# EXCLUSIVE $B \rightarrow K^{*} \ell^{+} \ell^{-}$DECAY WITH POLARIZED $K^{*}$ AND FOURTH GENERATION EFFECT 

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Polarization of $K^{*}$ in the rare $B \rightarrow K^{*} \ell^{+} \ell^{-}$decay governed by the quark level transitions $b \rightarrow s$, are investigated in the fourth quark generation model popularly known as SM4. We find that in this model due to the additional contributions coming from the heavy $t^{\prime}$ quark in the loop, the observables related to the polarization of $K^{*}$ deviate significantly from their SM values. Some of the physical observables are within the reach of LHCb experiment and search for such channels are strongly argued.

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## 1. Introduction

The flavor-changing neutral current (FCNC) processes induced by $b \rightarrow s(d)$ transitions are forbidden in SM at tree level [1, 2]. However, they can provide the most sensitive and stringiest test for the SM at one loop level. Despite the smallness of the branching ratios of FCNC decays, quite intriguing results have been obtained in ongoing experiments. The inclusive $B \rightarrow X_{s} \ell^{+} \ell^{-}$decay is observed in $\mathrm{BaBaR}[3]$ and Belle collaborations. Also, these collaborations measured exclusive modes $B \rightarrow K \ell^{+} \ell^{-}[4,5,6]$ and $B \rightarrow K^{*} \ell^{+} \ell^{-}$[7]. The experimental results on these decays are in good agreement with theoretical estimations [8, 9, 10] which can be used to constrain new physics (NP) effects.

In the past few years, we have seen some kind of deviations from the SM results in the CP violating observables of $B$ and $B_{s}$ meson decays involving $b \rightarrow s$ transitions [11, 12, 13, 14]. Several new physics scenarios are proposed in literature to account for these deviations [15]. In particular, it has been shown that SM4 can successfully explain several anomalies observed in the CP violation parameters of $B$ and $B_{s}$ mesons [15]. Therefore, it is quite
natural to expect that if there is some new physics present in the $b \rightarrow s$ transitions of $B$ meson decays, it must also affect the observables which are corresponding to the polarization of $K^{*}$ in the rare $B \rightarrow K^{*} \ell^{+} \ell^{-}$decay. Accordingly, this study can serve as a good tool to obtain an unambiguous signal of new physics.

The fourth family quarks and leptons can decay into the lighter fermions, and thus either the tree level charged-current decays or some loop induced (FCNC) decays can be affected by the existence of the 4 th generation top like quark $\left(t^{\prime}\right)$ (i.e., see $\left.[16,17,18,19,20,21,22]\right)$.

In this connection the following question can be presented. How sensitive are observables when $K^{*}$ meson is polarized longitudinally or transversally in the $B \rightarrow K^{*} \ell^{+} \ell^{-}$decay? The aim of the present work is to find an answer to this question.

The paper is organized as follows: in Sec. 2, we present the necessary theoretical background, for the $B \rightarrow K^{*} \ell^{+} \ell^{-}$decay in the SM with 4 th generations, for the longitudinal, transversal and normal polarization of leptons. In Sec. 3, we investigate the dependence of the branching ratio on 4th generation effects when $K^{*}$ meson is polarized longitudinally or transversally.

## 2. Theoretical results

With a sequential fourth generation, the Wilson coefficients $C_{7} C_{9}$ and $C_{10}$ receive contributions from the $t^{\prime}$ quark loop, which we will denote as $C_{7,9,10}^{\text {new }}$. Because the sequential fourth generation couples in a similar way to the photon and $W$, the effective Hamiltonian relevant for $b \rightarrow s \ell^{+} \ell^{-}$decay has the following form

$$
\begin{equation*}
\mathcal{H}_{\mathrm{eff}}=\frac{4 G_{\mathrm{F}}}{\sqrt{2}} V_{t b} V_{t s}^{*} \sum_{i=1}^{10} \mathcal{C}_{i}(\mu) \mathcal{O}_{i}(\mu) \tag{1}
\end{equation*}
$$

where the full set of the operators $\mathcal{O}_{i}(\mu)$ and the corresponding expressions for the Wilson coefficients $\mathcal{C}_{i}(\mu)$ in the SM are given in [23,24, 25]. As it has already been noted, the fourth generation up type quark $t^{\prime}$ is introduced in the same way as $u, c, t$ quarks introduced in the SM, so new operators do not appear and clearly the full operator set is exactly the same as in SM. The fourth generation changes the values of the Wilson coefficients $C_{7}(\mu), C_{9}(\mu)$ and $C_{10}(\mu)$, via virtual exchange of the fourth generation up type quark $t^{\prime}$. The above mentioned Wilson coefficients will explicitly change as

$$
\begin{equation*}
\lambda_{t} C_{i} \rightarrow \lambda_{t} C_{i}^{\mathrm{eff}}+\lambda_{t^{\prime}} C_{i}^{\text {new }} \tag{2}
\end{equation*}
$$

where $\lambda_{f}=V_{f b}^{*} V_{f s}$. The unitarity of the $4 \times 4$ CKM matrix leads to

$$
\begin{equation*}
\lambda_{u}+\lambda_{c}+\lambda_{t}+\lambda_{t^{\prime}}=0 \tag{3}
\end{equation*}
$$

Using Eq. (3) and ignoring the $\lambda_{u}$, Eq. (2) can be written as follows

$$
\begin{equation*}
\lambda_{t} C_{i}^{\mathrm{eff}}+\lambda_{t^{\prime}} C_{i}^{\mathrm{new}}=\lambda_{c} C_{i}^{\mathrm{eff}}+\lambda_{t^{\prime}}\left(C_{i}^{\mathrm{new}}-C_{i}^{\mathrm{eff}}\right) . \tag{4}
\end{equation*}
$$

It is clear that, for the $m_{t^{\prime}} \rightarrow m_{t}$ or $\lambda_{t^{\prime}} \rightarrow 0, \lambda_{t^{\prime}}\left(C_{i}^{\text {new }}-C_{i}^{\text {eff }}\right)$ term vanishes, as it is required by the GIM mechanism. One can also write $C_{i} \mathrm{~S}$ in the following form

$$
\begin{align*}
& C_{7}^{\mathrm{tot}}(\mu)=C_{7}^{\mathrm{eff}}(\mu)+\frac{\lambda_{t^{\prime}}}{\lambda_{t}} C_{7}^{\mathrm{new}}(\mu), \\
& C_{9}^{\mathrm{tot}}(\mu)=C_{9}^{\mathrm{eff}}(\mu)+\frac{\lambda_{t^{\prime}}}{\lambda_{t}} C_{9}^{\mathrm{new}}(\mu), \\
& C_{10}^{\mathrm{tot}}(\mu)=C_{10}^{\mathrm{eff}}(\mu)+\frac{\lambda_{t^{\prime}}}{\lambda_{t}} C_{10}^{\mathrm{new}}(\mu), \tag{5}
\end{align*}
$$

where the last terms in these expressions describe the contributions of the $t^{\prime}$ quark to the Wilson coefficients. $\lambda_{t^{\prime}}$ can be parametrized as

$$
\begin{equation*}
\lambda_{t^{\prime}}=V_{t^{\prime} b}^{*} V_{t^{\prime} s}=r_{s b} e^{i \phi_{s b}} \tag{6}
\end{equation*}
$$

In deriving Eq. (5) we factored out the term $V_{t b}^{*} V_{t s}$ in the effective Hamiltonian given in Eq. (1). The explicit forms of the $C_{i}^{\text {new }}$ can easily be obtained from the corresponding expression of the Wilson coefficients in SM by substituting $m_{t} \rightarrow m_{t^{\prime}}$ (see [23,24]). Neglecting the terms of $O\left(m_{q}^{2} / m_{\mathrm{W}}^{2}\right)$, $q=u, d, c$, the analytic expressions for all Wilson coefficients, except $C_{9}^{\text {eff }}$, can be found in [26]. The values of $C_{7}^{\text {eff }}$ and $C_{10}^{\text {eff }}$ in leading logarithmic approximation are

$$
\begin{equation*}
C_{7}^{\mathrm{eff}}=-0.313, \quad C_{10}^{\mathrm{eff}}=-4.669 \tag{7}
\end{equation*}
$$

The effective coefficient $C_{9}^{\text {eff }}$ can be written in the following form

$$
\begin{equation*}
C_{9}^{\mathrm{eff}}=\xi_{1}+\frac{\lambda_{u}}{\lambda_{t}} \xi_{2}+Y\left(s^{\prime}\right), \tag{8}
\end{equation*}
$$

where $s^{\prime}=q^{2} / m_{b}^{2}$ and the function $Y\left(s^{\prime}\right)$ denotes the perturbative part coming from one-loop matrix elements of four quark operators [24,25]. The explicit expressions for $\xi_{1}, \xi_{2}$, and the values of $C_{i}$ in the SM can be found in $[24,25]$.

In addition to the short distance contribution, $Y_{\text {per }}\left(s^{\prime}\right)$ receives also long distance contributions, which have their origin in the real $c \bar{c}$ intermediate states, i.e., $J / \psi, \psi^{\prime}, \ldots$.. The $J / \psi$ family is introduced by the Breit-Wigner distribution for the resonances through the replacement [27, 28, 29]

$$
\begin{equation*}
Y\left(s^{\prime}\right)=Y_{\mathrm{per}}\left(s^{\prime}\right)+\frac{3 \pi}{\alpha^{2}} C^{(0)} \sum_{V_{i}=\psi_{i}} \kappa_{i} \frac{m_{V_{i}} \Gamma\left(V_{i} \rightarrow \ell^{+} \ell^{-}\right)}{m_{V_{i}}^{2}-s^{\prime} m_{b}^{2}-i m_{V_{i}} \Gamma_{V_{i}}}, \tag{9}
\end{equation*}
$$

TABLE I
The numerical values of the Wilson coefficients at $\mu=m_{b}$ scale within the SM. The corresponding numerical value of $C^{0}$ is 0.362 .

| $C_{1}$ | $C_{2}$ | $C_{3}$ | $C_{4}$ | $C_{5}$ | $C_{6}$ | $C_{7}^{\mathrm{SM}}$ | $C_{9}^{\mathrm{SM}}$ | $C_{10}^{\mathrm{SM}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -0.248 | 1.107 | 0.011 | -0.026 | 0.007 | -0.031 | -0.313 | 4.344 | -4.669 |

where $C^{(0)}=3 C_{1}+C_{2}+3 C_{3}+C_{4}+3 C_{5}+C_{6}$. The phenomenological parameters $\kappa_{i}$ can be fixed from $\mathcal{B}\left(B \rightarrow K^{*} V_{i} \rightarrow K^{*} \ell^{+} \ell^{-}\right)=\mathcal{B}(B \rightarrow$ $\left.K^{*} V_{i}\right) \mathcal{B}\left(V_{i} \rightarrow \ell^{+} \ell^{-}\right)$, where the data for the right-hand side is given in [30]. For the lowest resonances $J / \psi$ and $\psi^{\prime}$ one can use $\kappa=1.65$ and $\kappa=2.36$, respectively (see [31]).

The above effective Hamiltonian leads to following matrix element for the $b \rightarrow s \ell^{+} \ell^{-}$decay

$$
\begin{align*}
\mathcal{H}_{\mathrm{eff}}= & \frac{G_{\mathrm{F}} \alpha}{\sqrt{2} \pi} V_{t b} V_{t s}^{*}\left[C_{9}^{\mathrm{tot}} \bar{s} \gamma_{\mu} P_{\mathrm{L}} b \bar{\ell} \gamma_{\mu} \ell+C_{10}^{\mathrm{tot}} \bar{s} \gamma_{\mu} P_{\mathrm{L}} b \bar{\ell} \gamma_{\mu} \gamma_{5} \ell\right. \\
& \left.-2 C_{7}^{\mathrm{tot}} \bar{s} i \sigma_{\mu \nu} \frac{q^{\nu}}{q^{2}}\left(m_{b} P_{\mathrm{R}}+m_{s} P_{\mathrm{L}}\right) b \bar{\ell} \gamma_{\mu} \ell\right] \tag{10}
\end{align*}
$$

where $q^{2}=\left(p_{1}+p_{2}\right)^{2}$ and $p_{1}$ and $p_{2}$ are the final leptons four-momenta and the chiral projection operators $P_{\mathrm{L}}$ and $P_{\mathrm{R}}$ are defined as

$$
P_{\mathrm{L}}=\frac{1-\gamma_{5}}{2}, \quad P_{\mathrm{R}}=\frac{1+\gamma_{5}}{2}
$$

It follows from Eq. (10) that in order to calculate the decay rate width and other physical observables of the exclusive $B \rightarrow K^{*} \ell^{+} \ell^{-}$decay, the matrix elements $\left\langle K^{*}\right| \bar{s} \gamma_{\mu}\left(1-\gamma^{5}\right) b|B\rangle$ and $\left\langle K^{*}\right| \bar{s} i \sigma_{\mu \nu} q^{\nu}\left(1+\gamma^{5}\right) b|B\rangle$ have to be calculated. In other words, the exclusive $B \rightarrow K^{*} \ell^{+} \ell^{-}$decay which is described in terms of the matrix elements of the quark operators given in Eq. (10) over meson states, can be parametrized in terms of form factors. For the vector meson $K^{*}$ with polarization vector $\varepsilon_{\mu}$ semileptonic form factors of the $\mathrm{V}-\mathrm{A}$ current is defined as

$$
\begin{align*}
& \left\langle K^{*}\left(p_{K^{*}}, \varepsilon\right)\right| \bar{s} \gamma_{\mu}\left(1-\gamma^{5}\right) b\left|B\left(p_{B}\right)\right\rangle \\
& =-\epsilon_{\mu \nu \rho \sigma} \varepsilon^{* \nu} p_{K^{*}}^{\rho} q^{\sigma} \frac{2 V\left(q^{2}\right)}{m_{B}+m_{K^{*}}}-i \varepsilon_{\mu}\left(m_{B}+m_{K^{*}}\right) A_{1}\left(q^{2}\right) \\
& +i\left(p_{B}+p_{K^{*}}\right)_{\mu}\left(\varepsilon^{*} q\right) \frac{A_{2}\left(q^{2}\right)}{m_{B}+m_{K^{*}}}+i q_{\mu} \frac{2 m_{K^{*}}}{q^{2}}\left(\varepsilon^{*} q\right)\left[A_{3}\left(q^{2}\right)-A_{0}\left(q^{2}\right)\right] \tag{11}
\end{align*}
$$

where $\varepsilon$ is the polarization vector of $K^{*}$ meson and $q=p_{B}-p_{K^{*}}$ is the momentum transfer. Using the equation of motion, the form factor $A_{3}\left(q^{2}\right)$ can be written in terms of the form factors $A_{1}\left(q^{2}\right)$ and $A_{2}\left(q^{2}\right)$ as follows

$$
\begin{equation*}
A_{3}=\frac{m_{B}+m_{K^{*}}}{2 m K^{*}} A_{1}-\frac{m_{B}-m_{K^{*}}}{2 m K^{*}} A_{2} . \tag{12}
\end{equation*}
$$

In order to ensure finiteness of Eq. (12) at $q^{2}=0$, we demand that $A_{3}\left(q^{2}=0\right)$ $=A_{0}\left(q^{2}=0\right)$. The semileptonic form factors coming from the dipole operator $\sigma_{\mu \nu} q^{\nu}\left(1+\gamma^{5}\right) b$ are defined as

$$
\begin{align*}
& \left\langle K^{*}\left(p_{K^{*}}, \varepsilon\right)\right| \bar{s} i \sigma_{\mu \nu} q^{\nu}\left(1 \pm \gamma^{5}\right) b\left|B\left(p_{B}\right)\right\rangle \\
& =4 \epsilon_{\mu \nu \rho \rho} \varepsilon^{* \nu} p^{\rho} q^{\sigma} T_{1}\left(q^{2}\right) \\
& \pm 2 i\left[\varepsilon_{\mu}^{*}\left(m_{B}^{2}-m_{K^{*}}^{2}\right)-\left(p_{B}+p_{K^{*}}\right)_{\mu}\left(\varepsilon^{*} q\right)\right] T_{2}\left(q^{2}\right) \\
& \pm 2 i\left(\varepsilon^{*} q\right)\left[q_{\mu}-\left(p_{B}+p_{K^{*}}\right)_{\mu} \frac{q^{2}}{m_{B}^{2}-m_{K^{*}}^{2}}\right] T_{3}\left(q^{2}\right) . \tag{13}
\end{align*}
$$

Using the form factors, the matrix element of the $B \rightarrow K^{*} \ell^{+} \ell^{-}$decay takes the following form

$$
\begin{align*}
& \mathcal{M}=\frac{G_{\mathrm{F}} \alpha}{4 \sqrt{2} \pi} V_{t b} V^{*} t s\left\{\left[\left(C_{9}^{\mathrm{tot}}-C_{1} 0^{\mathrm{tot}}\right) \bar{\ell} \gamma_{\mu}\left(1-\gamma_{5}\right) \ell+\left(C_{9}^{\mathrm{tot}}+C_{1} 0^{\mathrm{tot}}\right) \bar{\ell} \gamma_{\mu}\left(1+\gamma_{5}\right) \ell\right]\right. \\
& \times\left[-\epsilon_{\mu \nu \rho \sigma} \varepsilon^{* \nu} p_{K^{*}}^{\rho} q^{\sigma} \frac{2 V\left(q^{2}\right)}{m_{B}+m_{K^{*}}}-i \varepsilon_{\mu}^{*}\left(m_{B}+m_{K^{*}}\right) A_{1}\left(q^{2}\right)\right. \\
& \left.+i\left(p_{B}+p_{K^{*}}\right)_{\mu}\left(\varepsilon^{*} q\right) \frac{A_{2}\left(q^{2}\right)}{m_{B}+m_{K^{*}}}+i q_{\mu} \frac{2 m_{K^{*}}}{q^{2}}\left(\varepsilon^{*} q\right)\left[A_{3}\left(q^{2}\right)-A_{0}\left(q^{2}\right)\right]\right] \\
& -4 C_{7}^{\text {tot }} \frac{m_{b}}{q^{2}}\left[4 \epsilon_{\mu \nu \rho \sigma} \varepsilon^{* \nu} p_{K^{*}}^{\rho} q^{\sigma} T_{1}\left(q^{2}\right)+2 i\left[\varepsilon_{\mu}^{*}\left(m_{B}^{2}-m_{K^{*}}^{2}\right)\right.\right. \\
& \left.\left.+\left(p_{B}+p_{K^{*}}\right)_{\mu}\left(\varepsilon^{*} q\right)\right] T_{2}\left(q^{2}\right)+2 i\left(\varepsilon^{*} q\right)\left(q_{\mu}-\left(p_{B}+p_{K^{*}}\right)_{\mu} \frac{q^{2}}{m_{B}^{2}-m_{K^{*}}^{2}}\right) T_{3}\left(q^{2}\right)\right] \bar{\ell} \gamma_{\mu} \ell \\
& -4 C_{7}^{\text {tot }} \frac{m_{s}}{q^{2}}\left[4 \epsilon_{\mu \nu \rho \sigma} \varepsilon^{* \nu} p_{K^{*}}^{\rho} q^{\sigma} T_{1}\left(q^{2}\right)-2 i\left[\varepsilon_{\mu}^{*}\left(m_{B}^{2}-m_{K^{*}}^{2}\right)+\left(p_{B}+p_{K^{*}}\right)_{\mu}\left(\varepsilon^{*} q\right)\right] T_{2}\left(q^{2}\right)\right. \\
& \left.\left.-2 i\left(\varepsilon^{*} q\right)\left(q_{\mu}-\left(p_{B}+p_{K^{*}}\right)_{\mu} \frac{q^{2}}{m_{B}^{2}-m_{K^{*}}^{2}}\right) T_{3}\left(q^{2}\right)\right] \bar{\ell} \gamma_{\mu} \ell\right\} . \tag{14}
\end{align*}
$$

From Eqs. (11), (13) and (14) it can be seen that in calculating the physical observables at hadronic level, i.e., for the $B \rightarrow K^{*} \ell^{+} \ell^{-}$decay, we face the problem of computing the form factors. This problem is related to the nonperturbative sector of QCD and it can be solved only in the framework of
a nonperturbative approach. In the present work, we choose light cone QCD sum rules method prediction for the form factors. In what follows we will use the results of the work $[32,33,34]$ in which the form factors are described by a three-parameter fit where the radiative correction up to leading twist contribution and $\mathrm{SU}(3)$-breaking effects are taken in to account. Letting
$F\left(q^{2}\right) \in\left\{V\left(q^{2}\right), A_{0}\left(q^{2}\right), A_{1}\left(q^{2}\right), A_{2}\left(q^{2}\right), A_{3}\left(q^{2}\right), T_{1}\left(q^{2}\right), T_{2}\left(q^{2}\right), T_{3}\left(q^{2}\right)\right\}$
the $q^{2}$-dependence of any of these form factors could be parametrized as [37,38]

$$
F(s)=\frac{F(0)}{1-a_{F} s+b_{F} s^{2}},
$$

where $s=\frac{q^{2}}{m_{B}^{2}}$ and the parameters $F(0), a_{F}$ and $b_{F}$ are listed in Table II for each form factors.

TABLE II
$B$ meson decay form factors in a three-parameter fit, where the radiative correction to the leading twist contribution and $\mathrm{SU}(3)$ breaking effects are taken in to account.

|  | $F(0)$ | $a_{F}$ | $b_{F}$ |
| :---: | :---: | :---: | ---: |
| $A_{1}^{B \rightarrow K^{*}}$ | $0.34 \pm 0.05$ | 0.60 | -0.023 |
| $A_{2}^{B \rightarrow K^{*}}$ | $0.28 \pm 0.04$ | 1.18 | 0.281 |
| $V^{B \rightarrow K^{*}}$ | $0.46 \pm 0.07$ | 1.55 | 0.575 |
| $T_{1}^{B \rightarrow K^{*}}$ | $0.19 \pm 0.03$ | 1.59 | 0.615 |
| $T_{2}^{B \rightarrow K^{*}}$ | $0.19 \pm 0.03$ | 0.49 | -0.241 |
| $T_{3}^{B \rightarrow K^{*}}$ | $0.13 \pm 0.02$ | 1.20 | 0.098 |

The next task to be considered is the calculation of the branching ratio of the $B \rightarrow K^{*} \ell^{+} \ell^{-}$decay, when $K^{*}$ is polarized transversally or longitudinally. From matrix element Eq. (14) it is easy to derive the invariant dilepton mass spectrum for the $B \rightarrow K^{*} \ell^{+} \ell^{-}$decay corresponding to the transversally or longitudinally polarized $K^{*}$ meson

$$
\begin{equation*}
\frac{d \Gamma_{ \pm}}{d s}=\frac{G_{\mathrm{F}}^{2} \alpha^{2}}{2^{14} \pi^{5}}\left|V_{t b} V_{t s}^{*}\right|^{2} m_{B} v \sqrt{\lambda(1, r, s)} \Delta_{ \pm} \tag{15}
\end{equation*}
$$

where

$$
\begin{equation*}
\Delta_{ \pm}=\frac{4}{3} m_{B}^{2} s\left[\left(3-v^{2}\right)\left|B \mp \sqrt{\lambda} m_{B}^{2} A\right|^{2}+2 v^{2}\left|F \mp \sqrt{\lambda} m_{B}^{2} E\right|^{2}\right] \tag{16}
\end{equation*}
$$

and

$$
\begin{equation*}
\left(\frac{d \Gamma_{0}}{d s}\right)_{0}=\frac{G_{\mathrm{F}}^{2} \alpha^{2}}{2^{14} \pi^{5}}\left|V_{t b} V_{t s}^{*}\right|^{2} m_{B} v \sqrt{\lambda(1, r, s)} \Delta_{0} \tag{17}
\end{equation*}
$$

where

$$
\begin{align*}
& \Delta_{0}=\frac{1}{r} \lambda m_{B}^{4}\left\{\frac{\lambda m_{B}^{2}}{3}\left(3-v^{2}\right)|C|^{2}-\frac{2}{3}(1-r-s)\left(3-v^{2}\right) \operatorname{Re}\left(B C^{*}\right)\right. \\
& -\frac{2}{3}\left[(1-r-s)\left(3-v^{2}\right)+3 s\left(1-v^{2}\right)\right] \operatorname{Re}\left(F G^{*}\right)-2 s\left(1-v^{2}\right) \operatorname{Re}\left(F H^{*}\right) \\
& \left.+m_{B}^{2} s^{2}\left(1-v^{2}\right)|H|^{2}+2 m_{B}^{2} s(1-r)\left(1-v^{2}\right) \operatorname{Re}\left(G H^{*}\right)\right\} \\
& +\frac{m_{B}^{2}}{3 r}\left\{(\lambda+4 r s)\left(3-v^{2}\right)|B|^{2}+\lambda m_{B}^{4}\left[\lambda\left(3-v^{2}\right)-3 s(s-2 r-2)\left(1-v^{2}\right)\right]|G|^{2}\right. \\
& \left.+\left[\lambda\left(3-v^{2}\right)+8 r s v^{2}\right]|F|^{2}\right\} . \tag{18}
\end{align*}
$$

In Eqs. (15) and (17) subscripts $\pm$ and 0 denote polarization of $K^{*}$ and unpolarized cases. Meson, $v=\sqrt{1-4 m_{\ell}^{2} /\left(m_{B}^{2} s\right)}$ is the lepton velocity, $\lambda(1, r, s)=1+r^{2}+s^{2}-2 r-2 s-2 r s, r=m_{K^{*}}^{2} / m_{B}^{2}$ and $s=q^{2} / m_{B}^{2}$. The auxiliary function in Eqs. (16) and (18) are given by

$$
\begin{align*}
A & =\frac{2 V}{m_{B}+m_{K^{*}}} C_{9}^{\mathrm{tot}}+8\left(m_{s}+m_{b}\right) \frac{T_{1}}{q^{2}} C_{7}^{\mathrm{tot}} \\
B & =2\left(m_{B}+m_{K^{*}}\right) A_{1} C_{9}^{\mathrm{tot}}-8\left(m_{s}-m_{b}\right)\left(m_{B}^{2}-m_{K^{*}}^{2}\right) \frac{T_{2}}{q^{2}} C_{7}^{\mathrm{tot}} \\
C & =\frac{2 A_{2}}{m_{B}+m_{K^{*}}} C_{9}^{\mathrm{tot}}-\frac{8\left(m_{s}-m_{b}\right)}{q^{2}}\left[T_{2}+\frac{q^{2}}{m_{B}^{2}-m_{K^{*}}^{2}} T_{3}\right] \\
D & =\frac{2 m_{K^{*}}\left(A_{3}-A_{0}\right)}{q^{2}} C_{9}^{\mathrm{tot}}+\frac{8\left(m_{s}-m_{b}\right) T_{3}}{q^{2}} C_{7}^{\mathrm{tot}} \\
E & =\frac{2 V}{m_{B}+m_{K^{*}}} C_{10}^{\mathrm{tot}} \\
F & =2\left(m_{B}+m_{K^{*}}\right) A_{1} C_{10}^{\mathrm{tot}} \\
G & =\frac{2 A_{2}}{m_{B}+m_{K^{*}}} C_{10}^{\mathrm{tot}} \\
H & =\frac{4 m_{K^{*}}\left(A_{3}-A_{0}\right)}{q^{2}} C_{10}^{\mathrm{tot}} \tag{19}
\end{align*}
$$

## 3. Numerical analysis

In this section we will study the dependence of the total branching ratio to the fourth quark mass $\left(m_{t^{\prime}}\right)$ and the product of quark mixing matrix elements $\left(V_{t^{\prime} b}^{*} V_{t^{\prime} s}=r_{s b} e^{i \phi_{s b}}\right)$ when $K^{*}$ meson is polarized. The main input parameters in the calculations are the form factors, that presented in previous section. The other input parameters used in our numerical analysis are as follows [35]:

$$
\begin{align*}
m_{B} & =5.28 \mathrm{GeV}, & & m_{K^{*}}=0.892 \mathrm{GeV},
\end{aligned} \quad \begin{aligned}
& m_{b}=4.8 \mathrm{GeV} \\
& m_{c}
\end{align*}=1.5 \mathrm{GeV}, \quad \begin{array}{lll}
m_{\rho} & =0.77 \mathrm{GeV}, & \\
m_{\tau}=1.77 \mathrm{GeV}, & m_{\mu}=0.105 \mathrm{GeV} \\
m_{e} & =0.511 \mathrm{MeV}, & \\
G_{\mathrm{F}} & =1.166 \times 10^{-5} \mathrm{GeV}^{-2}, & \\
\left.\mathrm{Ge}_{t b} V_{t s}^{*}\right|^{2}=0.0385, & \tau_{B}=1.56 \times 10^{-12} \mathrm{~s} . & \alpha^{-1}=129 \tag{20}
\end{array}
$$

For numerical evaluations we need to know the values of the new parameters of this model. We use the allowed range for the new CKM elements as $r_{s b}=(0.08 \rightarrow 1.4) \times 10^{-2}$ and $\phi_{s b}=(0 \rightarrow 90)^{\circ}$ for $m_{t^{\prime}}=400 \mathrm{GeV}$, extracted using the available observables which are mediated through $b \rightarrow s$ transitions [36, 37]. The mentioned values are also consistent with the results of CDF and DO collaborations [38,39], where they found that masses of fourth generation quarks are larger than 300 GeV .

We present our numerical results in a series of graphs. It should be noted here, that the dependency for various $\phi_{s b} \sim\left\{0^{\circ}-90^{\circ}\right\}$ is a decreasing function, but it is not strong. Then we show the results just for $\phi_{s b}=90^{\circ}$. Also, we take $r_{s b}=0.01$ and $m_{t^{\prime}}=400 \mathrm{GeV}$.

Looking at these figures the following consequences are in order:

- In Figs. 1 (a) and 1 (b) we depict the dependence of polarized branching ratio for muon channel on $q^{2}$ when $K^{*}$ meson is longitudinally $\left(B_{\mathrm{L}}=\left(\Delta_{+}-\Delta_{-}\right) / \Delta_{0}\right)$ and transversally $\left(B_{\mathrm{T}}=\left(\Delta_{+}+\Delta_{-}\right) / \Delta_{0}\right)$ polarized, respectively. From these figures we observe that the polarized branching ratio in both cases enhance one order of magnitude comparing with correspondent SM values.


Fig. 1. The dependence of the longitudinally polarized branching ratio $B_{\mathrm{L}}$ (a) and transversally polarized branching ratio $B_{\mathrm{T}}(\mathrm{b})$ in $\left(B \rightarrow K^{*} \mu^{+} \mu^{-}\right)$decay on $q^{2}$ for $r_{s b}=0.01, m_{t^{\prime}}=400 \mathrm{GeV}$ and $\phi_{s b}=90^{\circ}$.

- In Figs. 2 (a) and 2 (b) we illustrate the dependence of of polarized branching ratio for tau channel on $q^{2}$ when $K^{*}$ meson is longitudinally $\left(B_{\mathrm{L}}=\left(\Delta_{+}-\Delta_{-}\right) / \Delta_{0}\right)$ and transversally $\left(B_{\mathrm{T}}=\left(\Delta_{+}+\Delta_{-}\right) / \Delta_{0}\right)$ polarized, respectively. While sizeable discrepancy is in the $14 \leq q^{2} \leq$ $16 \mathrm{GeV}^{2}$ region for $B_{\mathrm{L}}$, the sizeable discrepancy happens for $B_{\mathrm{T}}$ in the $14 \leq q^{2} \leq 19 \mathrm{GeV}^{2}$ region.


Fig. 2. The same as in Fig. 1, but for tau lepton channel.

- Finally, in Figs. 3 (a), 3 (b) we present the dependence of another physically measurable quantity, namely the ratio of the polarized branching ratio $\mathcal{B}_{\mathrm{L}} / \mathcal{B}_{\mathrm{T}}$ on the fourth generation effects. For $\tau$ channel it can be seen a valuable deviation with respect to the SM values. For $\mu$ channel, the sizeable discrepancy can be obtained in the low dileptonic invariant mass region.


Fig. 3. The dependence of $B_{\mathrm{L}} / B_{\mathrm{T}}$ in $\left(B \rightarrow K^{*} \ell^{+} \ell^{-}\right)$decay, where $\ell=\mu, \tau$, on $q^{2}$ for $r_{s b}=0.01, m_{t^{\prime}}=400 \mathrm{GeV}$ and $\phi_{s b}=90^{\circ}$.

We eliminate $q^{2}$ dependence by performing integration over $\hat{s}$ in the allowed region, i.e., we consider the averaged polarized branching ratio. The average gained, here, over $s$ is defined as

$$
\left\langle\mathcal{P}_{i}\right\rangle=\frac{\int_{4 \hat{m}_{\ell}^{2}}^{(1-\sqrt{r})^{2}} \mathcal{P}_{i} \frac{d \mathcal{B}}{d s} d s}{\int_{4 \hat{m}_{\ell}^{2}}^{(1-\sqrt{r})^{2}} \frac{d \mathcal{B}}{d s} d s},
$$

where $\hat{m}_{\ell}=m_{\ell} / m_{B}$.
Our quantitative analysis indicate that for $\mu$ channel the $\left\langle\mathcal{P}_{i}\right\rangle$ are less sensitive to the 4 th generation parameters; i.e., the maximum deviation from the SM3 are $\sim 1 \%$. We do not present those dependencies on 4 th generation parameters with relevant figures. We present our analysis for $\tau$ channel. We find that integrated polarized branching ratio can deviate sizeable from the corresponding SM3 values (see Fig. 4).


Fig. 4. The dependence of the averaged longitudinally polarized branching ratio $B_{\mathrm{L}}$ (a) and the averaged transversally polarized branching ratio $B_{\mathrm{T}}(\mathrm{b})$ in $(B \rightarrow$ $K^{*} \tau^{+} \tau^{-}$) decay on $m_{t^{\prime}}$ for $r_{s b}=0.01$ and $\phi_{s b}=90^{\circ}$.

In conclusion, in this study we present the systematic analysis of the $B \rightarrow K^{*} \ell^{-} \ell^{+}$decay, when $K^{*}$ is longitudinally and transversally polarized by using the SM with four generations of quarks. The sensitivity of the branching ratio, when $K^{*}$ meson is polarized, on the new parameters that come out of fourth generations are studied. We find out that the above mentioned physical observable depicts a strong dependence on the fourth generation parameters in the experimentally allowed regions where quark mass is $\left(m_{t^{\prime}} \simeq 400 \mathrm{GeV}\right)$. Likely, the product of quark mixing matrix elements can be estimated as $\left(V_{t^{\prime} b}^{*} V_{t^{\prime} s}=0.01 e^{i 90^{\circ}}\right)$. We find that the study of
these readily measurable quantities especially for both $(\mu, \tau)$ cases can serve as a good tool to look for physics beyond the SM. More precisely, the results can be used as a good tool for indirect search for the fourth generation of quarks.

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