QCD FOR THE LHC* **

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We discuss the new era of precision QCD as it relates to the physics requirements of the LHC for both the signal and background type processes. Some attention is paid to the issue of the theoretical error associated with any given theoretical prediction. In the cases considered, we present where the theory precision is at this writing and where it needs to go in order that it not impede the discovery potential of the LHC physics program. To complete the discussion, we also discuss possible paradigms the latter program may help us understand and some new developments that may play a role in achieving that respective understanding.

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1. Introduction

As the start-up of the LHC has precipitated the era of precision QCD, by which we mean predictions for QCD processes at the total precision tag of 1% or better, it is appropriate for any discussion of the requirements on QCD for the LHC to set its framework by recalling, at least in generic terms, why we need the LHC in the first place. In the following discussion of the QCD for the LHC, we shall begin with such recollection. In this way, the entire effect of the effort required to realize precision QCD for the LHC in a practical way can be more properly assessed.

Thus, we ask, "Why do we need the LHC?". Many answers can be found in the original justifications for the colliding beam device and its detectors in Refs. [1, 2, 3, 4, 5]. We will call attention to a particular snap shot of the latter discussions with some eye toward the requirements of precision

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QCD from the theoretical standpoint. More precisely, the LHC is a crucial step toward resolving fundamental outstanding issues in elementary particle physics: the big and little hierarchy problems, the number for families, the origin of Lagrangian fermion (and gauge boson) masses, baryon stability, the union of quantum mechanics and general theory of relativity, the origin of CP violation, the origin of the cosmological constant Λ, dark matter *etc.* Much theory effort has been invested in the "New Physics" (NP) that would seem to be needed to solve all of these outstanding issues, that is to say, in the physics beyond the Standard Model 't Hooft–Veltman renormalized Glashow–Salam–Weinberg EW × Gross–Wilczek–Politzer QCD theory that seems to describe the quantum loop corrections in the measurements of electroweak and strong interactions at the shortest distances so far achieved in laboratory-based experiments.

We mention that superstring theory [6, 7] solves everything in principle but has trouble in practice: for example it has more than 10^{500} candidate solutions for the vacuum state [8]. The ideas in superstring theory have helped to motivate many so-called string inspired models of NP such as [9] string-inspired GUTs, large extra dimensions, Kaluza–Klein excitations *etc.* We list supersymmetric extensions of the SM, such as the MSSM and the CMSSM [9], as separate proposals from superstring motivated ideas, as historically this was the case. Modern approaches to the dynamical EW symmetry breaking (technicolor) such as little Brout–Englert–Higgs models [9], obtain as well. The list is quite long and LHC will help us shorten it, no doubt.

Perhaps, one of the most provocative ideas is the one which some superstring theorists [8] invoke to solve the problem of the large number of candidate superstring vacua: the anthropic principle, by which the solution is the one that allows us to be in the state in which we find ourselves. In the view of some [10], this would be the end of reductionist physics as we now know it. Can LHC even settle this discussion? Perhaps.

More recently, even newer paradigms are emerging. In Ref. [11], the UV limit of theories such as quantum gravity is solved by the dynamical generation of non-perturbative large distance excitations called classicalons, which provide the necessary damping of the naively divergent UV behavior. When discussed in general terms, possible new signatures for the LHC obtain [11].

In Ref. [12], the $E_8 \times E_8 \equiv E_{8a} \times E_{8b}$ symmetry group suggested by the heterotic string theory [13] is abstracted to apply to the fundamental symmetry group physics for GUT's and it is shown that, if one presumes that the known light leptons are in the 3-families of SO(10) $\underline{16}$'s with three sets of new quarks, $\{u', d'; c', s'; t', b'\}$ while the known quarks are in three families of SO(10) $\underline{16}$'s with three new sets of heavy leptons $\{\nu_{\ell'}, \ell', \ell' = e', \mu', \tau'\}$, where the two sets of three SO(10) families can either be generated by break-

ing the $E_{8a} \times E_{8b}$ such that the first (second) set transforms non-trivially only under E_{8a} (E_{8b}) or be generated by having both sets of three families transform non-trivially under E_{8a}, leaving open the possibility of an unspecified number of families, as yet unseen, to transform under non-trivially under E_{8b}. The proton is stable for purely kinematic reasons — all the leptons to which it could decay are too heavy for the decay to occur. The mixing matrix for the low energy EW gauge bosons from the GUT scale breaking of the $E_{8a} \times E_{8b}$ symmetry down to the Standard Model gauge group then allows the GUT scale $M_{\rm GUT}$ to obtain at $\lesssim 200$ TeV, in reach of the VLHC colliding beam device as discussed in Refs. [14]. Many of the new heavy quarks and leptons in this paradigm could already be visible at the LHC.

If one does not use string theory for the unification of the EW and QCD theories with quantum gravity, then one needs a remedy for the UV sector of quantum gravity. Recently, in addition to the ideas in Ref. [11], more progress has been made on solving this problem in the context of local Lagrangian field theory methods [15, 16, 17]. Specifically, following the suggestion by Weinberg [18] that quantum gravity might have a non-trivial UV fixed point, with a finite dimensional critical surface in the UV limit, so that it would be asymptotically safe with an S-matrix that depends on only a finite number of observable parameters, in Refs. [15] strong evidence has been calculated using Wilsonian [19] field-space exact renormalization group methods to support Weinberg's asymptotic safety hypothesis for the Einstein–Hilbert theory. In a parallel but independent development [16], we have shown [20] that the extension of the amplitude-based, exact resummation theory of Ref. [21] to the Einstein–Hilbert theory leads to UV-fixed-point behavior for the dimensionless gravitational and cosmological constants with the bonus that the resummed theory is actually UV finite when expanded in the resummed propagators and vertices's to any finite order in the respective improved loop expansion. We refer to the resummed theory as resummed quantum gravity. In addition, more evidence for Weinberg's asymptotic safety behavior has been calculated using causal dynamical triangulated lattice methods in Ref. [22][1]. At this point, there is no known inconsistency between our analysis and those of the Refs. [15, 22] or the leg renormalizability arguments in Ref. [17]. We note further that, in Refs. [24, 25], it has been argued that the approach in Refs. [15] to quantum gravity may indeed provide a realization[2] of the successful inflationary model [27, 28] of cosmology without the need of the as yet unseen inflaton

[1] We also note that the model in Ref. [23] realizes many aspects of the effective field theory implied by the anomalous dimension of 2 at the UV-fixed point but it does so at the expense of violating Lorentz invariance.

[2] The attendant choice of the scale $k \sim 1/t$ used in Refs. [24, 25] was also proposed in Ref. [26].

scalar field: the attendant UV fixed point solution allows one to develop Planck scale cosmology that joins smoothly onto the standard Friedmann–Walker–Robertson classical descriptions so that then one arrives at a quantum mechanical solution to the horizon, flatness, entropy and scale free spectrum problems. In Ref. [20], we have shown that, in the new resummed theory [16] of quantum gravity, we recover the properties as used in Refs. [24,25] for the UV fixed point of quantum gravity with the added results that we get "first principles" predictions for the fixed point values of the respective dimensionless gravitational and cosmological constants in their analysis. In Ref. [29] we carry the analysis one step further and arrive at a prediction for the observed cosmological constant Λ, $\rho_\Lambda \cong (2.400 \times 10^{-3}\ \mathrm{eV})^4$, in the context of the Planck scale cosmology of Refs. [24, 25], which is reasonably close to the observed value [30, 31] $(2.368 \times 10^{-3}\ \mathrm{eV}(1 \pm 0.023))^4$.

It follows that the new paradigms, which we have illustrated admittedly only in part in a limited way to set the stage of our discussion here, must be taken seriously in analyzing the new LHC data. In particular, we must be able to distinguish higher order SM processes from New Physics and we must be able to probe New Physics precisely to distinguish among different New Physics scenarios. This necessitates the era of precision QCD for the LHC.

Our discussion is organized as follows. We first discuss in the next section the issue of QCD at high energies from the standpoint of precision theory. Section 3 deals with applications of such theory to the LHC scenario and concludes with a look toward the future.

2. QCD at high energies

At high energies when we have sufficiently large momentum transfer interactions, such as we have at the hard scattering processes at the LHC, we have the master formula for the respective fully differential cross-sections as

$$d\sigma = \sum_{i,j} \int dx_1 dx_2 F_i(x_1) F_j(x_2) d\hat{\sigma}_{\mathrm{res}}(x_1 x_2 s)\,, \tag{1}$$

using a standard notation so that the $\{F_j\}$ and $d\hat{\sigma}_{\mathrm{res}}$ are the respective parton densities and reduced hard differential cross-section where we indicate that latter has been resummed for all large EW and QCD higher order corrections in a manner consistent with achieving a total precision tag of 1% or better for the total theoretical precision of (1) as we discuss in more detail presently — this latter precision tag will be our definition of precision QCD theory. See Refs. [32] where an example of such simultaneous QCD×EW resummation is presented — such resummation will be reviewed briefly in the following as well in the interest of completeness[3].

[3] For an alternative approach to such simultaneous QCD×EW resummation, see Refs. [33].

At high energies, we may have the hadron–hadron colliding beam paradigm, such as what we have at the LHC and at the Tevatron, the e^+e^- colliding beam paradigm, as it is proposed now for the ILC, and the lepton–hadron colliding beam paradigm, as we had until recently at HERA. How do we assess the precision of a theoretical result for (1) in these paradigms? The respective theoretical precisions, $\Delta\sigma_{\rm th}$, can be decomposed as follows

$$\begin{aligned} \text{Hadron–Hadron} \quad &: \quad \Delta\sigma_{\rm th} = \Delta F \oplus \Delta\hat{\sigma}_{\rm res}\,, \\ e^+e^- \quad &: \quad \Delta\sigma_{\rm th} = 0 \oplus \Delta\hat{\sigma}_{\rm res}\,, \end{aligned}$$

where the lepton–hadron case is covered by the hadron–hadron case if we interpret the parton density theory error, ΔF, as that for just one factor of the F_j in the cross-section accordingly and where we note that, in the e^+e^- high energy colliding beam case the analoga of the $\{F_j\}$ can be computed, on an event-by-event basis by MC methods as a part of the resummed cross-section [34], $\hat{\sigma}_{\rm res}$, so that we can set the analog of ΔF to zero accordingly.

We stress that the theoretical precision, $\Delta\sigma_{\rm th}$, validates the application of a given theoretical prediction to precision experimental observations, for the discussion of backgrounds for both SM and NP studies and for the signals for both SM and NP studies, and more specifically for the overall normalization of the cross-sections in such studies. NP can be missed if a calculation with an unknown value of $\Delta\sigma_{\rm th}$ is used to assess theoretical expectations for such studies. This point cannot be emphasized too much.

Here, we define $\Delta\sigma_{\rm th}$ as the total theoretical uncertainty coming from the physical precision contribution and the technical precision contribution [35]: the physical precision contribution, $\Delta\sigma_{\rm th}^{\rm phys}$, arises from such sources as missing graphs, approximations to graphs, truncations, *etc.*; the technical precision contribution, $\Delta\sigma_{\rm th}^{\rm tech}$, arises from such sources as bugs in codes, numerical rounding errors, convergence issues, *etc.* The total theoretical error is then generically given by

$$\Delta\sigma_{\rm th} = \Delta\sigma_{\rm th}^{\rm phys} \oplus \Delta\sigma_{\rm th}^{\rm tech}\,. \tag{2}$$

The desired value for $\Delta\sigma_{\rm th}$ depends on the specific requirements of the observations. As a general rule, one would like that $\Delta\sigma_{\rm th} \leq f\Delta\sigma_{\rm expt}$, where $\Delta\sigma_{\rm expt}$ is the respective experimental error and $f \lesssim \frac{1}{2}$ so that the theoretical uncertainty does not significantly affect the analysis of the data for physics studies in an adverse way.

For illustration we note the following examples that have been obtained. At the Tevatron one generally had the luminosity experimental uncertainty [36] of $\Delta\sigma_{\rm expt}^{\rm norm} \cong 6$–$7\%$ so that theoretical predictions for cross-sections at the level of $\Delta\sigma_{\rm th} \sim 10\%$ were acceptable in general. At LEP1, the

observation of 20M Zs necessitated that the normalization error from the theoretical cross-section [34] was $\Delta\sigma_{\rm th}^{\rm norm} = 0.061\%$ or better. What do we need for the LHC physics in this context?

3. Applications of precision QCD for LHC physics

When we consider the LHC, we already see the effect of much better detectors and much larger statistics compared to the Tevatron for example. Indeed, already, the experiments [37] are reporting a normalization error from experiment at the 3–4% level, with the expectation [38] that 1–2% will be achieved. This defines a new set of goals for the theoretical uncertainty in QCD calculations for the LHC.

Specifically, the goals for the theoretical uncertainty $\Delta\sigma_{\rm th}$ for precision QCD calculations (henceforward we take it as understood that we include the corresponding higher order EW and mixed EW⊗QCD corrections as well) can be illustrated as follows: for the so-called standard candle processes, we have

$$\begin{aligned} \text{Single } Z, W \text{ production} &: \Delta\sigma_{\rm th} \lesssim 1\%\,, \\ t\bar{t} \text{ production} &: \Delta\sigma_{\rm th} \lesssim 1\%\,. \end{aligned} \tag{3}$$

For the Les Houches [39] list

$$2 \to n \text{ processes to } \mathcal{O}(\alpha_{\rm s}), \quad n \geq 3 : \quad \Delta\sigma_{\rm th} \cong 10\%\,. \tag{4}$$

Exactness of the theoretical results is essential to have any chance of achieving these goals in a practical way. What is the current state-of-the-art (SOTA) for published results on such goals?

3.1. SOTA for $\Delta\sigma_{\rm th}$ for LHC physics

There has been significant progress on the goals outlined for $\Delta\sigma_{\rm th}$. We now summarize some of this progress[4]. On single Z and W production, the situation has been analyzed in Ref. [40] where the values found for $\Delta\sigma_{\rm th}$ are as follows for single Z production at 14 TeV, using a standard notation for effects from Ref. [40], $\Delta\sigma_{\rm th} = (4.91 \pm 0.38)\% = (2.45 \pm 0.73)\%$ (QCD + EW) $\oplus\, 4.11\%$ (PDF) $\oplus\, (1.10 \pm 0.44)\%$ (QCD Scale) and for single $W^+[W^-]$ production at the same cms energy the corresponding value is $\Delta\sigma_{\rm th} = (5.05 \pm 0.58)\%$ $[(5.24 \pm 0)\%]$. The error due to the PDF uncertainty shown here quantifies the type of value we have for the error ΔF in (2). We see that in all three cases, there is still considerable effort that remains to be done to reach the goals presented in (3).

[4] We apologize if we omit some of references that should just as well be cited but we had to make some choices due to the limitations of space for this report.

For $t\bar{t}$ production, the situation was recently reviewed by Salam in Ref. [41] as we reproduce here in Fig. 1. The results shown do not contain any contribution from the respective PDF uncertainty, what we referred to as above as ΔF, so that the total value of $\Delta_{\rm th}$ is $\Delta_F \oplus \Delta\sigma_{\rm th}^{\rm rest}$, where here $\Delta\sigma_{\rm th}^{\rm rest}$ refers to the errors shown in Fig. 1 for the cited calculation. If we use the basic estimator of the actual error via [31] the standard formula ($\bar{\sigma}$ is the usual naive average)

$$\Delta_{\rm th}^2 \cong \frac{1}{N-1}\sum_{i=1}^{N}(\sigma_i - \bar{\sigma})^2 \tag{5}$$

for the results in Fig. 1, we arrive at the optimistic result $\Delta_{\rm th} \cong 4.8\%$ for the current SOTA for $t\bar{t}$ production at the LHC at 7 TeV. This again is significantly larger than the goal in (3).

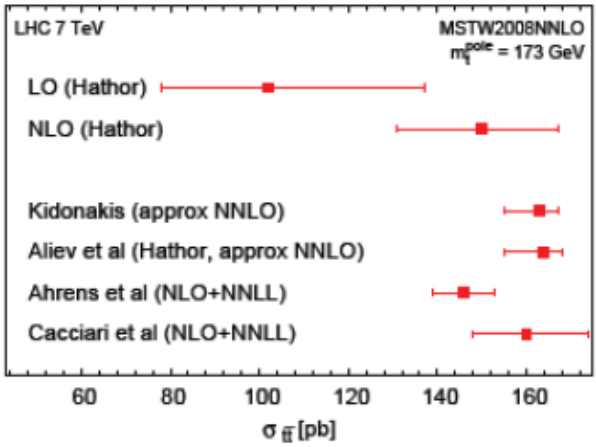


Fig. 1. Results on $t\bar{t}$ production at the LHC as reviewed in Ref. [41].

For the $2 \to n$, $n \geq 3$ processes, the applications are to backgrounds to NP and to more precision tests for the SM processes, wherein the $\mathcal{O}(\alpha_{\rm s})$ correction is essential. There has been great progress in achieving these $\mathcal{O}(\alpha_{\rm s})$ corrections: for $2 \to 5$, the BlackHat group has recently reported a result in Ref. [42]; for $2 \to 4$, there are many published results, some of which involve automation — see Ref. [41] for a good review. What is the value of $\Delta\sigma_{\rm th}$ for these impressive results?

To illustrate the situation, we show in Fig. 2 the application of the result in Ref. [42] for the $W + 4\,\text{jets} + X$ process at LHC for the $p_{\rm T}$ spectra of the leading 4 jets. The theoretical uncertainty of the NLO result is at the 10% level, optimistically, if we only use the scale dependence variation.

For precision LHC applications, we need to know true value of $\Delta\sigma_{\rm th}$, including the contributions of the PDF uncertainty, the technical precision uncertainty *etc.*

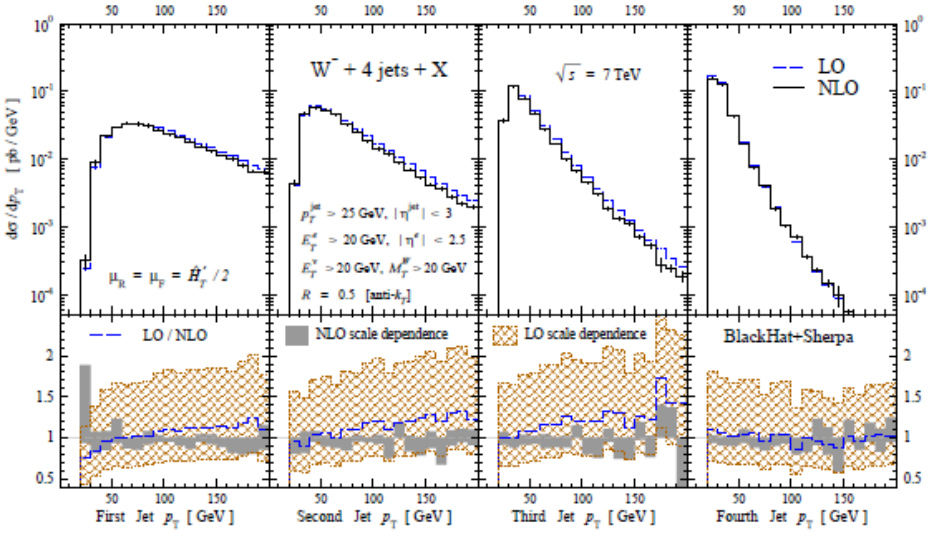


Fig. 2. Results on the jet $p_{\rm T}$ spectra in $W + 4\,\text{jets} + X$ production at the LHC as reported in Ref. [42] and reviewed in part in Ref. [41]. The calculation is NLO in the leading color approximation for the virtual corrections.

3.2. $\Delta\sigma_{\rm th}$ *in LHC physics*

The basic paradigm in which we need to be able to prove the value of $\Delta\sigma_{\rm th}$ is the following one: we have, when arbitrary detector cuts are allowed, the combination

$$\text{MC} \cup \text{NLO} \cup \text{NNLO/NNLL}\,, \tag{6}$$

where the EW and mixed EW⊗QCD corrections [32, 33, 43] at the corresponding precision level are included here. In addition, this means that the quark masses $m_q \neq 0$ are general required for ISR at $\mathcal{O}(\alpha_{\rm s}^n)$, $n \geq 2$ so that an approach such as that in Ref. [44] is needed.

There are by now some very standard tools available in the paradigm.The MC is generically one of parton shower type [45], where in the traditional FORTRAN we have reference to HERWIG6.5 [46] and PYTHIA6.4 [47] and, more recently, in C++ we have SHERPA [48], HERWIG++ [49] and PYTHIA8 [50], to be specific. Again, we can get an estimate of the values of $\Delta\sigma_{\rm th}$

from Fig. 3, where we see that the MC's PYTHIA8, SHERPA1.2 and HERWIG++2.4 have uncertainties that very from 10% to 50% depending on the value of the Z p_{T} in single Z production at the Tevatron. This shows that the exact NLO corrections are necessary for precision studies already at the Tevatron.

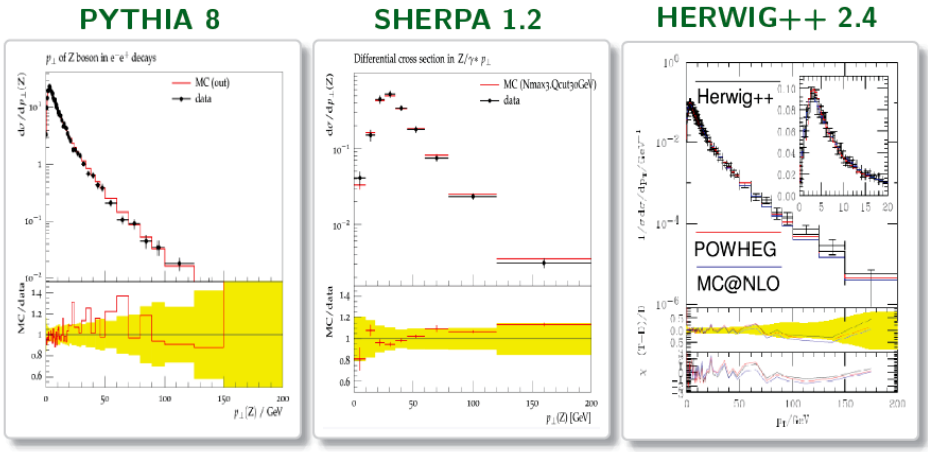


Fig. 3. Results on $Z + X$ production at the Tevatron as reviewed in Ref. [41].

Indeed, the parton shower MC's with exact $\mathcal{O}(\alpha_{\mathrm{s}})$ corrections, MC@NLO [51] and POWHEG [52], provide realizations of the needed exact NLO corrections. The resulting improvement in the value of the $\Delta\sigma_{\mathrm{th}}$ when including these exact corrections can also be seen in Fig. 3 where the comparison of the variation of the Z p_{T} spectrum between MC@NLO and POWHEG is given — it reduces the variation between the theory and the data considerably and shows a difference between the two theory predictions at the level of $\sim$ 5–10%. This is definite improvement but still leaves us quite a bit of work to reach our goal in this process. We stress that while the two calculations in MC@NLO and POWHEG have both the exact $\mathcal{O}(\alpha_{\mathrm{s}})$ corrections, they differ in the corrections at $\mathcal{O}(\alpha_{\mathrm{s}}^{n})$, $n \geq 2$.

To control all aspects of the contributions to $\Delta\sigma_{\mathrm{th}}$ it is important for the observables to be infrared safe, as it is well-known. The new infrared-safe anti-k_{T} jet algorithm of Ref. [53] allows us to have infrared-safe jet definition in practice at the LHC, as it is apparently being adopted by the LHC collaborations [41]. The basic idea [53] is that one repeatedly combines pairs of objects with the smallest values of $d_{ij} = R_{ij}^2/\max(k_{\mathrm{T}i}^2, k_{\mathrm{T}j}^2)$, where R_{ij} is an appropriately normalized "distance" between the two objects i,j. This yields cones in an infrared safe manner [41, 53].

Further results on realizing exact $\mathcal{O}(\alpha_s)$ with n jets obtain: the MENLOPS [54] project with NLO Z production and LO $Z + n$ jets + parton showers (using the CKKW and MLM merging methods [55]) adds in the multi-leg corrections. The combination of NLO Z and NLO $Z/\gamma + n$ jets with parton showers has been done in Ref. [56], for $n = 1$. In all cases, one needs to prove one knows the corresponding values of $\Delta\sigma_{\rm th}$.

What is needed here is an NNLO with (resummed) parton shower MC for the complete realization of the LHC discovery potential. One needs specifically resummation of all large collinear effects, so that one can use DGLAP-CS [57, 58] evolution in which $p_{\rm T}$ is integrated out and one can use evolutions in which $p_{\rm T}$ is alive [59]. One needs as well resummation of all large soft effects, including the Regge limit [60, 61], so that one needs these effects in both the collinear regime and in the non-collinear regime. One needs exact treatment of differential distributions through NNLO, with exact phase space and no miss-counting of efforts, including the effect of non-zero quark masses. The goal is event-by-event realization of these effects with exclusive exact NNLO with parton showers to yield a proof of the value of $\Delta\sigma_{\rm th}$.

There is some progress in this effort as well. In (1), resummation of collinear evolution is realized in the evolution of the $\{F_j\}$ and soft resummation (non-collinear) is realized in the calculation of $d\hat{\sigma}_{\rm res}$. For example, from Ref. [32] we have the representation

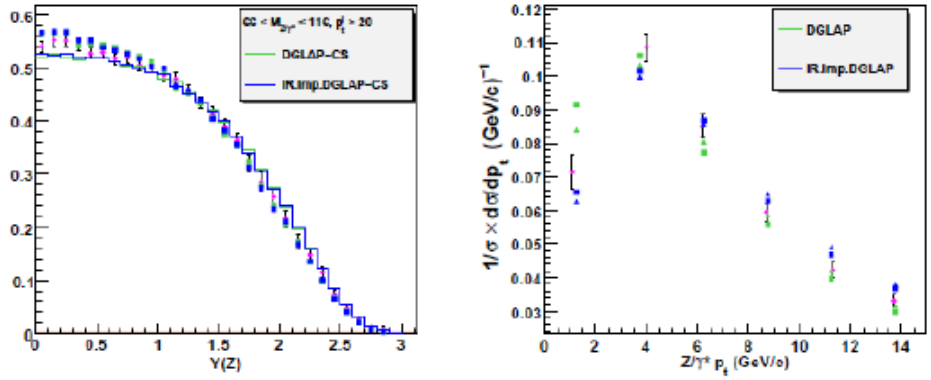
$$
\begin{aligned}
d\hat{\sigma}_{\rm res} \;=\; & e^{{\rm SUM_{IR}(QCED)}} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^{n} \frac{d^3k_{j_1}}{k_{j_1}} \\
& \times \prod_{j_2=1}^{m} \frac{d^3k'_{j_2}}{k'_{j_2}} \int \frac{d^4y}{(2\pi)^4} e^{iy\left(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2}\right)+D_{\rm QCED}} \\
& \times \tilde{\tilde{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m) \frac{d^3p_2}{p_2^0} \frac{d^3q_2}{q_2^0}\,, \qquad (7)
\end{aligned}
$$

where the new YFS-style [21] (non-Abelian) residuals $\tilde{\tilde{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$ have n hard gluons and m hard photons and we show the final state with two hard final partons with momenta p_2, q_2 specified for a generic $2f$ final state for definiteness. The infrared functions ${\rm SUM_{IR}(QCED)}$, $D_{\rm QCED}$ are defined in Refs. [32, 62, 63]. This simultaneous resummation of QED and QCD large IR effects is exact. Moreover, the residuals $\tilde{\tilde{\beta}}_{n,m}$ allow a rigorous parton shower/ME matching via their shower-subtracted counterparts $\hat{\tilde{\tilde{\beta}}}_{n,m}$ [32]. When the formula in (7) is applied to the calculation of the kernels, P_{AB}, in the DGLAP-CS theory itself, we get an improvement of the IR limit of these kernels, an IR-improved DGLAP-CS theory [62,63] in which

large IR effects are resummed for the kernels themselves. The resulting new resummed kernels, $P_{AB}^{\exp}$ as given in Ref. [62, 63] and as illustrated below, yield a new resummed scheme for the PDFs and the reduced cross-section

$$F_j,\ \hat{\sigma} \ \rightarrow\ F_j',\ \hat{\sigma}' \text{ for}$$
$$P_{gq}(z) \ \rightarrow\ P_{gq}^{\exp}(z) = C_{\rm F} F_{\rm YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q}\,,\ etc.\,, \tag{8}$$

with the same value for σ in (1) with improved MC stability as discussed in Ref. [64]. Here, the YFS [21] infrared factor is given by $F_{\rm YFS}(a) = e^{-C_{\rm E}a}/\Gamma(1+a)$, where $C_{\rm E}$ is Euler's constant and we refer the reader to Refs. [62, 63] for the definition of the infrared exponents γ_q, δ_q. $C_{\rm F}$ is the quadratic Casimir invariant for the quark color representation. The new MC HERWIRI1.031 [64] gives the first realization of the new IR-improved kernels in the HERWIG6.5 [46] environment. We illustrate it in comparison with HERWIG6.510, both with and without the MC@NLO exact $\mathcal{O}(\alpha_{\rm s})$ correction, in Fig. 4 in relation to D0 data [65] on the Z boson $p_{\rm T}$ in single Z production and the CDF data [66] on the Z boson rapidity in the same



(a) (b)

Fig. 4. From Ref. [64], comparison with FNAL data: (a) CDF rapidity data on (Z/γ^*) production to e^+e^- pairs, the circular dots are the data, the light grey (dark grey) (green (blue)) lines are HERWIG6.510(HERWIRI1.031); (b), D0 $p_{\rm T}$ spectrum data on (Z/γ^*) production to e^+e^- pairs, the circular dots are the data, the dark grey (blue) triangles are HERWIRI1.031, the light grey (green) triangles are HERWIG6.510. In both (a) and (b) the dark grey (blue) squares are MC@NLO/HERWIRI1.031, and the light grey (green) squares are MC@NLO/HERWIG6.510, where MC@NLO/X denotes the realization by MC@NLO of the exact $\mathcal{O}(\alpha_{\rm s})$ correction for the generator X. These are untuned theoretical results.

process all at the Tevatron. We see [64] that the IR improvement improves the χ^2/d.o.f in comparison with the data in both cases for the soft $p_{\rm T}$ data and that for the rapidity data it improves the χ^2/d.o.f before the application of the MC@NLO exact $\mathcal{O}(\alpha_{\rm s})$ correction and that with the latter correction the χ^2/d.o.fs are statistically indistinguishable. More importantly, this theoretical paradigm can be systematically improved in principle to reach any desired $\Delta\sigma_{\rm th}$. The suggested accuracy at the 10% level shows the need for the NNLO extension of MC@NLO, in view of our goals for this process.

We also note the developments in Refs. [59] aimed at exclusive realization of the NLO correction to the DGLAP-CS kernels P_{AB}. We show in Fig. 5 numerical results that demonstrate the proof of concept for the non-singlet analysis as reported in Ref. [59] for the case that one NLO insertion is added anywhere in the standard LL ladder representation of the solution for the respective distribution function. In this approach the modifications to the

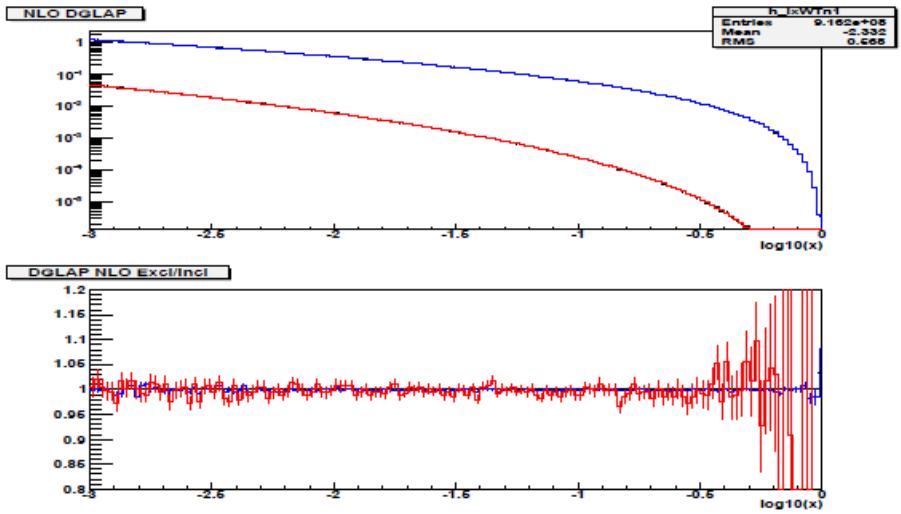


Fig. 5. Numerical cross check of the approach in Ref. [59].

usual LL ladder for the respective distribution function $\bar{D}_B$ can be seen in the formula

$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{\rm ISR}}\left\{\delta_{x=1}+\sum_{n=1}^{\infty}\left(\prod_{i=1}^{n}\int\limits_{Q>a_i>a_{i-1}} d^3\eta_i\rho_{1B}^{(1)}(k_i)\right)\right.$$
$$\left.\times\left[\sum_{p=1}^{n}\beta_0^{(1)}(z_p)+\sum_{p=1}^{n}\sum_{j=1}^{p-1}W\left(\tilde{k}_p,\tilde{k}_j\right)\right]\delta_{x=\prod\limits_{j=1}^{n}x_j}\right\}, \quad (9)$$

where the residuals $\beta_0^{[1]}$ and W allow one to include the exclusive effects for the NLO correction to the usual ladder solution, as expounded in Ref. [59], where the Sudakov exponent $S_{\rm ISR}$ and the real emission kinematics in (9) are all defined. Similar results have been obtained for FSR. The next step is to add more NLO insertions, 2,3, and so on. This is in progress. This theoretical paradigm can in principle also be systematically improved to a given value of $\Delta\sigma_{\rm th}$.

We then can prescribe a future QCD for the LHC as follows: it needs exact amplitude-based resummation with NNLO hard corrections ($\mathcal{O}(\alpha_{\rm s}^2, \alpha\alpha_{\rm s}, \alpha^2 L^2)$) on an event-by-event basis via MC methods, with IR and collinearly improved showers, exact phase space and complete mass effects. The result will be provable control on $\Delta\sigma_{\rm th}$ for LHC physics.

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