ON HIGH ENERGY FACTORIZATION: THEORETICAL BASICS AND PHENOMENOLOGICAL APPLICATIONS*

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We overview some of theory and phenomenology aspects of high energy factorization. In the theory part we focus on basic equations of high energy factorization *i.e.* BFKL, CCFM, BK. In the phenomenology part we focus on forward-central jets correlations at Large Hadron Collider and on production of charged particles in HERA.

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1. Introduction

Large Hadron Collider (LHC) is already operational and Quantum Chromodynamics (QCD) is the basic theory which is used to set up the initial conditions for the collisions at the LHC but also to calculate hadronic observables. Application of perturbative QCD relies on so-called factorization theorems which allow to decompose given process into long distance part called parton density and short distance part called matrix element. Here we will focus on high energy factorization [1] (there exists also collinear factorization scheme but we will not discuss it here) which applies when both momentum scale and energy scale involved in scattering process are high. The evolution equations of high energy factorization sum up logarithms of energy accompanied by coupling constant $\alpha_s^n \ln^m s$. Depending on the energy range and observable one may use: BFKL [2,3,4], BK [5,6] or CCFM [7,8,9] evolution equation. When the energies of the collision are of the order of 10^3 GeV and one considers inclusive processes in electron–proton Deep Inelastic Scattering as for example at HERA the BFKL approximation applies.

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CCFM equation since it depends on the hardness of the probe allows additionally for studies of exclusive final states. However, if one would like to account for formation of dense system like in nuclei–nuclei collision, where partons eventually overlap the BK, equation or some nonlinear extension of CCFM has to be considered since it, apart from splittings of gluons, allows for their recombination.

The paper is organized as follows. In the following Section 2 we introduce framework of high energy factorization and basic evolution equations. In Section 3 we present phenomenological applications on two examples: production of forward-central jets at Large Hadron Collider and production of charged particles at HERA.

2. High energy factorization and evolution equations in pQCD

The high energy factorization formula while applied to jet production in hadron–hadron scattering reads

$$\begin{aligned}\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} &= \sum_{a,b,c,d} \int \frac{d^2k_1}{\pi}\frac{d^2k_2}{\pi}\frac{1}{16\pi^2\left(x_1x_2S\right)^2}\overline{|\mathcal{M}_{ab\to cd}|}^2 \\ &\times\, \delta^2\left(\vec{k}_1+\vec{k}_2-\vec{p}_{3t}-\vec{p}_{4t}\right)\mathcal{A}_{a/A}\left(x_1,k_1^2,\mu^2\right) \\ &\times \mathcal{A}_{b/B}\left(x_2,k_2^2,\mu^2\right)\frac{1}{1+\delta_{cd}}\,, \qquad (2.1)\end{aligned}$$

where $k_1 \equiv |\boldsymbol{k}_1|$, $k_2 \equiv |\boldsymbol{k}_2|$ and x_1, x_2 are longitudinal momentum fractions see Fig 1. The functions $\mathcal{A}_{a/A}(x_1,k_1^2,\mu^2)$ and $\mathcal{A}_{b/B}(x_2,k_2^2,\mu^2)$ are the unintegrated distributions which are solutions of high energy factorisable

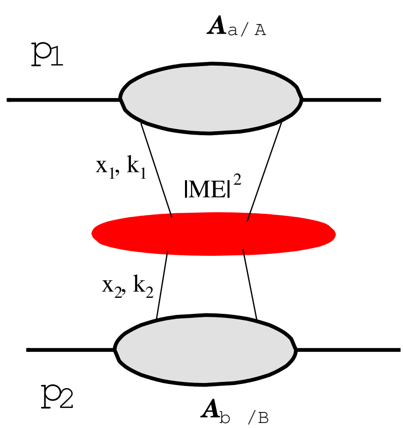

Fig. 1. Factorized structure of the cross-section.

evolution equations. They describe distributions of transversal and longitudinal momenta of partons in the incoming protons A and B respectively. This factorization formula, apart from summing up logarithms of energy, accounts also for hardness of the process. The sum is made over all flavors of initial and final partons. Similar formula can be written for DIS process.

2.1. BFKL equation

The simplest of high energy factorizable evolution equations is the BFKL equation. At leading order in $\ln 1/x$ (LLln$1/x$) it reads

$$\frac{\partial\phi\left(x,k^2\right)}{\partial\ln 1/x}=\overline{\alpha}_{\rm s}\int\limits_0^\infty dl^2\left[\frac{l^2\phi\left(x,l^2\right)-k^2\phi\left(x,k^2\right)}{|l^2-k^2|}+\frac{k^2\phi\left(x,k^2\right)}{\sqrt{4l^2+k^2}}\right], \qquad (2.2)$$

where we used standard notation $\phi(x,k^2)$ for factorization scale independent unintegrated gluon density. The real emission part of the kernel describes radiation of gluons which are strongly ordered in longitudinal momentum fraction *i.e.* they are well separated in rapidity. The virtual part removes singularity when $k=l$ and is called the Regge trajectory of the gluon. This equation predicts strong rise of gluon density at small x $\phi(x,k^2)=x^{-\lambda}$ and this tendency is not changed even if subleading logarithms in $\ln 1/x$ are taken into account [10]. Recently the BFKL equation with special Ansatz for running coupling constant and renormalization group improved kernel has been used to describe F_2 data and very good description of proton structure function has been achieved [11].

2.2. CCFM equation

The CCFM is an equation which sums up cascades of gluons under the assumption that subsequent emissions are ordered in an emission angle. It assumes the following form

$$\begin{aligned}\mathcal{A}(x,k^2,p) \;=\; &\bar{\alpha}_{\rm s}\int\limits_x^1 dz\int\frac{d^2\bar{q}}{\pi\bar{q}^2}\,\theta(p-z\bar{q})\,\Delta_{\rm s}(p,z\bar{q})\\ &\times\left(\frac{\Delta_{\rm ns}(z,k,q)}{z}+\frac{1}{1-z}\right)\mathcal{A}\left(\frac{x}{z},k',\bar{q}\right). \qquad (2.3)\end{aligned}$$

The momentum variable p is defined via $\bar{\xi}=p^2/(x_n^2 s)$, and $k'=|\boldsymbol{k}+(1-z)\bar{\boldsymbol{q}}|$ the momentum $\bar{q}$ is the rescaled momentum of the real gluon, and is related to q by $\bar{q}=q/(1-z)$. Here $\Delta_{\rm s}$ is the Sudakov form factor which regularizes the singularity of the $1/(1-z)$ pole, while $\Delta_{\rm ns}$ is the so-called non-Sudakov form factor and it corresponds to virtual contribution in the BFKL equation (their detailed form is not important in this note).

For recent theoretical works on this equation we refer reader to [12,13] while for phenomenological applications to [14,15]. The CCFM equation has been derived after observation of coherence effects in emission of gluons [9] and it combines information from BFKL and DGLAP and reduces to each of them in appropriate limits: BFKL in the limit when $x \to 0$ and DGLAP when $x \to 1$. Since it depends on hardness of the probe and also on k^2 of the incoming gluon it might be used in studies of final states. In particular, one can study physics of production of jets in forward direction which we overview in Section 3.

2.3. Saturation effects: BK equation

As it has been already remarked, if one wants to study physics at largest energies available at LHC one eventually has to go beyond BFKL, CCFM. This is because these equations were derived in an approximation of dilute partonic system where partons do not overlap or, to put it differently, do not recombine. Because of this, those equations cannot be safely extrapolated towards high energies, as this is in conflict with unitarity requirements. To account for dense partonic systems one has to introduce a mechanism which allows partons to recombine and therefore to saturate [16]. Existing data suggest that the phenomenon of saturation occurs in nature. The seminal example is provided by the discovery of geometrical scaling in HERA data [17] and more recently by geometrical scaling in production of inclusive jets at the LHC data [18,19]. Also the recently observed ridge-like structure in p–p collision at he LHC [20] has been described within approach including saturation [21]. There are various ways to approach the problem of evolution allowing for formation of dense system [22,23,24,25,26,27,28,29,30], here we are interested in the one which can be directly formulated within high energy factorization approach [1]. In this approach one can formulate momentum space version [31] of the Balitsky–Kovchegov equation which sums up large part of important terms for saturation and which is a nonlinear extension of the BFKL equation. The equation reads

$$\frac{\partial \phi\left(x,k^2\right)}{\partial \ln 1/x} = \frac{N_c \alpha_{\rm s}}{\pi} \int\limits_0^\infty \frac{dl^2}{l^2}\left[\frac{l^2\phi\left(x,l^2\right) - k^2\phi\left(x,k^2\right)}{|k^2-l^2|} + \frac{k^2\phi\left(x,k^2\right)}{\sqrt{(4l^4+k^4)}}\right]$$
$$-\frac{\alpha_{\rm s}^2}{R^2}\left\{\left[\int\limits_{k^2}^\infty \frac{dl^2}{l^2}\phi\left(x,l^2\right)\right]^2 + \phi\left(x,k^2\right)\int\limits_{k^2}^\infty \frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\phi\left(x,l^2\right)\right\}\,, \quad (2.4)$$

where R is the radius of the proton. The nonlinear term being convolution of the triple pomeron vertex [32] with gluon density allows gluons to merge apart from gluons splitting. Due to the interplay between splitting and merging of gluons the equation above generates dynamically a scale which is called saturation scale $Q_{\rm s}$ (equation (2.4) can be also obtained by Fourier transform of coordinate version of BK equation as has been done in [33]). This scale acts effectively as a mass of gluons and therefore regulates bad infrared behavior of gluon density [34]. It also selects the most probable k of gluon to be of the order of saturation scale. It follows from the fact that at $k = Q_{\rm s}$ the gluon density has a maximum

$$Q_{\rm s} \equiv \partial_{\ln k^2}\phi\left(x, k^2\right) = 0\,. \tag{2.5}$$

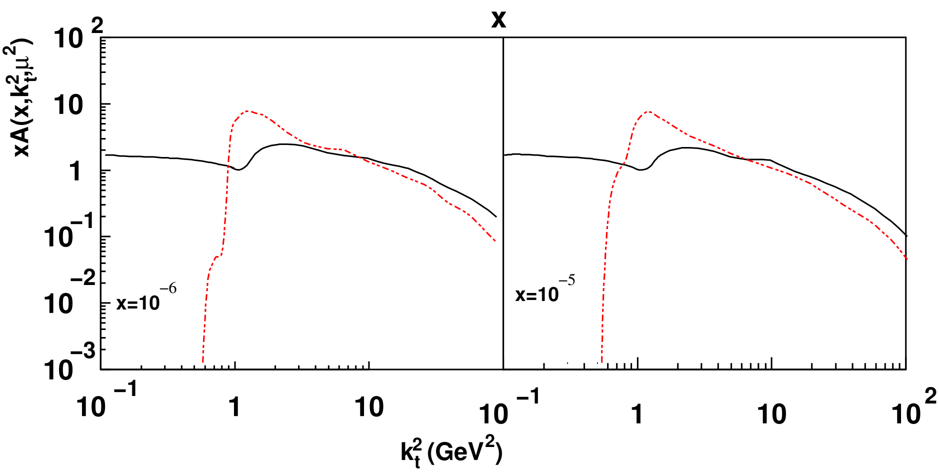

Fig. 2. Gluon density obtained from CCFM with saturation to gluon density from CCFM as a function of k^2 for $x = 10^{-5}$, $x = 10^{-6}$.

In order to have flexible approach and be able to simulate the scattering process in detail one uses Monte Carlo implementation of evolution equations, this is the case for DGLAP, CCFM, BFKL. As BK is a nonlinear equation it is not of straightforward usage in Monte Carlo generators. However, one can avoid complications coming from nonlinearity by applying absorptive boundary conditions [35] which mimics the nonlinear term in the BK equation.

2.4. Saturation effects: CCFM equation with absorptive boundary

The basic principle of this method is that the absorptive boundary limits the phase space for gluons and therefore effectively acts as nonlinear term in the BK equation. In the original formulation it was required that the

BFKL amplitude should be equal to unity for a certain combination of k^2 and x. In discussed here approach the energy dependent cutoff on transverse gluon momenta has been imposed. It acts as absorptive boundary and slows down the rate of growth of the gluon density. In order to have description of exclusive processes and account for saturation effects one can use CCFM evolution equation together with absorptive boundary [35]. Its certain variation has been implemented in CASCADE Monte Carlo event generator [36] (for adaptation to CCFM of method developed in [35] we refer the reader to [12, 13]). In the approach of [37] as the absorptive boundary the GBW [38] critical line has been used. The condition for saturation was provided by the GBW saturation scale $Q_{\rm s} = k_0(x_0/x)^{\lambda/2}$ *i.e.* density of gluons with momenta generated at given x with transversal momenta which satisfied condition $k < Q_{\rm s}$ was set to go to zerto as k^2 what is in agreement with numerical solution of BK. This prescription gives gluon density which has a maximum as a function of k in agreement with results obtained from BK. However, one should add that this method is quite simplistic and within this approach one can not find the effect of saturation of saturation scale itself which has been found in [13].

3. Phenomenological applications

3.1. Production of central-forward jets at LHC

Physics in the forward region at hadron colliders is traditionally dominated by soft particle production. With the advent of the LHC, forward physics phenomenology turns into a largely new field [39, 40, 41] involving both soft and hard production processes, because of the phase space opening up at high center-of-mass energies. Forward jet production enters the LHC physics program in an essential way both for QCD studies since one can probe dense parton systems [42] and for new particle searches, *e.g.* in vector boson fusion search channels for the Higgs boson [43, 44]. The forward production of high-$p_{\rm T}$ particles brings jet physics into a region characterized by multiple energy scales and asymmetric parton kinematics. Here we overview results of [15] where the study of forward-central jet correlations of two jets has been done. The results of such investigations can serve to estimate the size of backgrounds from QCD radiation between jets at large rapidity separations for Higgs boson searches in vector boson fusion channels. High-energy factorization allows one to decompose the cross-section for the forward-central jet production of Fig. 3 into partonic distributions and hard-scattering kernels, obtained via the high-energy projectors [1] from the amplitudes for the process $p_1 + p_2 \to p_3 + p_4 + 2$ massless partons. In Fig. 4 the prediction of differential cross-section $\frac{d\sigma}{dp_\perp}$ is shown as obtained from CASCADE and PYTHIA. The cross-sections predicted from both simulations

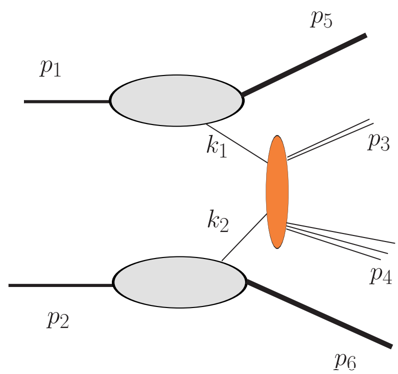

Fig. 3. Production of forward-central jets.

at low momentum are of the similar order, however, at larger transverse momentum the CASCADE predicts a larger cross-section what is clearly visible for central jets (Fig. 4, right). This behavior is expected since CASCADE

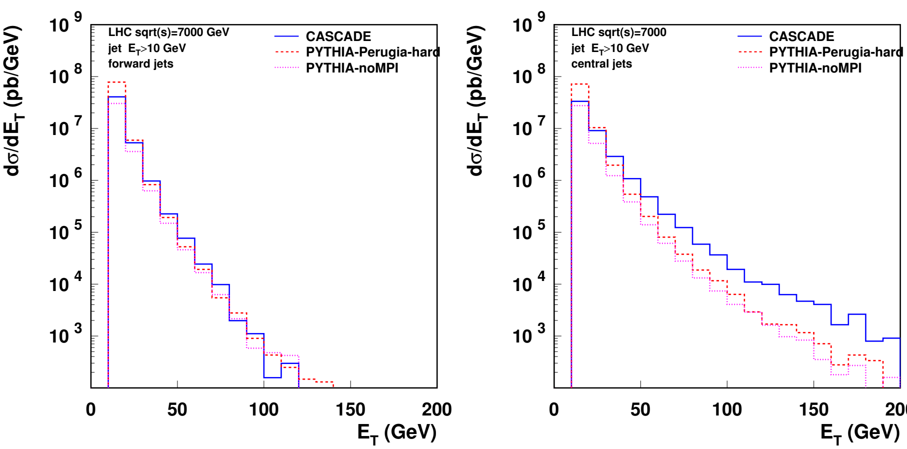

Fig. 4. Transversal momentum spectra of produced jets at total collision energy $\sqrt{s} = 7$ TeV with requirement that $p_\perp > 10$ GeV. We compare predictions obtained from CASCADE and PYTHIA running in a multiple interactions mode and no multiple interactions mode. Spectrum of forward jets (left); spectrum of central jets (right).

uses matrix elements which are calculated within high energy factorization scheme allowing for harder transversal momentum dependence as compared to collinear factorization. Moreover, CASCADE applies CCFM parton shower utilizing angle dependent evolution kernel which at small x does not lead to ordering in transverse momentum, and thus allows for more hard radiation during evolution as compared to based on leading order DGLAP splitting functions Monte Carlo generator PYTHIA. The parton shower has major influence on the side where the small x gluon enters the hard interaction, thus the jets in the central region are mainly affected by the parton shower.

3.2. Production of charged particles at HERA

Another exclusive observable that is interesting to look at is the $p_{\rm T}$ spectrum of produced charged particles in DIS. Here we review application of the unintegrated gluon density from CCFM with introduced saturation effects via energy dependent cut off mimicking nonlinear effects as has been done in [37]. In the results, we see a clear difference between the approach which includes saturation and the one which does not include it. The description with saturation is closer to data suggesting the need for saturation effects [45]. We compare our calculation with calculation based on CCFM (CASCADE) and on DGLAP (RapGAP) evolution equations. From the plots, Fig. 5, we see that the CCFM with saturation describes data better then the other approaches. CCFM overestimates the cross-section for very low x data while DGLAP underestimates it. This is easy to explain, in CCFM one can get large contributions from larger momenta in the chain due to lack of ordering in k^2 while in DGLAP large k^2 in the chain is suppressed. On the other hand, CCFM with saturation becomes ordered for small x both in k^2 and rapidity and therefore interpolates between these two.

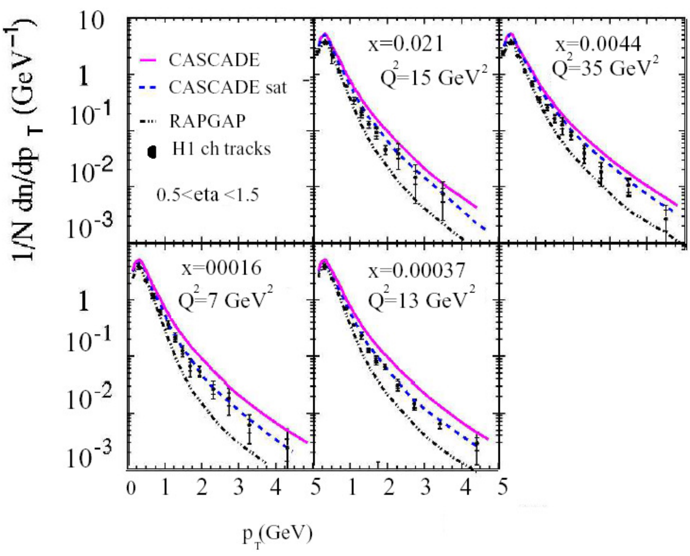

Fig. 5. Differential cross-section for transverse momentum distribution of charged hadrons calculated within CCFM (violet continuous line), CCFM with saturation (dashed blue line) and DGLAP (dotted black line).

4. Conclusions

We have reviewed basic theoretical aspects of high energy factorization and gave examples of phenomenological applications to hadron–hadron collisions and lepton–proton collisions. We also stressed the uniqueness of the high energy factorization as a framework in which both hard processes and formation of dense system can be studied.

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