TWISTED ACCELERATION-ENLARGED NEWTON–HOOKE SPACE-TIMES AND CONSERVATIVE FORCE TERMS

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There are analyzed two classical systems defined on twist-deformed acceleration-enlarged Newton–Hooke space-times — non-relativistic particle moving in constant field force $\vec{F}$ and harmonic oscillator model. It is demonstrated that only in the case of canonical twist deformation the force terms generated by space-time non-commutativity remain conservative for both models.

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Recently, there appeared a lot of papers dealing with non-commutative classical and quantum mechanics (see *e.g.* [1, 2, 3]) as well as with field theoretical models (see *e.g.* [4, 5]), in which the quantum space-time is employed. The suggestion to use non-commutative coordinates goes back to Heisenberg and was firstly formalized by Snyder in [6]. Recently, there were also found formal arguments based mainly on Quantum Gravity [7] and String Theory models [8], indicating that space-time at Planck scale should be non-commutative, *i.e.* it should have a quantum nature. On the other side, the main reason for such considerations follows from the suggestion that relativistic space-time symmetries should be modified (deformed) at Planck scale, while the classical Poincaré invariance still remains valid at larger distances [9, 10].

Presently, it is well known, that in accordance with the Hopf-algebraic classification of all deformations of relativistic and non-relativistic symmetries, one can distinguish three basic types of quantum spaces [11, 12]:

1. Canonical ($\theta^{\mu\nu}$-deformed) space-time
$$[x_\mu, x_\nu] = i\theta_{\mu\nu}\,, \qquad \theta_{\mu\nu} = \text{const.}\,, \tag{1}$$
introduced in [13,14] in the case of Poincaré quantum group and in [15] for its Galilean counterpart.

2. Lie-algebraic modification of classical space
$$[x_\mu, x_\nu] = i\theta^{\rho}_{\mu\nu} x_\rho\,, \tag{2}$$
with particularly chosen coefficients $\theta^{\rho}_{\mu\nu}$ being constants. This type of non-commutativity has been obtained as the representations of the κ-Poincaré [16] and κ-Galilei [17] as well as the twisted relativistic [18] and non-relativistic [15] symmetries, respectively.

3. Quadratic deformation of Minkowski and Galilei space
$$[x_\mu, x_\nu] = i\theta^{\rho\tau}_{\mu\nu} x_\rho x_\tau\,, \tag{3}$$
with coefficients $\theta^{\rho\tau}_{\mu\nu}$ being constants. This kind of deformation has been proposed in [18,19,20] at relativistic and in [15] at non-relativistic level.

   Besides, it has been demonstrated in [21], that in the case of so-called acceleration-enlarged Newton–Hooke Hopf algebras $\mathcal{U}_0(\widehat{\mathrm{NH}}_\pm)$ the (twist) deformation provides the new space-time non-commutativity, which is expanding ($\mathcal{U}_0(\widehat{\mathrm{NH}}_+)$) or periodic ($\mathcal{U}_0(\widehat{\mathrm{NH}}_-)$) in time, *i.e.* it takes the form[1,2]

4. $$[t, x_i] = 0\,, \qquad [x_i, x_j] = if_\pm\left(\frac{t}{\tau}\right)\theta_{ij}(x)\,, \tag{4}$$
with time-dependent functions
$$f_+\left(\frac{t}{\tau}\right) = f\left(\sinh\left(\frac{t}{\tau}\right), \cosh\left(\frac{t}{\tau}\right)\right)\,,$$
$$f_-\left(\frac{t}{\tau}\right) = f\left(\sin\left(\frac{t}{\tau}\right), \cos\left(\frac{t}{\tau}\right)\right)\,,$$
and $\theta_{ij}(x) \sim \theta_{ij} = \text{const.}$ or $\theta_{ij}(x) \sim \theta^{k}_{ij} x_k$. Such a kind of non-commutativity follows from the presence in acceleration-enlarged

[1] The $\mathcal{U}_0(\widehat{\mathrm{NH}}_\pm)$ acceleration-enlarged Newton–Hooke Hopf structures are obtained by adding to the $\widehat{\mathrm{NH}}_\pm$ algebras (see [22,23]) the trivial coproduct $\Delta_0(a) = a \otimes 1 + 1 \otimes a$.

[2] $x_0 = ct$.

Newton–Hooke symmetries $\mathcal{U}_0(\widehat{\mathrm{NH}}_\pm)$ of the time scale parameter (cosmological constant) τ. As it was demonstrated in [21] this very parameter is responsible for oscillation or expansion of space-time noncommutativity.

It should be noted that both Hopf structures $\mathcal{U}_0(\widehat{\mathrm{NH}}_\pm)$ contain, apart from rotation (M_{ij}), boost (K_i) and space-time translation (P_i, H) generators, the additional ones denoted by F_i, responsible for constant acceleration. Consequently, if all generators F_i are equal zero we obtain the twisted Newton–Hooke quantum space-times [24], while for time parameter τ running to infinity we get the acceleration-enlarged twisted Galilei Hopf structures proposed in [21]. In particular, due to the presence of generators F_i, for $\tau \to \infty$ we get the new cubic and quartic type of space-time non-commutativity

$$[x_\mu, x_\nu] = i\alpha_{\mu\nu}^{\rho_1 \ldots \rho_n} x_{\rho_1} \ldots x_{\rho_n}\,, \tag{5}$$

with $n = 3$ and 4 respectively, whereas for $F_i \to 0$ and $\tau \to \infty$ we reproduce the canonical (1), Lie-algebraic (2) and quadratic (3) (twisted) Galilei spaces provided in [15]. Finally, it should be noted, that all mentioned above non-commutative space-times have been defined as the quantum representation spaces, the so-called Hopf modules (see [25, 26, 13, 14]), for quantum acceleration-enlarged Newton–Hooke Hopf algebras, respectively.

Recently, in the series of papers [27,28,29,30,31,32,33,34,35,36] there has been discussed the impact of different kinds of space-time non-commutativity on the structure of physical systems. More preciously, it has been demonstrated that in the case of classical Newtonian models there are generated by quantum spaces additional force terms, which appear in Newton equation. Such an observation permitted to analyze the Pioneer anomaly phenomena [30] with use of the classical non-relativistic particle model defined on κ-Galilei quantum space-time [17]. Besides, there has been suggested in [33], that deformation of classical systems can be identified with their non-inertial transformation, while the forces of inertia should be identical with force terms produced by space-time non-commutativity.

In this article, we check which forces generated by mentioned above acceleration-enlarged Newton–Hooke quantum spaces remain conservative. We perform our investigations in the context of two simplest physical systems — non-relativistic particle moving in constant external field force $\vec{F}$ and the classical oscillator model. Besides, it should be noted that we consider only non-commutative space-times equipped with classical time and quantum spatial directions, *i.e.* we consider spaces of the form

$$[t, x_i] = 0\,, \qquad [x_1, x_2] = if(t)\,, \qquad [x_1, x_3] = 0 = [\,x_2, x_3]\,, \qquad i = 1, 2, 3\,, \tag{6}$$

with function $f(t)$ given by

$$f(t) = f_{\kappa_1}(t) = f_{\pm,\kappa_1}\left(\frac{t}{\tau}\right) = \kappa_1\, C_{\pm}^2\left(\frac{t}{\tau}\right), \tag{7}$$

$$f(t) = f_{\kappa_2}(t) = f_{\pm,\kappa_2}\left(\frac{t}{\tau}\right) = \kappa_2\tau\, C_{\pm}\left(\frac{t}{\tau}\right) S_{\pm}\left(\frac{t}{\tau}\right), \tag{8}$$

$$f(t) = f_{\kappa_3}(t) = f_{\pm,\kappa_3}\left(\frac{t}{\tau}\right) = \kappa_3\tau^2\, S_{\pm}^2\left(\frac{t}{\tau}\right), \tag{9}$$

$$f(t) = f_{\kappa_4}(t) = f_{\pm,\kappa_4}\left(\frac{t}{\tau}\right) = 4\kappa_4\tau^4\left(C_{\pm}\left(\frac{t}{\tau}\right) - 1\right)^2, \tag{10}$$

$$f(t) = f_{\kappa_5}(t) = f_{\pm,\kappa_5}\left(\frac{t}{\tau}\right) = \pm\kappa_5\tau^2\left(C_{\pm}\left(\frac{t}{\tau}\right) - 1\right) C_{\pm}\left(\frac{t}{\tau}\right), \tag{11}$$

$$f(t) = f_{\kappa_6}(t) = f_{\pm,\kappa_6}\left(\frac{t}{\tau}\right) = \pm\kappa_6\tau^3\left(C_{\pm}\left(\frac{t}{\tau}\right) - 1\right) S_{\pm}\left(\frac{t}{\tau}\right); \tag{12}$$

$$C_{+/-}\left(\frac{t}{\tau}\right) = \cosh/\cos\left(\frac{t}{\tau}\right), \qquad \text{and} \qquad S_{+/-}\left(\frac{t}{\tau}\right) = \sinh/\sin\left(\frac{t}{\tau}\right).$$

As it was already mentioned, in $\tau \to \infty$ limit the above quantum spaces reproduce the canonical (1), Lie-algebraic (2), quadratic (3) as well as cubic and quartic (5) type of space-time non-commutativity, with[3]

$$f_{\kappa_1}(t) = \kappa_1, \tag{13}$$

$$f_{\kappa_2}(t) = \kappa_2\, t, \tag{14}$$

$$f_{\kappa_3}(t) = \kappa_2\, t^2, \tag{15}$$

$$f_{\kappa_4}(t) = \kappa_4\, t^4, \tag{16}$$

$$f_{\kappa_5}(t) = \tfrac{1}{2}\kappa_5\, t^2, \tag{17}$$

$$f_{\kappa_6}(t) = \tfrac{1}{2}\kappa_6\, t^3. \tag{18}$$

Of course, for all parameters κ_a running to zero the above deformations disappear.

Let us now turn to the mentioned above dynamical models. Firstly, we start with following phase space[4]

$$\{t, \bar{x}_i\} = 0, \qquad \{\bar{x}_1, \bar{x}_2\} = f_{\kappa_a}(t), \qquad \{\bar{x}_1, \bar{x}_3\} = 0 = \{\bar{x}_2, \bar{x}_3\}, \tag{19}$$

$$\{\bar{x}_i, \bar{p}_j\} = \delta_{ij}, \qquad \{\bar{p}_i, \bar{p}_j\} = 0, \tag{20}$$

[3] Space-times (13)–(15) correspond to the twisted Galilei Hopf algebras provided in [15], while the quantum spaces (16)–(18) are associated with acceleration-enlarged Galilei Hopf structures [21].

[4] We use the correspondence relation $\{a, b\} = \frac{1}{i}[\hat{a}, \hat{b}]$ ($\hbar = 1$).

corresponding to the quantum space-time (6). One can check that the relations (19), (20) satisfy the Jacobi identity and for deformation parameters κ_a running to zero become classical. Next, we define the Hamiltonian function for non-relativistic particle moving in constant field force $\vec{F}$ as follows

$$H(\bar{p},\bar{x}) = \frac{1}{2m}\left(\bar{p}_1^2+\bar{p}_2^2+\bar{p}_3^2\right) - \sum_{i=1}^{3} F_i\bar{x}_i\,. \tag{21}$$

In order to analyze the above system, we represent the non-commutative variables $(\bar{x}_i,\bar{p}_i)$ on classical phase space (x_i,p_i) as (see *e.g.* [34, 35, 36])

$$\bar{x}_1 = x_1 - \frac{f_{\kappa_a}(t)}{2}p_2\,, \qquad \bar{x}_2 = x_2 + \frac{f_{\kappa_a}(t)}{2}p_1\,, \qquad \bar{x}_3 = x_3\,, \qquad \bar{p}_i = p_i\,, \tag{22}$$

where

$$\{x_i,x_j\} = 0 = \{p_i,p_j\}\,, \qquad \{x_i,p_j\} = \delta_{ij}\,. \tag{23}$$

Then, the Hamiltonian (21) takes the form

$$\begin{aligned} H(p,x) &= H_f(t) \\ &= \frac{1}{2m}\left(p_1^2+p_2^2+p_3^2\right) - \sum_{i=1}^{3} F_i x_i + F_1\frac{f_{\kappa_a}(t)}{2}p_2 - F_2\frac{f_{\kappa_a}(t)}{2}p_1\,. \end{aligned} \tag{24}$$

Using the formulas (23) and (24) one gets the following canonical Hamiltonian equations of motions ($\dot{o}_i = \frac{d}{dt}o_i = \{o_i,H\}$)

$$\dot{x}_1 = \frac{p_1}{m} - \frac{f_{\kappa_a}(t)}{2}F_2\,, \qquad \dot{p}_1 = F_1\,, \tag{25}$$

$$\dot{x}_2 = \frac{p_2}{m} + \frac{f_{\kappa_a}(t)}{2}F_1\,, \qquad \dot{p}_2 = F_2\,, \tag{26}$$

$$\dot{x}_3 = \frac{p_3}{m}\,, \qquad \dot{p}_3 = F_3\,, \tag{27}$$

which, when combined, yield the equations

$$\left\{ \begin{array}{lll} m\ddot{x}_1 &=& F_1 - \frac{m\dot{f}_{\kappa_a}(t)}{2}F_2 = G_1(t)\,, \\ m\ddot{x}_2 &=& F_2 + \frac{m\dot{f}_{\kappa_a}(t)}{2}F_1 = G_2(t)\,, \\ m\ddot{x}_3 &=& F_3 = G_2\,. \end{array} \right. \tag{28}$$

First of all, by trivial integration one can find the solution of above system; it looks as follows

$$x_1(t) = \frac{F_1}{2m}t^2 + v_1^0 t - \frac{F_2}{2}\int_0^t f_{\kappa_a}(t')dt' \,,$$

$$x_2(t) = \frac{F_2}{2m}t^2 + v_2^0 t + \frac{F_1}{2}\int_0^t f_{\kappa_a}(t')dt' \,,$$

$$x_3(t) = \frac{1}{2m}F_3 t^2 + v_2^0 t + x_3^0 \,, \tag{29}$$

with v_i^0 and x_3^0 denoting the initial velocity and position of particle, respectively. Further, one should observe that the non-commutativity (6) generates the new, time-dependent force term $\vec{G}(t) = [G_1(t), G_2(t), G_3]$ which, for deformation parameters κ_a approaching zero, reproduces undeformed force $\vec{F}$. Besides, it should be noted that for $f(t) = \kappa_1 = \theta$ and $f(t) = \kappa_2 t$ (see formulas (13) and (14) respectively) we recover two models provided in [31]. First of them does not introduce any modification of Newton equation, while the second one generates the constant acceleration of particle. Finally, let us notice that for arbitrary function $f_{\kappa_a}(t)$ the rotation of force $\vec{G}(t)$ vanishes

$$\vec{\nabla} \times \vec{G}(t) = 0 \,, \tag{30}$$

i.e. the generated by space-time non-commutativity (6) force term (28) remains conservative, and the corresponding (non-stationary) potential function takes the form[5]

$$V(\vec{x}, t) = -\sum_{i=1}^{3} F_i x_i - \frac{\dot{f}_{\kappa_a}(t)}{2}\left(F_1 x_2 - F_2 x_1\right) \,. \tag{31}$$

Let us now turn to the second dynamical system — to the harmonic oscillator model described by

$$H\left(\bar{p}, \bar{x}\right) = \frac{1}{2m}\left(\bar{p}_1^2 + \bar{p}_2^2 + \bar{p}_3^2\right) + \frac{m\omega^2}{2}\left(\bar{x}_1^2 + \bar{x}_2^2 + \bar{x}_3^2\right) \,, \tag{32}$$

where m and ω denote the mass of particle and frequency of oscillation, respectively. Using transformation rules (22) one can rewrite the above Hamiltonian function in terms of commutative variables as follows

$$H_f(t) = \frac{\left(p_1^2 + p_2^2\right)}{2M_f(t)} + \frac{m\omega^2}{2}\left(x_1^2 + x_2^2\right) + \frac{f_{\kappa_a}(t) m\omega^2 L_3}{2} + \frac{p_3^2}{2m} + \frac{m\omega^2 x_3^2}{2} \,, \tag{33}$$

[5] $\vec{G}(t) = -\mathrm{grad}\, V(\vec{x}, t)$.

with

$$M_f(t) = \frac{m}{1 + \frac{m^2\omega^2}{4} f_{\kappa_a}^2(t)}\,, \qquad L_3 = x_1p_2 - x_2p_1\,. \tag{34}$$

Next, we find the corresponding Newton equation which takes the form [32]

$$\left\{\begin{array}{rcl} m\ddot{x}_1 &=& \frac{m^2\omega^2 f_{\kappa_a}(t)}{2}\left(\dot{f}_{\kappa_a}(t)M_f(t)\dot{x}_1 + 2\dot{x}_2\right) \\ && + \frac{m^2\omega^2 \dot{f}_{\kappa_a}(t)}{2}\left(1 - \frac{m\omega^2 M_f(t)}{2} f_{\kappa_a}^2(t)\right) x_2 - m\omega^2 x_1 = H_1(\vec{x}, \dot{\vec{x}}, t)\,, \\ m\ddot{x}_2 &=& \frac{m^2\omega^2 f_{\kappa_a}(t)}{2}\left(\dot{f}_{\kappa_a}(t)M_f(t)\dot{x}_2 - 2\dot{x}_1\right) \\ && + \frac{m^2\omega^2 \dot{f}_{\kappa_a}(t)}{2}\left(\frac{m\omega^2 M_f(t)}{2} f_{\kappa_a}^2(t) - 1\right) x_1 - m\omega^2 x_2 = H_2(\vec{x}, \dot{\vec{x}}, t)\,, \\ m\ddot{x}_3 &=& -m\omega^2 x_3 = H_3(\vec{x})\,. \end{array}\right. \tag{35}$$

Firstly, it should be noted that the solution of the above system has been studied numerically in [32] only for most simple (canonical) case. Besides, one can observe that for function $f(t)$ approaching zero the discussed deformation disappears. Finally, by simple calculation one can find the rotation of generated by space-time non-commutativity (6) force $\vec{H}(\vec{x}, \dot{\vec{x}}, t)$; it looks as follows

$$\vec{\nabla} \times \vec{H}\left(\vec{x}, \dot{\vec{x}}, t\right) = \hat{e}_3 m^2\omega^2 \dot{f}_{\kappa_a}(t)\left[\frac{m\omega^2 M_f(t)}{2} f_{\kappa_a}^2(t) - 1\right]. \tag{36}$$

It is easy to see, that r.h.s. of the above identity vanishes only for canonical deformation (13), and then, the generated force term takes the form

$$H_1\left(\vec{x}, \dot{\vec{x}}\right) = -m\omega^2 x_1 + m^2\omega^2\theta\dot{x}_2\,, \tag{37}$$

$$H_2\left(\vec{x}, \dot{\vec{x}}\right) = -m\omega^2 x_2 - m^2\omega^2\theta\dot{x}_1\,, \tag{38}$$

$$H_3(\vec{x}) = -m\omega^2 x_3\,. \tag{39}$$

Of course, in the case of remaining spaces the obtained forces become non-conservative.

Let us summarize our results. In this article we investigate the simple property of force terms generated by different types of quantum spaces, *i.e.* we check which of them remain conservative. We perform our investigations in context of two basic systems: non-commutative particle moving in external constant field force $\vec{F}$ and harmonic oscillator model. Particularly, we demonstrate that in the case of first dynamical system all considered quantum space-times produce conservative force terms, while for the second model such a situation appears only for the canonical type of noncommutativity. This result confirms that the canonical deformation is close to the undeformed one, *i.e.* for canonically deformed quantum space both analyzed models do not change the conservative nature of its dynamics. Finally, it should be noted that the obtained results describe only two basic (mentioned above) classical systems. Obviously, our kind of investigations can be applied to much more complicated physical models. However, the main aim of this article is only to signalize and to illustrate an interesting problem, and the choice of such simple systems is dictated by technical transparency of performed calculations.

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