SPECTRUM OF STRANGE AND NONSTRANGE BARYONS BY USING GENERALIZED GÜRSEY RADICATI MASS FORMULA AND HYPERCENTRAL POTENTIAL

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(Received December 20, 2010; revised version received February 14, 2011)

In this work, we study the spin and flavor dependent SU(6) violations in the strange and nonstrange baryons spectrum using a simple approach based on the Gürsey Radicati mass formula (GR). The average energy value of each SU(6) multiplet is described using the SU(6) invariant interaction given by a hypercentral potential. In this paper the hypercentral potential is regarded as a combination of the Coulombic-like term plus a linear confining term and we have added the harmonic oscillator potential. In fact, we have built up a potential scheme for the internal baryon structure which has three-body forces among three quarks. The results of our model (the combination of our proposed hypercentral Potential and generalized GR mass formula to description of the spectrum) show that the strange and nonstrange baryons spectrum are, in general, fairly well reproduced. The overall good description of the spectrum which we obtain shows that our model can also be used to give a fair description of the energies of the excited multiplets at least up to 2 GeV and negative-parity resonance. Moreover, we have shown that our model reproduces the position of the Roper resonances of the nucleon.

DOI:10.5506/APhysPolB.42.1247
PACS numbers: 12.39.Pn, 21.60.Fw

1. Introduction

Constituent Quark Models (CQMs) have been recently widely applied to the description of baryon properties [1, 2, 3, 4] and most attention has been devoted to the spectrum [5, 6, 7]. The baryon spectrum is usually described well, although the various models are quite different. Common to these models is the fact that the three quark interaction can be divided

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in two parts: the first one, containing the confinement interaction, is spin and flavor independent which is therefore SU(6) invariant, while the second violates the SU(6) symmetry [8, 9, 10]. One of the most popular ways to violate the SU(6) invariance was the introduction of a hyperfine (spin–spin) interaction [11, 12], however in many studies a spin and isospin [1, 13, 14] or a spin and flavor dependent interaction [1, 13] has been considered. It is well known that the Gürsey Radicati mass formula [15] describes quite well the way SU(6) symmetry is broken, at least in the lower part of the baryon spectrum. In this paper we applied the generalized Gürsey Radicati (GR) mass formula which is presented by Giannini *et al.* [16] to obtain the best description of the strange and nonstrange baryons spectrum. The model we used is a simple CQM, where the SU(6) invariant part of the Hamiltonian is the same as in the hypercentral Constituent Quark Model (hCQM) [17, 18] and where the SU(6) symmetry is broken by a generalized GR mass formula. The main point in our model is that not only the confining potential is characterized by the presence of a long range confinement part but also by a short-range potential, which is a Coulombic one, depending on the color charge.

In Sec. 2 we briefly remind the hypercentral Constituent Quark Model and introduce the interaction potentials between three quark in baryons. In Sec. 3 we present the exact solution of the radial Schrödinger equation for our proposed potential. In Sec. 4, in order to describe the splitting within the SU(6) multiplets, we introduce the Gürsey Radicati mass formula and generalized GR mass formula in the hCQM, then we give the results obtained by fitting the generalized GR mass formula parameters to the strange and nonstrange baryons energies and we compare the spectrum with the experimental data. Finally, in Sec. 5 there are some discussions and conclusions.

2. The hypercentral potential

We consider baryons as bound states of three quarks. After removing the center of mass coordinate R, the internal quark motion is described by the Jacobi coordinates, ρ and λ:

$$\vec{\rho} = \frac{1}{\sqrt{2}}\left(\vec{r_1} - \vec{r_2}\right) , \qquad \vec{\lambda} = \frac{1}{\sqrt{6}}\left(\vec{r_1} + \vec{r_2} - 2\vec{r_3}\right) \tag{1}$$

or, equivalently, ρ, Ω_ρ, λ, Ω_λ. Such that

$$m_\rho = \frac{2m_1 m_2}{m_1 + m_2} , \qquad m_\lambda = \frac{3m_3(m_1 + m_2)}{2(m_1 + m_2 + m_3)} . \tag{2}$$

Here m_1, m_2 and m_3 are the constituent quark masses.

In order to describe three-quark dynamics, it is convenient to introduce the hyperspherical coordinates, which are obtained by substituting the absolute values ρ and λ by:

$$x = \sqrt{\rho^2 + \lambda^2}\,, \qquad \xi = \arctan\left(\frac{\rho}{\lambda}\right)\,, \tag{3}$$

where x is the hyperradius and ξ the hyperangle. The quark dynamics has a dominant SU(6) invariant part, which accounts for the average multiplet energies. In the Hypercentral Constituent Quark Model it is assumed to be given by the hypercentral potential. The potentials could be of any confining form (*e.g.* linear, log, power law, *etc.*). In many practical applications a harmonic oscillator potential produces spectra not much different from those found from potentials such as Coulombic plus linear [6, 19]. Since harmonic oscillator (h.o.) models have nice mathematical properties, they have often been employed as the confining potential. Isgur and Karl [6] have used such two-body confinement. On the other side, the Coulombic term alone is not sufficient because it would allow free quarks to ionize from the system. In our model there are three hypercentral interacting potentials. First, the six-dimensional hyper-Coulomb potential [20] which is attractive for small separations

$$V_{\mathrm{hyc}}(x) = -\tau x^{-1} \tag{4}$$

while at large separations a hyper-linear term gives rise to quark confinement [13]:

$$V_{\mathrm{con}}(x) = \kappa x\,. \tag{5}$$

From Eqs. (4) and (5), the interaction potential can be taken as Coulomb term plus confining term ($\kappa x - \frac{\tau}{x}$) as suggested by the lattice QCD calculations [6, 19]. In this paper, we have added the six dimensions harmonic oscillator potential, which has a two-body character, and turns out to be exactly hypercentral since

$$V_{\mathrm{h.o.}} = \sum_{i<j}^{i=3} \tfrac{1}{2} k (r_i - r_j)^2 = \tfrac{3}{2} k x^2 = \eta x^2\,. \tag{6}$$

Here, the interaction potential is assumed as follows (from Eqs. (4)–(6))

$$V(x) = \eta x^2 + \kappa x - \tau x^{-1}\,, \tag{7}$$

where η, κ and τ are constants. The quark potential V, is supposed to depend on the hyperradius x only, that is to be hypercentral. Therefore, $V = V(x)$ is in general a three-body potential, since the hyperradius x depends on the coordinates of all three quarks. This potential has interesting properties since it can be solved analytically, with a good correspondence to physical results. First, we have solved the Schrödinger equation exactly and find eigenvalue and eigenfunction of the potential then by using the generalized GR mass formula we can try to find the baryons spectrum.

3. Exact analytical solution of the Schrödinger radial equation for the hypercentral potential

For hypercentral potentials, the Schrödinger equation, in the hyperspherical coordinates, is simply reduced to a single hyperradial equation, while the angular and hyperangular parts of the $3q$-states are the known hyperspherical harmonics [21].

Therefore, the Hamiltonian will be

$$H = \frac{P_\rho^2}{2m} + \frac{P_\lambda^2}{2m} + V(x) \tag{8}$$

and the hyperradial wave function $\psi_{\nu\gamma}(x)$ is determined by the hypercentral Schrödinger equation

$$\left(\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{\gamma(\gamma+4)}{x^2}\right)\psi_{\nu\gamma}(x) = -2m[E - V(x)]\psi_{\nu\gamma}(x)\,, \tag{9}$$

where γ is the grand angular quantum number and given by $\gamma = 2n + l_\rho + l_\lambda$, $n = 0, 1, \ldots$; l_ρ and l_λ are the angular momenta associated with the $\vec{\rho}$ and $\vec{\lambda}$ variables and ν denotes the number of nodes of the space three-quark wave functions. In equation (9) m is the reduced mass [22] which is defined as

$$m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda}\,. \tag{10}$$

Now, we want to solve the hyperradial Schrödinger equation for the three-body potential interaction (7). The transformation

$$\psi_{\nu\gamma}(x) = x^{-\frac{5}{2}}\varphi_{\nu\gamma}(x) \tag{11}$$

reduces Eq. (9) to the form

$$\varphi''_{\nu\gamma}(x) + \left[\varepsilon - \eta_1 x^2 - \kappa_1 x + \frac{\tau_1}{x} - \frac{(2\gamma+3)(2\gamma+5)}{4x^2}\right]\varphi_{\nu\gamma}(x) = 0\,. \tag{12}$$

The hyperradial wave function $\varphi_{\nu\gamma}(x)$ is a solution of the reduced Schrödinger equation for each of the three identical particles with the mass m and interacting potential (7), where

$$\varepsilon = 2mE\,, \qquad \eta_1 = 2m\eta\,, \qquad \kappa_1 = 2m\kappa\,, \qquad \tau_1 = 2m\tau\,. \tag{13}$$

We suppose the following form for the wave function

$$\varphi_{\nu\gamma}(x) = h(x)e^{g(x)} \,. \tag{14}$$

Now, for the functions $h(x)$ and $g(x)$ we make use of the ansatz [23, 24, 25]:

$$\begin{aligned} h(x) &= \begin{cases} \prod\limits_{i=1}^{\upsilon}(x - a_i^{\nu}) & \nu = 1, 2, \ldots n \,, \\ 1 & \nu = 0 \,, \end{cases} \\ g(x) &= -\tfrac{1}{2}\alpha x^2 - \beta x + \delta \ln x \,, \end{aligned} \tag{15}$$

where α and β are positive. From Eq. (14) we obtain

$$\varphi^{''}(x) = (g^{''}(x) + g'^2(x) + \frac{h^{''}(x) + 2g'(x)h'(x)}{h(x)})\varphi(x) \,. \tag{16}$$

Comparing Eqs. (12) and (16), it can be found that

$$\begin{aligned} &\left[\eta_1 x^2 + \kappa_1 x - \tau_1 x^{-1} + \frac{(2\gamma+3)(2\gamma+5)}{4x^2} - \varepsilon\right] \\ &= \frac{h^{''}(x) + 2g'(x)h'(x)}{h(x)} + g^{''}(x) + g'^2(x) \,. \end{aligned} \tag{17}$$

By substituting Eq. (15) into Eq. (17) we obtained the following equation

$$\begin{aligned} &-\varepsilon + \eta_1 x^2 + \kappa_1 x - \tau_1 x^{-1} + \frac{(2\gamma+3)(2\gamma+5)}{4x^2} \\ &= \alpha^2\beta^2 - 2\alpha\beta x - \alpha(1+2\delta) + \beta^2 - \frac{2\beta\delta}{x} + \frac{\delta(\delta-1)}{x^2} \,. \end{aligned} \tag{18}$$

By equating the corresponding powers of x on both sides of Eq. (18), we can obtain

$$\begin{aligned} \alpha &= \sqrt{\eta_1} \,, \qquad \beta = \frac{\kappa_1}{2\sqrt{\eta_1}} \,, \qquad \tau_1 = 2\beta\delta \,, \qquad \delta = \gamma + \tfrac{5}{2} \,, \\ \delta &= -\gamma - \tfrac{3}{2} \,, \qquad \varepsilon = \alpha(1+2\delta) - \beta^2 \,. \end{aligned} \tag{19}$$

Since $\eta = \frac{m\omega^2}{2}$, we have:

$$\alpha = \sqrt{2m\eta} = m\omega \,, \qquad \beta = \frac{\kappa}{\omega} = \frac{2m\tau}{(2\gamma+5)} \,. \tag{20}$$

We have taken δ from Eq. (19) as $\delta = \gamma + 5/2$ for the well behaved solution at the origin and infinity. By this selection and from Eqs. (13) and (19), we can find a restriction on the coefficient of potential parameters κ and τ as follows

$$\tau = \frac{\kappa}{m\omega}\left(\gamma + \tfrac{5}{2}\right) . \tag{21}$$

The energy eigenvalues for the mode $\nu = 0$ and grand angular momentum γ from Eqs. (13) and (19) are given as follows

$$E_{0\gamma} = (2\gamma + 6)\frac{\omega}{2} - \frac{2m\tau^2}{(2\gamma+5)^2} . \tag{22}$$

By using Eq. (19) for α and β, then from Eqs. (14), (15) and (19), the ground state normalized eigenfunctions are given as

$$\psi_{0\gamma} = N_\gamma x^{-\frac{5}{2}} \varphi_{0\gamma} = N_\gamma x^\gamma \exp\left(-\frac{m\omega}{2}x^2 - \frac{2m\tau}{(2\gamma+5)}x\right) . \tag{23}$$

In a similar manner we can continue for other modes ($\nu = 1, 2, 3, \ldots$).

4. Mass spectrum of baryons resonances and the Gürsey Radicati mass formula

The description of the strange and nonstrange baryons spectrum obtained by the hypercentral Constituent Quark Model (hCQM) [11] is fairly good and comparable to the results of other approaches, but in some cases the splitting within the various SU(6) multiplets are too low and not all adequately described by the hyperfine interaction. This is particularly true for the Roper resonances. The preceding results [14, 17, 26] show that both spin and isospin dependent terms in the quark Hamiltonian are important. Description of the splitting within the SU(6) baryon multiplets is provided by the Gürsey Radicati mass formula [15]

$$M = M_0 + CS(S+1) + DY + E\left[T(T+1) - \tfrac{1}{4}Y^2\right] , \tag{24}$$

where M_0 is the average energy value of the SU(6) multiplet, S is the total spin, Y is the hypercharge and T is the total isospin of the baryon. Eq. (24) can be rewritten in terms of Casimir operators in the following way

$$\begin{aligned} M &= M_0 + CC_2[\mathrm{SU}_S(2)] + DC_1[\mathrm{U}_Y(1)] \\ &\quad + E\left[C_2[\mathrm{SU}_I(2)] - \tfrac{1}{4}(C_1[\mathrm{U}_Y(1)])^2\right] , \end{aligned} \tag{25}$$

where $C_2[\mathrm{SU}_S(2)]$ and $C_2[\mathrm{SU}_I(2)]$ are the SU(2) (quadratic) Casimir operators for spin and isospin, respectively, and $C_1[\mathrm{U}_Y(1)]$ is the Casimir for the U(1) subgroup generated by the hypercharge Y. This mass formula has been tested to be successful in the description of the ground state baryon masses, however, as stated by the authors themselves, Eq. (25) is not the most general mass formula that can be written on the basis of a broken SU(6) symmetry.

In order to generalize Eq. (25), Giannini *et al.* considered a dynamical spin-flavor symmetry $\mathrm{SU}_{SF}(6)$ [16] and described the $\mathrm{SU}_{SF}(6)$ symmetry breaking mechanism by generalizing Eq. (25) as

$$\begin{aligned} M &= M_0 + AC_2[\mathrm{SU}_{SF}(6)] + BC_2[\mathrm{SU}_F(3)] + CC_2[\mathrm{SU}_S(2)] \\ &\quad + DC_1[\mathrm{U}_Y(1)] + E\left[C_2[\mathrm{SU}_I(2)] - \tfrac{1}{4}(C_1[\mathrm{U}_Y(1)])^2\right] . \end{aligned} \tag{26}$$

In Eq. (26) The spin term represents spin–spin interactions, the flavor term denotes the flavor dependence of the interactions, and the $\mathrm{SU}_{SF}(6)$ term depends on the permutation symmetry of the wave functions, represents signature-dependent interactions. The signature-dependent (or exchange) interactions were extensively investigated years ago within the framework of Regge theory [27]. The last two terms represent the isospin and hypercharge dependence of the masses.

The generalized Gürsey Radicati mass formula Eq. (25) can be used to describe the strange and nonstrange baryons spectrum, provided that two conditions are fulfilled. The first condition is the feasibility of using the same splitting coefficients for different SU(6) multiplets. This seems actually to be the case, as shown by the algebraic approach to the baryon spectrum, where a formula similar to Eq. (25) has been applied. The second condition is given by the feasibility of getting reliable values for the unperturbed mass values M_0 [16]. For this goal we regarded the SU(6) invariant part of the hCQM, which provides a good description of the baryons spectrum and used the Gürsey Radicati inspired SU(6) breaking interaction to describe the splitting within each SU(6) multiplet.

Therefore, the nonstrange baryons masses are obtained by three quark masses and the eigenenergies ($E_{\nu\gamma}$) of the radial Schrödinger equation with the expectation values of H_{GR} as follows

$$M = 3m_q + E_{\nu\gamma} + \langle H_{\mathrm{GR}} \rangle \,, \tag{27}$$

where m_q is the constituent quarks mass. It must be noticed that, in order to simplify the solving procedure, the constituent quarks masses are assumed to be the same for all the quark flavors ($m_u = m_d = m_s = m_q$). Therefore, within this approximation, the SU(3) symmetry is only broken dynamically by the spin and flavor dependent terms in the Hamiltonian. In previous section we determined eigenenergies ($E_{\nu\gamma}$) by exact solution of the Schrödinger radial equation for the hypercentral Potential (7). In the above equation

$H_{\rm GR}$ is in the following form

$$\begin{aligned} H_{\rm GR} &= AC_2[{\rm SU}_{SF}(6)] + BC_2[{\rm SU}_F(3)] + CC_2[{\rm SU}_S(2)] + DC_1[{\rm U}_Y(1)] \\ &\quad +E\left[C_2[{\rm SU}_I(2)] - \tfrac{1}{4}(C_1[{\rm U}_Y(1)])^2\right] . \end{aligned} \tag{28}$$

The expectation values of $H_{\rm GR}$, ($\langle H_{\rm GR}\rangle$), is completely identified by the expectation values of the Casimir operators [28]:

$$\begin{aligned} \langle C_2[{\rm SU}_{SF}(6)]\rangle &= \begin{cases} 45/4 & \text{for} \quad [56]\,, \\ 33/4 & \text{for} \quad [70]\,, \\ 21/4 & \text{for} \quad [20]\,, \end{cases} \\ \langle C_2[{\rm SU}_F(3)]\rangle &= \begin{cases} 3 & \text{for} \quad [8]\,, \\ 6 & \text{for} \quad [10]\,, \\ 0 & \text{for} \quad [1]\,, \end{cases} \\ \langle C_2[{\rm SU}_I(2)]\rangle &= T(T+1)\,, \\ \langle C_1[{\rm U}_Y(1)]\rangle &= Y\,, \\ \langle C_2[{\rm SU}_S(2)]\rangle &= S(S+1)\,. \end{aligned} \tag{29}$$

In the algebraic description of baryon properties [1], the space part of the mass operator is written in terms of the generators of the U(7) group, while for the internal degrees of freedom the Güersey Radicati mass formula [15] is used. So in this work we do not consider interaction terms that mix the spatial and internal degrees of freedom. Therefore, the model is expected to be unsuccessful at the description of all those observables where an excellent description of the three quark wave function is crucial. For calculating the baryons mass according to Eq. (27), we need to find the unknown parameters. To specify parameters D and E we choose a limited number of well known strange and nonstrange resonances and express their mass differences using $H_{\rm GR}$ and the Casimir operator expectation values given in Eq. (29):

$$\begin{aligned} (\varSigma(1193)P11 - N(938)P11) &= \tfrac{3}{2}E - D\,, \\ (\varLambda(1116)P01 - N(938)P11) &= -D - \tfrac{1}{2}E \end{aligned} \tag{30}$$

we determined m_q, τ, ω (in Eq. (22)) and the three coefficients (A, B, C) of Eq. (28) in a simultaneous fit to the 3 and 4 star resonances of Table I which have been assigned as octet and decuplet states. The fitted parameters are reported in Table II, while the resulting spectrums are shown in Figs. 1 and 2. The corresponding numerical values are given in Table I, column $M_{\rm Our\ calc}$. In Table I, column $M_{[29]\ \rm calc}$, we reported the numerical values of the calculated masses of baryons resonances by Bijker *et al.* [29]. They used the collective U(7) model for studying the spectrum of strange and

TABLE I

Mass spectrum of baryons resonances (in MeV) calculated with the mass formula Eq.(27). The column $M_{\text{Our calc}}$ contains our calculations with the parameters of Table II and the columns $M_{[29]\text{ calc}}$ and $M_{[16]\text{ calc}}$ show calculations of Bijker and Giannini, respectively.

Baryon	Status	$\text{Mass}^{(\text{exp})}_{[30]}$	State	$M_{[29]\text{ calc}}$	$M_{[16]\text{ calc}}$	$M_{\text{Our calc}}$
$N(938)P11$	$****$	938	$^28_{1/2}[56,0^+]$	—	938.0	938.4
$N(1440)P11$	$****$	1420–1470	$^28_{1/2}[56,0^+]$	1444	1448.7	1422.5
$N(1520)D13$	$****$	1515–1525	$^28_{3/2}[70,1^-]$	1563	1543.7	1529.1
$N(1535)S11$	$****$	1525–1545	$^28_{1/2}[70,1^-]$	1563	1543.7	1529.1
$N(1650)S11$	$****$	1645–1670	$^48_{1/2}[70,1^-]$	1683	1658.6	1641.6
$N(1675)D15$	$****$	1670–1680	$^48_{5/2}[70,1^-]$	1683	1658.6	1641.6
$N(1700)D13$	$***$	1650–1750	$^48_{3/2}[70,1^-]$	1683	1658.6	1641.6
$N(1710)P11$	$***$	1680-1-740	$^28_{1/2}[70,0^+]$	1683	1795.4	1748.0
$\Delta(1232)P33$	$****$	1231–1233	$^410_{3/2}[56,0^+]$	1246	1232.0	1231.8
$\Delta(1600)P33$	$***$	1550–1700	$^410_{3/2}[56,0^+]$	1660	1683.0	1707.6
$\Delta(1620)S31$	$****$	1600–1660	$^210_{1/2}[70,1^-]$	1649	1722.8	1710.0
$\Delta(1700)D33$	$****$	1670–1750	$^210_{3/2}[70,1^-]$	1649	1722.8	1710.0
$\Delta(1905)F35$	$****$	1865–1915	$^410_{5/2}[56,2^+]$	1921	1945.4	1877.5
$\Delta(1910)P31$	$****$	1870–1920	$^410_{1/2}[56,2^+]$	1921	1945.4	1877.5
$\Delta(1920)P33$	$***$	1900–1970	$^410_{3/2}[56,0^+]$	—	2089.4	2041.4
$\Delta(1950)F37$	$****$	1915–1950	$^410_{7/2}[56,2^+]$	1921	1945.4	1877.5
$\Delta(2420)H3,11$	$****$	2300–2500	$^410_{11/2}[56,4^+]$	2414	—	2363.3
$\Lambda(1116)P01$	$****$	1116	$^28_{1/2}[56,0^+]$	1133	1116.0	1116.5
$\Lambda(1600)P01$	$***$	1560–1700	$^28_{1/2}[56,0^+]$	1577	1626.7	1592.3
$\Lambda(1670)S01$	$****$	1660–1680	$^28_{1/2}[70,1^-]$	1686	1721.7	1707.1
$\Lambda(1690)D03$	$****$	1685–1695	$^28_{3/2}[70,1^-]$	1686	1721.7	1707.1
$\Lambda(1800)S01$	$***$	1720–1850	$^48_{1/2}[70,1^-]$	1799	1836.6	1819.6
$\Lambda(1810)P01$	$***$	1750–1850	$^28_{1/2}[70,0^+]$	1799	1973.4	1762.1
$\Lambda(1820)F05$	$****$	1815–1825	$^28_{5/2}[56,2^+]$	1849	1829.4	1764.9
$\Lambda(1830)D05$	$****$	1810–1830	$^48_{5/2}[70,1^-]$	1799	1836.6	1819.6
$\Lambda(1890)P03$	$****$	1850–1910	$^28_{3/2}[56,2^+]$	1849	1829.4	1764.9
$\Lambda(2110)F05$	$****$	2090–2140	$^48_{5/2}[70,2^+]$	2074	1995.0	2087.6
$\Lambda^*(1405)S01$	$****$	1402–1410	$^21_{1/2}[70,1^-]$	1641	1657.5	1612.9
$\Lambda^*(1520)D01$	$****$	1518–1520	$^21_{3/2}[70,1^-]$	1641	1657.5	1612.9
$\Sigma(1193)P11$	$****$	1193	$^28_{1/2}[56,0^+]$	1170	1193.0	1193.5
$\Sigma(1660)P11$	$***$	1630–1690	$^28_{1/2}[56,0^+]$	1604	1703.7	1669.3
$\Sigma(1670)D13$	$****$	1665–1685	$^28_{3/2}[70,1^-]$	1711	1798.7	1784.1
$\Sigma(1750)S11$	$***$	1730–1800	$^28_{1/2}[70,1^-]$	1711	1798.7	1784.1
$\Sigma(1775)D15$	$****$	1770–1780	$^48_{5/2}[70,1^-]$	1822	1913.6	1896.6
$\Sigma(1915)F15$	$****$	1900–1935	$^28_{5/2}[56,2^+]$	1872	1906.4	1839.1
$\Sigma(1940)D13$	$***$	1900–1950	$^28_{3/2}[56,1^-]$	1974	1913.6	1996.6
$\Sigma^*(1385)P13$	$****$	1383–1385	$^410_{3/2}[56,0^+]$	1382	1371.6	1371.4
$\Sigma^*(2030)F17$	$****$	2025–2040	$^410_{7/2}[56,2^+]$	2012	2085.0	2061.4
$\Xi(1318)P11$	$****$	1314–1316	$^28_{1/2}[56,0^+]$	1334	1332.6	1332.0
$\Xi(1820)D13$	$***$	1818–1828	$^28_{3/2}[70,1^-]$	1828	1938.3	1923.7
$\Xi^*(1530)P13$	$****$	1531–1532	$^410_{3/2}[56,0^+]$	1524	1511.1	1511.0
$\Omega(1672)P03$	$****$	1672–1673	$^410_{3/2}[56,0^+]$	1670	1650.7	1650.5

TABLE II

The fitted values of the parameters of the Eq. (27), obtained with resonances mass differences and global fit to the experimental resonance masses [30].

Parameter	Value
m_q	360 MeV
τ	3.6
ω	0.4 fm^{-1}
A	-9.6 MeV
B	21.8 MeV
C	37.5
D	-197.3 MeV
E	38.5 MeV

nonstrange baryons and applied the following formula to give the spectrum of baryons

$$\begin{aligned} M^2 \;=\; & M_0^2 + \kappa_1\nu_1 + \kappa_2\nu_2 + \alpha L + a[2f_1(f_1+5) + 2f_2(f_2+3) + 2f_3(f_3+1) \\ & -\tfrac{1}{3}(f_1+f_2+f_3)^2 - 45] + b\left[\tfrac{3}{2}\left(g_1(g_1+2) + g_2^2 - \tfrac{1}{3}(g_1+g_2)^2\right) - 9\right] \\ & + c\left[S(S+1) - \tfrac{3}{4}\right] + d[Y-1] + e[Y^2-1] + f\left[I(I+1) - \tfrac{3}{4}\right], \end{aligned} \tag{31}$$

where the coefficient M_0^2 is determined by the nucleon mass $M_0^2 = 0.882\,\mathrm{GeV}^2$, ν_1 and ν_2 are the vibrational quantum numbers corresponding to the symmetric stretching vibration along the direction of the strings (breathing mode) and two degenerate bending vibrations of the strings and L is a linear term. Here, $[\ f_1f_2f_3\ ]$ and $[\ g_1g_2\ ]$ represent the Young tableaux [29]. Also, in Table I, column $M_{[16]\ \mathrm{calc}}$ we have shown the numerical values of the calculated masses of baryons resonances by Giannini *et al.*, where they regarded the confinement potential as the Cornell potential, that is

$$V(x) = -\frac{\tau}{x} + \alpha x\,. \tag{32}$$

The solution of the hypercentral Eq. (9) with this potential Eq. (32) cannot be obtained analytically [12], therefore Giannini *et al.* used the dynamic symmetry O(7) of the hyperCoulomb problem to obtain the hyperCoulomb Hamiltonian and eigenfunctions analytically and they regarded the linear term as a perturbation. Comparison between our results and Bijker's results show that in some cases such as $\Delta(1232)P33$, $\Delta(1700)D33$, $\Lambda(1830)D05$, $\Sigma(1193)P11$, $\Sigma(1660)P11$ (refer to Table I) our model has improved the results of [29]. Our applied model is to some extent similar to that of Giannini's *et al.* Comparison between our results ($M_{\mathrm{Our\ calc}}$) and the experimental masses [30] show that our model has certainly improved the

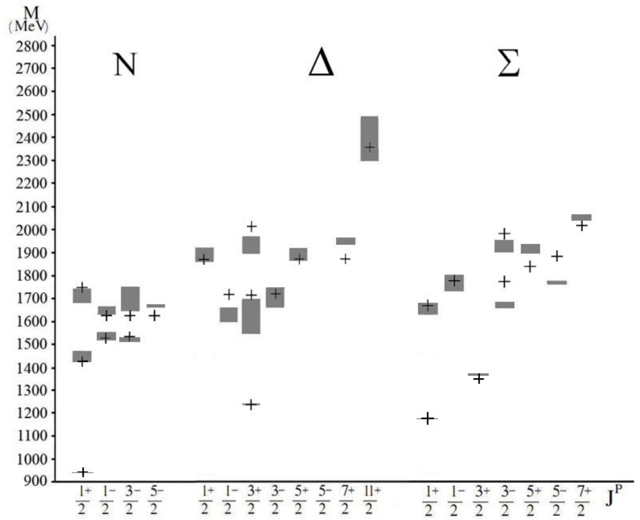

Fig. 1. Comparison between the experimental mass spectrum of three and four star N, Δ and Σ resonances [30] (gray boxes) and our calculated masses (+) which obtained with the equation (27) fixing the mass relation parameters by a fitting procedure.

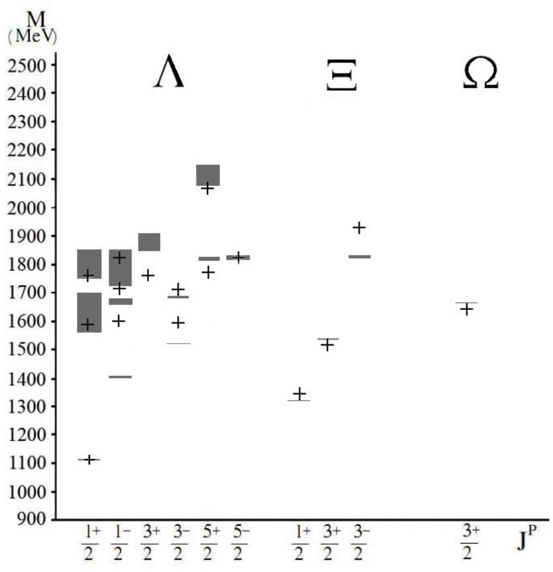

Fig. 2. Comparison between the experimental mass spectrum of three and four star Λ, Ξ and Ω resonances [30] (gray boxes) and our calculated masses (+) which obtained with the equation (27) fixing the mass relation parameters by a fitting procedure.

results of model in Ref. [17], particularly in $\Lambda(1810)P01$ (about 150 MeV), $\Lambda(2110)F05$ (about 90 MeV), $\Lambda^*(1405)S01$, $\Lambda^*(1520)D01$ (about 40 MeV) and $\Delta(1905)F35$ (about 20 MeV) (refer to Table I). These improvements in reproduction of baryons resonance masses obtained by using a suitable form for confinement potential and exact analytical solution of the radial Schrödinger equation for our proposed potential.

The authors would like to thanks the referee for his useful comments and suggestions.

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