NUCLEAR MATTER EQUATION OF STATE AND INCOMPRESSIBILITY IN THE RELATIVISTIC DENSITY DEPENDENT HADRON FIELD THEORY

S. Haddad

Physics Department, Atomic Energy Commission of Syria
P.O. Box 6091, Damascus, Syria

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Nuclear matter equation of state and incompressibility are determined utilizing the relativistic density dependent Hadron field theory. Nuclear matter is studied at symmetric ground-state and at supernova collapse conditions, and pressure density of isentropic nuclear matter is determined as a function of the density at supernova collapse conditions. The value of the ground-state nuclear matter incompressibility is within the interval determined by isoscalar giant monopole resonance measurements and relativistic calculations, and the dependency of the coupling parameters on density leads to results closer to the results of calculations used in the study of supernova explosion than the results of other relativistic calculations.

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1. Introduction

Nuclear matter incompressibility is of interest in intermediate energy heavy-ion collisions, neutron star structure, and supernova explosion calculations [1, 2]. The nuclear matter incompressibility may be deduced from measurements of the isoscalar giant monopole resonance in medium and heavy nuclei, but the resulting value turns out to be model dependent [3], and hydrodynamical calculations of supernova explosions determine ranges for the stiffness of the nuclear matter equation of state and incompressibility [4].

The relativistic Brueckner–Hartree–Fock theory (RBHF) is generally accepted as one of the most reliable and feasible microscopic methods for the description of effective interactions in the nuclear medium [5, 6]. Unfortunately, the application of the RBHF approach is highly complicated. In order to overcome the complexity of RBHF calculations, attempts have been made to use effective interactions with density dependent meson–nucleon couplings to describe the medium dependency of the nuclear interaction. Density dependent meson coupling has been deduced in Ref. [7] from RBHF calculations by reproducing the nucleon self-energy resulting from RBHF calculations of symmetric nuclear matter, utilizing the one-boson-exchange (OBE) potentials Bonn A, B, and C of Refs. [5, 6]. The parametrizations of Ref. [7] have then been used by Ref. [8] in a relativistic density dependent hadron field approach, with the result that the Bonn A parameters reproduce experimental binding energies, charge radii, and measured charge distributions of finite nuclei better than Bonn B and C parameters. The parametrization given in Ref. [7] for the RBHF Bonn A potential has been refined and extended in Refs. [9, 10] to reproduce the nucleon self-energy resulting from RBHF nuclear matter calculations in the general case of different proton and neutron densities.

The idea of utilizing density dependent meson–nucleon couplings has also been used to extend the relativistic mean field (RMF) framework in a phenomenological approach, where the density dependent couplings are adjusted to the properties of a set of spherical nuclei and nuclear matter. See, as an example, the DD-ME2 parametrization of Ref. [11]. RMF approaches have the trend of predicting a rather stiff equation of state with a high value for the incompressibility at supernova collapse conditions [12]. Nuclear equation of state and incompressibility are the basic input quantities necessary for solving the stellar structure equations [13, 14].

Nuclear matter equation of state and incompressibility are determined in this work utilizing the relativistic density dependent hadron field theory of Refs. [9, 10], where the dependency of the coupling parameters on the density is deduced by reproducing the nucleon self-energy resulting from RBHF nuclear matter calculations in the general case of different proton and neutron densities.

Section 2 introduces the nuclear matter equation of state and incompressibility. Section 3 reviews briefly the general theory of the effective nuclear interaction with density dependent coupling parameters used in the relativistic density dependent hadron field theory of Refs. [9, 10]. The nuclear matter equation of state and incompressibility are determined in Sec. 4 at the ground-state of symmetric nuclear matter, and at supernova collapse conditions, where a neutron to proton ratio of 2 stays almost constant during the collapse time and one should consider an adiabatic process at the constant value $S = 1$ for the entropy per particle in k_{B} units [14]. The pressure density as a function of the density is also determined in Sec. 4 at supernova collapse conditions. Results are compared with experimental data, results of relativistic and non-relativistic calculations, and results of hydrodynamical calculations of supernova explosions. The main conclusions are summarized in the last section.

2. Nuclear matter equation of state and incompressibility

The nuclear equation of state gives the nucleon energy e as a function of the density ρ at definite values for the asymmetry parameter β and the entropy per particle S

$$e = e(\rho)|_{\beta,S} \,, \tag{1}$$

where the density is the sum of the neutron and proton densities,

$$\rho = \rho_n + \rho_p \,, \tag{2}$$

and the asymmetry parameter is defined by

$$\beta = \frac{\rho_n - \rho_p}{\rho} \,. \tag{3}$$

The ground-state of nuclear matter is the state of symmetric nuclear matter $\beta = 0$ at zero entropy $S = 0$. The neutron density is twice the proton density at supernova collapse conditions $\rho_n = 2\rho_p$, and hence $\beta = 0.33$, and since the collapse takes less than one second, it should be considered as an adiabatic process at the constant entropy value $S = 1$, and the entropy is used instead of the temperature in describing the collapse mechanism of the supernova [12, 14].

The incompressibility describes the curvature of the nuclear equation of state at the nuclear matter saturation density under the defined conditions

$$K = 9 \left(\rho^2 \frac{\partial^2 e}{\partial \rho^2} \right) \Bigg|_{\rho_{\rm s}(\beta,S)} \,, \tag{4}$$

where the nuclear matter saturation density under the defined conditions $\rho_{\rm s}(\beta, S)$ is the nuclear matter density at which the nucleon energy takes its minimum at the defined values for the asymmetry parameter β and the entropy S, *i.e.*

$$\frac{\partial e}{\partial \rho} \Bigg|_{\rho_{\rm s}(\beta,S)} = 0 \,. \tag{5}$$

3. The effective density dependent interaction

The effective nucleon–nucleon interaction is described by the electromagnetic field between protons and the exchange of four mesons: the isoscalar scalar meson σ, the isoscalar vector meson ω, the isovector scalar meson δ, and the isovector vector meson ρ. Density dependent coupling parameters for the isoscalar mesons are introduced by

$$\frac{g_i(\rho)}{g_i(\rho_0)} - 1 = a_i \left(\exp \left[b_i \left(1 - \left(\frac{\rho}{\rho_0} \right)^{1/3} \right) \right] - 1 \right) , \qquad i = \sigma, \omega \,, \tag{6}$$

where ρ_0 is the symmetric nuclear matter saturation density and a_i, b_i, and $g_i(\rho_0)$ are the coefficients of the density dependent function $g_i(\rho)$. Density dependent coupling parameters for the isovector mesons are introduced by

$$g_i(\rho) = g_i(\rho_0) \exp\left[b_i\left(1 - \frac{\rho}{\rho_0}\right)\right], \qquad i = \delta, \rho\,, \tag{7}$$

where b_i and $g_i(\rho_0)$ are the coefficients of the density dependent function $g_i(\rho)$.

The coefficients a_i, b_i, and $g_i(\rho_0)$ $(i = \sigma, \omega)$ and b_i and $g_i(\rho_0)$ $(i = \delta, \rho)$ are adjusted to the outcome of the RBHF calculations of the nucleon self-energy in nuclear matter of Refs. [15, 16]. The coefficients of the resulting density dependent parametrization of the RBHF potential Bonn A [6] are given in Table I. The masses m_N, m_σ, m_ω, m_δ, and m_ρ and the saturation density ρ_0 are those of the Bonn A potential. See Refs. [9, 17], for instance, for a detailed description of the relativistic density dependent hadron field theory, and Refs. [12, 18] for more details on the introduction of the temperature and the calculation of energy and entropy.

TABLE I

The density dependent parameter set. m_i is the mass of the i-meson. a_i, b_i, and $g_i(\rho_0)$ are the coefficients of the parametrization of the density dependent coupling parameters $(i = \sigma, \omega, \delta, \rho)$. $m_N = 938.926$ MeV is the average nucleon mass used by Ref. [6] and $\rho_0 = 0.185$ fm^{-3} is the saturation density resulting from the RBHF potential Bonn A [6].

Meson i	σ	ω	δ	ρ
m_i (MeV)	550	782.6	983	769
$g_i(\rho_0)$	9.297	11.269	4.701	2.370
a_i	0.2941	0.3451	—	—
b_i	2.217	2.113	1.223	1.634

4. Results and discussion

Figure 1 shows the nuclear matter equation of state at ground-state conditions (GS) ($\beta = 0$, $S = 0$) and at supernova collapse conditions (SC) ($\beta = 0.33$, $S = 1$), and Table II summarizes nuclear matter saturation properties at each of these conditions, compared with the values given in Ref. [6] for the nuclear matter ground-state resulting from RBHF calculations with the Bonn A potential.

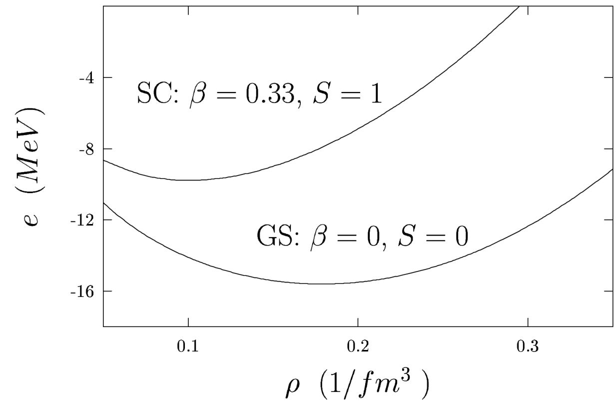

Fig. 1. Nuclear matter equation of state at ground-state conditions GS: $\beta = 0$, $S = 0$, and at supernova collapse conditions SC: $\beta = 0.33$, $S = 1$.

TABLE II

Nuclear matter saturation properties at ground-state conditions GS and at supernova collapse conditions SC resulting from this work, and ground-state nuclear matter saturation properties resulting from RBHF calculations with the Bonn A potential.

		$\rho_s(\beta, S)$ (1/fm^3)	$e(\rho_s)$ (MeV)	K (MeV)
RBHF / Bonn A	GS	0.185	-15.59	290
This work	GS	0.179	-15.60	264
	SC	0.099	-9.78	137

The value of the GS nuclear matter incompressibility, deduced from measurements of the isoscalar giant monopole resonance in medium and heavy nuclei, is model dependent. The presently available experimental data set does not limit the range of the GS incompressibility to better than 200–300 MeV [3, 19]. Theoretical investigations show clear discrepancy between the values of the GS nuclear matter incompressibility predicted by relativistic and non-relativistic models. Non-relativistic calculations predict the value in the range 210–230 MeV [20], while relativistic 250–270 MeV [21].

The flattening of the nucleon energy curve $e(\rho)$ around saturation density in the relativistic density dependent hadron field theory, as depicted in Fig. 1, compensates the effect of the, in comparison with the experimental value, larger value produced for the nuclear matter ground-state saturation density, which enters quadratically in the relation determining the incompressibility, see Eq. (4), such that the resulting value of the GS nuclear matter incompressibility lies within the ranges determined from measurements of the isoscalar giant monopole resonance and from relativistic calculations.

Table III compares the values resulting from this work for the nuclear matter incompressibility at ground-state and at supernova collapse conditions with the values determined from calculations used in supernova studies: the lowest order constrained variational approach [22] with realistic potentials like the AV_{18} potential [23] and the Δ-Reid coupled-channels model with Δ-excitation [24], the finite temperature Brueckner–Bethe–Goldstone approach with the Paris potential [25], and the hydrodynamical calculations [4].

TABLE III

Nuclear matter incompressibility values in the unit of MeV at ground-state GS and at supernova collapse conditions SC resulting from this work, and the values determined from calculations used in supernova studies.

	GS	SC
This work	264	137
AV_{18}	258	132
Δ-Reid	220	124
Paris	144	99
Hydrodynamics		90

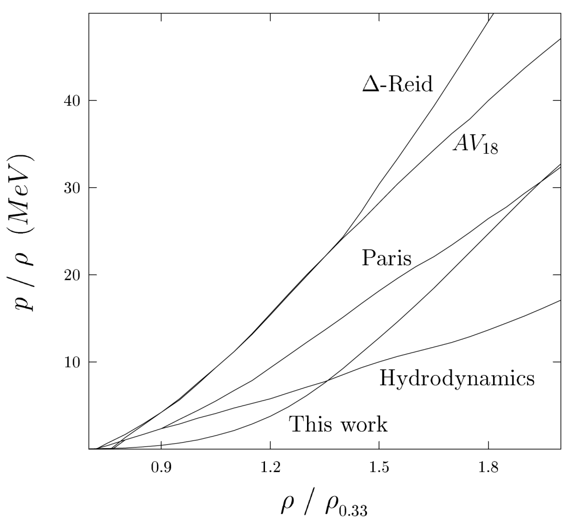

Fig. 2. Nuclear matter pressure density at supernova collapse conditions $\beta = 0.33$ and $S = 1$, as a function of the density in the units of the nuclear matter saturation density $\rho_{0.33}$ at $\beta = 0.33$. The figure compares the results of this work with the results of calculations used in supernova studies.

Figure 2 compares the nuclear matter pressure density as a function of the density at supernova collapse conditions, *i.e.*

$$p\left(\rho, \beta = 0.33, S = 1\right)/\rho = \rho \left.\frac{de}{d\rho}\right|_{\beta,S} , \tag{8}$$

with the results from calculations used in supernova studies.

It can be inferred from Table III and Fig. 2 that the results obtained for the nuclear matter incompressibility and pressure density using the relativistic density dependent hadron field theory are closer to the results of calculations used in supernova studies than the results of other relativistic models like the RMF theory, which produces values in the range of 150–175 MeV for the incompressibility at supernova collapse conditions, as summarized in Ref. [12].

5. Summary

Nuclear matter equation of state and incompressibility are determined utilizing the relativistic density dependent hadron field theory, at nuclear matter ground-state and at supernova collapse conditions. The pressure density at supernova collapse conditions is also determined as a function of the density.

The value of the ground-state nuclear matter incompressibility resulting from the relativistic density dependent hadron field theory lies within the interval determined by isoscalar giant monopole resonance measurements and relativistic calculations. The comparison with the results of calculations used in supernova studies shows that the dependency of the coupling parameters on density in the relativistic density dependent hadron field theory, deduced by reproducing the nucleon self-energy resulting from RBHF calculations, leads to results closer to the calculations used in supernova studies than the results of other relativistic models like the RMF theory.

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