)}(r)}\right)'\right. \\ &\quad \left.\left. -\left(\frac{A(r)}{B^{d/(D-2)}(r)}\right)\left(\frac{A(r)}{B^{\tilde{d}/(D-2)}(r)}\right)''\right]\right|_{r=r_{\rm H}} .\end{aligned} \tag{47}$$

Because on the horizon $A(r_{\rm H}) = 0$, we obtain

$$\sqrt{-g_2}T^r_{t(\rm H)}(r_{\rm H}) = \frac{1}{96\pi}\left.\left[\left(\frac{A(r)}{B^{d/(D-2)}(r)}\right)'\left(\frac{A(r)}{B^{\tilde{d}/(D-2)}(r)}\right)'\right]\right|_{r=r_{\rm H}} . \tag{48}$$

The combination of (48) and (39) yields

$$a_{\rm H} = \frac{1}{96\pi}\left.\left[\left(\frac{A(r)}{B^{d/(D-2)}(r)}\right)'\left(\frac{A(r)}{B^{\tilde{d}/(D-2)}(r)}\right)'\right]\right|_{r=r_{\rm H}} . \tag{49}$$

To substitute (49) and (42) into (41) and make use of (18) and (7), we obtain

$$a_{\rm o} = \frac{1}{4\pi}\left(\sum_{i=1}^{N} m_i\Omega_i\right)^2 + \frac{1}{192\pi}\left[\left(\frac{A(r)}{B^{d/(D-2)}(r)}\right)'\left(\frac{A(r)}{B^{\tilde{d}/(D-2)}(r)}\right)'\right]\Bigg|_{r=r_{\rm H}} . \tag{50}$$

Thus we have obtained the energy-momentum flux of the two-dimensional black hole (17). The first term of (50) is due to the radiation of the U(1) gauge currents of the two-dimensional black hole (17).

We can write (50) in the form

$$a_{\rm o} = \frac{1}{4\pi}\left(\sum_{i=1}^{N} m_i\Omega_i\right)^2 + \frac{\pi}{12}T_{\rm H}^2 , \tag{51}$$

where $T_{\rm H}$ is the presumed Hawking temperature of metric (17). To compare (51) with (50) and because $A(r_{\rm H}) = 0$, we obtain

$$T_{\rm H} = \frac{1}{4\pi}\frac{A'(r)}{B^{1/2}(r)}\Bigg|_{r=r_{\rm H}} . \tag{52}$$

Equation (52) is the Hawking temperature of metric (17) obtained from the method of anomaly cancellation. For the original rotating D-brane metric (2), from the mode decomposition of scalar field in terms of (12), we can deduce that the distribution of the spectrum does not change. In (29), because m_i are just the N angular momentum quantum numbers of the partial waves of the rotating D-brane, (29) are just the angular momentum fluxes of the rotating D-brane. Therefore, we can deduce that the energy-momentum flux of the rotating D-brane is just given by (51), the Hawking temperature of the rotating D-brane is given by (52) which is just equal to that of (10) obtained from the black brane thermodynamics. Thus, we have derived the Hawking temperature of the rotating D-brane from the method of anomaly cancellation.

4. Hawking temperatures of rotating *D*-branes

In this section, we give the Hawking temperatures of all of the rotating D-branes in 10-dimensional space-time in terms of metric (2) to use (52) obtained in Sec. 3, where the two functions $A(r)$ and $B(r)$ have been given by (9). Most of these results have not been given in the literature before. The obtained results are the following

$$
\begin{aligned}
T_{D0} &= \frac{1}{4\pi m r_{\mathrm{H}}^2 \cosh\alpha}\Big[7mr_{\mathrm{H}} - 3\left(l_1^2l_2^2l_3^2 + l_1^1l_2^2l_4^2 + l_1^1l_3^2l_4^2 + l_2^1l_3^2l_4^2\right) r_{\mathrm{H}}^2 \\
&\quad - 2\left(l_1^2l_2^2 + l_1^2l_3^2 + l_1^2l_4^2 + l_2^2l_3^2 + l_2^2l_4^2 + l_3^2l_4^2\right) r_{\mathrm{H}}^4 \\
&\quad - \left(l_1^2 + l_2^2 + l_3^2 + l_4^2\right) r_{\mathrm{H}}^6 - 4l_1^2l_2^2l_3^2l_4^2\Big]\,, \qquad (53)\\
T_{D1} &= \frac{1}{4\pi m r_{\mathrm{H}}^3 \cosh\alpha}\Big[3r_{\mathrm{H}}^8 + 2\left(l_1^2 + l_2^2 + l_3^2 + l_4^2\right) r_{\mathrm{H}}^6 \\
&\quad + \left(l_1^2l_2^2 + l_1^2l_3^2 + l_1^2l_4^2 + l_2^2l_3^2 + l_2^2l_4^2 + l_3^2l_4^2\right) r_{\mathrm{H}}^4 - l_1^2l_2^2l_3^2l_4^2\Big]\,, \qquad (54)\\
T_{D2} &= \frac{1}{4\pi m r_{\mathrm{H}}^2 \cosh\alpha}\Big[5mr_{\mathrm{H}} - 2\left(l_1^2l_2^2 + l_1^2l_3^2 + l_2^2l_3^2\right) r_{\mathrm{H}}^2 \\
&\quad - \left(l_1^2 + l_2^2 + l_3^2\right) r_{\mathrm{H}}^4 - 3l_1^2l_2^2l_3^2\Big]\,, \qquad (55)\\
T_{D3} &= \frac{1}{4\pi m r_{\mathrm{H}}^3 \cosh\alpha}\Big[2r_{\mathrm{H}}^6 + \left(l_1^2 + l_2^2 + l_3^2\right) r_{\mathrm{H}}^4 - l_1^2l_2^2l_3^2\Big]\,, \qquad (56)\\
T_{D4} &= \frac{1}{4\pi m r_{\mathrm{H}}^2 \cosh\alpha}\Big[3mr_{\mathrm{H}} - \left(l_1^2 + l_2^2\right) r_{\mathrm{H}}^2 - 2l_1^2l_2^2\Big]\,, \qquad (57)\\
T_{D5} &= \frac{1}{4\pi m r_{\mathrm{H}}^3 \cosh\alpha}\Big[r_{\mathrm{H}}^4 - l_1^2l_2^2\Big]\,, \qquad (58)\\
T_{D6} &= \frac{1}{4\pi m r_{\mathrm{H}}^2 \cosh\alpha}\Big[mr_{\mathrm{H}} - l^2\Big]\,, \qquad (59)\\
T_{D7} &= \frac{l^2}{4\pi m r_{\mathrm{H}}^3 \cosh\alpha}\,. \qquad (60)
\end{aligned}
$$

For a $D8$-brane, because its number of rotating parameters is zero, it is not a rotating D-brane.

5. Conclusion

There are many different approaches for the derivation of Hawking radiation of a black hole [1, 2, 3, 4, 5, 6]. Recently, a new method for the derivation of Hawking radiation has been proposed by Robinson and Wilczek *et al.* which is called anomaly cancellation [7, 8]. Using such a method, Hawking radiation of a black hole can be derived from the near horizon two-dimensional chiral field theory of a black hole. Such a method has been widely used for the Hawking radiation of many different kinds of black holes [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35]. In this paper, we have studied the Hawking radiation of rotating D-branes of superstring theories [36, 37, 38, 39] from the method of anomaly cancellation. We obtain that their reduced field theories near their horizons are two-dimensional chiral field theories in a set of curved backgrounds. Then we calculate their angular momentum fluxes and energy-momentum fluxes from the method of anomaly cancellation. We obtain that the energy-momentum fluxes of the rotating D-branes are in accordance with thermal radiations, their thermal temperatures match with their Hawking temperatures obtained from black brane thermodynamics.

If the scalar field carries Ramond–Ramond charges, then it will couple with the Ramond–Ramond gauge fields of the rotating D-branes. However, since now the explicit forms of the covariant couplings of quantum fields with Ramond–Ramond gauge fields are not clear, we do not consider such a coupling and the Ramond–Ramond charge fluxes in this paper. However, we can expect that if we include the couplings of scalar field with Ramond–Ramond gauge fields, the curved backgrounds of the reduced two-dimensional field theories of the rotating D-branes are still given by metric (17), and the final results will not change.

Appendix A

Fluxes of two-dimensional black body radiations

In this appendix, we derive the fluxes of two-dimensional black body radiations with multiple chemical potentials for the purpose of comparison. Although the calculation of this paper is carried out with respect to a scalar field, the same treatment can be carried out with respect to a fermion field to supply some additional detailed analysis. We can obtain that for the radiation of a fermion particle, its U(1) gauge currents and energy-momentum flux are still given by (29), (50) and (51). In order to avoid the problem of superradiance that is related with the rotation of a black hole, we calculate the fluxes of the radiation of a fermion particle in this appendix.

To consider a two-dimensional black body radiation with thermal temperature T_{H} and N chemical potentials $\Omega_1, \Omega_2, \ldots, \Omega_N$, the distribution of a fermion particle carrying N U(1) charges $m_1, m_2, \ldots, m_N$ is given by

$$N(\omega, m_1, \ldots, m_N) = \frac{1}{e^{(\omega - m_1\Omega_1 - \cdots - m_N\Omega_N)/T_{\mathrm{H}}} + 1}\,. \tag{A.1}$$

The N U(1) gauge currents of this two-dimensional black body radiation are given by

$$
\begin{aligned}
F_{a_i} &= m_i \int_0^\infty \frac{d\omega}{2\pi} \left(N\left(\omega, m_1, \ldots, m_N\right) - N\left(\omega, -m_1, \ldots, -m_N\right)\right) \\
&= \frac{m_i}{2\pi} \sum_{j=1}^N m_j \Omega_j \,.
\end{aligned} \tag{A.2}
$$

The energy-momentum flux of this two-dimensional black body radiation is given by

$$
\begin{aligned}
F_E &= \int_0^\infty \frac{d\omega}{2\pi} \omega \left(N\left(\omega, m_1, \ldots, m_N\right) + N\left(\omega, -m_1, \ldots, -m_N\right)\right) \\
&= \frac{1}{4\pi} \left(\sum_{i=1}^N m_i \Omega_i\right)^2 + \frac{\pi}{12} T_{\mathrm{H}}^2 \,.
\end{aligned} \tag{A.3}
$$

In the context, we have obtained that the N U(1) gauge currents and the energy-momentum flux of the two-dimensional black hole (17) are given by (29) and (51). We can see that they are just equal to (A.2) and (A.3). Therefore, we can deduce that the distribution of the Hawking radiation of a fermion particle of the two-dimensional black hole (17) is just equal to that of (A.1). And we can deduce that for the original ten-dimensional metric (2), the spectrum of its Hawking radiation is still given by (A.1). However for metric (2), $\Omega_1, \Omega_2, \ldots, \Omega_N$ are just the angular velocities of its event horizon, $m_1, m_2, \ldots, m_N$ are just the N angular momentum quantum numbers of its radiated particles; therefore, (29) are just the angular momentum fluxes and (51) is just the energy-momentum flux of the rotating D-brane.

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