THE HEAVY BARYON MASSES IN VARIATIONAL APPROACH AND SPIN–ISOSPIN DEPENDENCE

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By using the variational approach, we have studied the strange, charmed and beauty baryons masses. The considered potential is Coulomb as well as linear confining terms and the spin–isospin dependent potential is regarded as a perturbation, too. Some numerical results are given for spectra of heavy baryons and compared with experiments or other works.

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1. Introduction

The investigation of hadrons containing heavy quarks is of great interest in understanding the dynamics of QCD at the hadronic scale. There is renewed interest both experimentally and theoretically in the static properties of heavy flavor baryons such as mass and magnetic moments [1, 2, 3, 4, 5, 6]. Ebert *et al.* [7] calculated the masses of baryons with two heavy quarks in the framework of the relativistic quasipotential quark model and computed baryons with $j^p = \frac{1}{2}^+, \frac{3}{2}^+$. Using the hypercentral description of the three-body problem, Patel *et al.* [8,9,10,11] studied heavy flavor baryons containing single and double charm (beauty) quarks with light flavor combinations and assumed the confinement potential as hypercentral Coulomb plus power potential with power index ν. Roncaglia *et al.* [12] studied baryons using Feynman–Hellmann theorem and semi-empirical mass formula within the framework of a non-relativistic constituent quark model.

Bowler *et al.* [13] presented the results of the first lattice study of semi-leptonic decay of baryons containing a b quark and computed the Isgur–Wise functions for heavy baryons. Kiselev and Likhoded [14] extended the effective theory of heavy quarks to systems with two heavy quarks and one light quark and calculated the masses, decay widths and yields of doubly heavy baryons. Aaltonen *et al.* [15] reported an observation of new bottom baryons produced in $p\overline{p}$ collision at the Tevatron. Jenkins [16] calculated unknown heavy baryons masses based on an expansion in $\frac{1}{m_Q}$, $\frac{1}{N_C}$ and SU(3) breaking. Bagan *et al.* [17] studied the masses and couplings of baryons made with two heavy quarks using QCD spectral sum rules (QSSR). Ferabetti *et al.* [18] presented evidence for Ω_c^0 in new mode $\Omega_c^0 \longrightarrow \Sigma^+ K^- K^- \Pi^+$. Gershtein *et al.* [19] calculated the spectra of masses for families of doubly heavy baryons in the framework of non-relativistic quark model with the QCD potential of Buchmüller and Tye.

The purpose of this paper is to calculate the masses of the heavy baryons in a simple approximation, developed in [20, 21, 22, 23, 24]. In this work, the considered potential is a long-range linear confinement part as well as a short-range potential, which is a coulomb one, depending on the color charge. Extra non-confining interquark potential, which contains spin dependent $H_{\mathrm{S}}(x)$, isospin dependent $H_{\mathrm{I}}(x)$ and spin–isospin dependent $H_{\mathrm{SI}}(x)$ parts, are also considered as perturbation terms [25, 26, 27]. Narodetskii and Trusov [24] studied the similar work but only calculated the masses of baryons containing two heavy quarks without spin–spin term. We extend it to calculate masses of baryons containing one, two and three heavy quarks with spin, isospin and spin–isospin dependent potential. However, isospin and spin–isospin has no effect on the masses of baryons containing two or three heavy quarks.

2. Interaction potential model

The Coulomb-plus-linear potential, $V(x) = -\frac{a}{x} + bx$, also known as the Cornell potential, has received a great deal of attention both in particle physics, more precisely in the context of meson spectroscopy where it is used to describe systems of quark and antiquark bound states, and in atomic and molecular physics where it represents a radial Stark effect in hydrogen. This potential was used with considerable success in models describing systems of bound heavy quarks [24, 28, 29]. The potential includes the short distance Coulombic interaction of quarks, known from perturbative quantum chromodynamics (QCD), and the large distance quark confinement, known from lattice QCD, via the linear term in a simple form. Coulombic term alone is not sufficient because it would allow free quarks to ionize from the system. All of our results presented in this work will be based on the Cornell potential, *i.e.*

$$V(x) = -\frac{a}{x} + bx\,, \tag{1}$$

where a and b are constants and x is the hyperradius. The non-confining potential contains a δ-like term that is modified by a Gaussian of the quark pair relative distance [30]

$$H_{\mathrm{S}} = A_{\mathrm{S}} \left(\frac{1}{\sqrt{\pi}\sigma_{\mathrm{S}}}\right)^3 \exp\left(\frac{-x^2}{\sigma_{\mathrm{S}}^2}\right) \left(\overrightarrow{S_1} \cdot \overrightarrow{S_2}\right)\,, \tag{2}$$

where $\overrightarrow{S_i}$ is the spin operator of the i-th quark. The non-confining potential (2) is provided by the interaction with the Goldstone bosons, which gives rise to a spin- and isospin-dependent part. This is good to describe the spectrum for energies lower than 1.7 GeV [25, 26]. Recently, it has also been pointed out that an isospin dependence of the quark potential can be obtained by means of quark exchange [31]. More generally, one can expect that the quark–quark pair production can lead to an effective quark interaction containing an isospin- (or flavor-)dependent term [31, 32]. With these motivations in mind, we have introduced isospin-dependent terms. Finally, we have added two terms in the Hamiltonian quark–quark pairs with hyperfine interaction similar to Eq. (2). The first one depends on the isospin only and has the form [30, 33]

$$H_{\mathrm{I}} = A_{\mathrm{I}} \left(\frac{1}{\sqrt{\pi}\sigma_{\mathrm{I}}}\right)^3 \exp\left(\frac{-x^2}{\sigma_{\mathrm{I}}^2}\right) \left(\overrightarrow{t_1} \cdot \overrightarrow{t_2}\right)\,, \tag{3}$$

where $\overrightarrow{t_i}$ is the isospin operator of the i-th quarks. The second one is a spin–isospin interaction, given by

$$H_{\mathrm{SI}} = A_{\mathrm{SI}} \left(\frac{1}{\sqrt{\pi}\sigma_{\mathrm{SI}}}\right)^3 \exp\left(\frac{-x^2}{\sigma_{\mathrm{SI}}^2}\right) \left(\overrightarrow{S_1} \cdot \overrightarrow{S_2}\right) \left(\overrightarrow{t_1} \cdot \overrightarrow{t_2}\right)\,, \tag{4}$$

where $\overrightarrow{S_i}$ and $\overrightarrow{t_i}$ are the spin and isospin operators of the i-th quark respectively. Then from Eqs. (2), (3), (4), the hyperfine interaction or non-confining potential is given by

$$H_{\mathrm{int}}(x) = H_{\mathrm{S}}(x) + H_{\mathrm{I}}(x) + H_{\mathrm{SI}}\,. \tag{5}$$

The values of the hyperfine interaction constants are displayed in Table I.

In the next section, we obtain the wave function and energy of system in the framework of the simple approximation with Cornell potential (1).

TABLE I

The values of the hyperfine interaction constants [30].

A_{S}	σ_{S}	A_{I}	σ_{I}	A_{SI}	σ_{SI}
67.4 $(\mathrm{fm})^2$	2.87 fm	51.7 $(\mathrm{fm})^2$	3.45 fm	-106.2 $(\mathrm{fm})^2$	2.31 fm

3. The wave function and energy for confining potential; hyper central model

To describe the baryon as a bound state of three constituent quarks, we define the configuration of three particles by the Jacobi coordinates ρ and λ as [11, 30]

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2})\,, \qquad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r_1} + \vec{r_2} - 2\vec{r_3}) \tag{6}$$

such that

$$m_\rho = \frac{2m_1m_2}{m_1+m_2}\,, \qquad m_\lambda = \frac{3m_3\,(m_1+m_2)}{2\,(m_1+m_2+m_3)}\,, \tag{7}$$

where m_1, m_2 and m_3 are the constituent quark masses. Instead of ρ and λ, one can introduce the hyperspherical coordinates, which are given by the angles $\Omega_\rho = (\theta_\rho, \phi_\rho)$ together with the hyperradius x, and the hyperangle ζ, defined respectively by [27, 34, 35, 36, 37]

$$x = \sqrt{\rho^2 + \lambda^2}\,, \qquad \xi = \arctan\left(\sqrt{\frac{\rho}{\lambda}}\right)\,. \tag{8}$$

Therefore, the Hamiltonian will be

$$H = \frac{P_\rho^2}{2m} + \frac{P_\lambda^2}{2m} + V(x)\,, \tag{9}$$

where m is the reduced mass as

$$m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda}\,. \tag{10}$$

In the hypercentral constituent quark model (hCQM), the quark potential v is assumed to depend on the hyperradius x only that is to be hypercentral. Therefore, $V = V(x)$ is, in general, a three-body potential, and the hyperradius x depends on the coordinates of all the three quarks. In the three-quark wave function one can factor out the hyperangular part, which is given by hyperspherical harmonics [20]. The remaining hyperradial part of the wave function is determined by hypercentral Schrödinger equation as follows [8, 11, 27, 30]

$$\left[\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{\gamma(\gamma+4)}{x^2}\right]\psi_\gamma(x) = -2m\left[E - V(x)\right]\psi_\gamma(x)\,, \tag{11}$$

where γ is the grand angular quantum number given by $\gamma = 2\upsilon + l_\rho + l_\lambda$; l_ρ and l_λ are the angular momenta associated with the $\vec{\rho}$ and $\vec{\lambda}$ variables and υ is a non-negative integer number. We investigate the ground state of baryons ($\gamma = 0$). For ground state of baryons and using the hyper radial approximation, where has been introduced in Ref. [24], the Schrödinger equation is given as

$$\frac{d^2\chi(x)}{dx^2} + 2\mu\left[E - V(x) - \frac{15}{8\mu x^2}\right]\chi(x) = 0\,, \tag{12}$$

where μ is an arbitrary parameter with the dimension of mass and $\chi(x)$ is the reduced function as follows

$$\chi(x) = x^{\frac{5}{2}}\psi(x) \tag{13}$$

and $V(x)$ is the three-quark potentials over the six-dimensional sphere that was defined in Eq. (1). We use a new variable $x' = \sqrt{\mu x}$, to eliminate an artificial dependence of Eq. (12) on μ, then the equation (12) becomes

$$\chi''(x') + 2\left[E - V(x') - \left(\frac{15}{8x'^2}\right)\right]\chi(x') = 0\,, \tag{14}$$

where

$$V(x') = -\frac{a\sqrt{\mu}}{x'} + \frac{b}{\sqrt{\mu}}x'\,. \tag{15}$$

We can solve Eq. (14) by the variational method. We introduce a simple variational ansatz for $\chi(x')$ as

$$\chi(x') = 2\sqrt{2}p^3 x'^{\frac{5}{2}} e^{-P^2 x'^2}\,, \tag{16}$$

where p is the variational parameter, and the numerical factor is chosen so that $\int \chi^2(x')dx' = 1$. The trial three-quark Hamiltonian admits explicit solutions for the wave function and the energy $E_0 = \min E(p)$ where

$$E(p) = \langle\chi|H|\chi\rangle = 3p^2 - a\sqrt{\mu}\,\frac{3}{4}\sqrt{\frac{\pi}{2}}\,p + \frac{b}{\sqrt{\mu}}\,\frac{15}{16}\sqrt{\frac{\pi}{2}}\,p^{-1}\,. \tag{17}$$

Now by using the condition $\frac{dE}{dp}|_{p=p_0} = 0$, the value of p_0 is found.

4. The heavy baryon masses

The baryon masses are given by three quark masses and the energy E_0 which is a function of a and b, with the hyperfine interaction potential $\langle H_{\rm int}\rangle$ treated as a perturbation. The first order energy correction from the non-confining potential $\langle H_{\rm int}\rangle$, as given in (5), can be obtained by using the unperturbed wave function (16) as follows

$$\langle H_{\rm int}\rangle = \int \chi H_{\rm int}\chi dx' \,. \tag{18}$$

Therefore, the Baryon mass then becomes the sum of quark masses and energy of perturbed system, thus [27]

$$M_{\rm baryon} = m_{q_1} + m_{q_2} + m_{q_3} + E_0 + \langle H_{\rm int}\rangle \,. \tag{19}$$

This depends on the constituent quark masses m_q and potential parameters a and b that are listed in Table II.

TABLE II

The value of the potential parameters and quark masses.

a	b	m_u	m_d	m_s	m_c	m_b
4.59 [33]	1.61 fm^{-2} [33]	330 MeV	335 MeV	469 MeV	1.6 GeV	4.98 GeV

We use the experimental masses of Σ^+, Σ^0, Σ^-, Ξ^-, Ξ_c^+ and Σ_b^- as input to determine the quark masses and the value of μ. The potential parameters are obtained from Refs. [34, 35, 36] that the static properties of the baryons with Cornell potential have investigated. We obtained the heavy baryons mass and compared with experimental data or other theoretical model predictions in Tables III–VIII. In Table III, we calculated heavy baryon masses containing one strange quark.

TABLE III

Strange baryon masses (in MeV).

Baryon	$I\,(j^p)$	Present work	Exp. [38]
$\Sigma^+(uus)$	$1\left(\frac{1}{2}^+\right)$	1189	1189
$\Sigma^0(uds)$	$1\left(\frac{1}{2}^+\right)$	1192	1192
$\Sigma^-(dds)$	$1\left(\frac{1}{2}^+\right)$	1197	1197
$\Xi^0(uss)$	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	1317.8	1314
$\Xi^-(dss)$	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	1321	1321

Single charm and single beauty baryon masses are presented in Tables IV and V, respectively.

TABLE IV

Single charm baryon masses (in GeV).

Baryon	$I\,(j^p)$	Present work	[9]	Others	Exp.
$\Sigma_c^{++}(uuc)$	$1\left(\frac{1}{2}^+\right)$	2.318	2.443	2.460[13]	2.452[38]
$\Sigma_c^{*++}(uuc)$	$1\left(\frac{3}{2}^+\right)$	2.446	2.506	2.440[13]	—
$\Lambda_c^{+}(udc)$	$0\left(\frac{1}{2}^+\right)$	2.303	—	2.290[11]	2.284[38]
$\Sigma_c^{+}(udc)$	$1\left(\frac{1}{2}^+\right)$	2.323	2.460	2.453[12]	2.451[38]
$\Sigma_c^{*+}(udc)$	$1\left(\frac{3}{2}^+\right)$	2.451	2.525	2.520[12]	2.518[39]
$\Sigma_c^{0}(ddc)$	$1\left(\frac{1}{2}^+\right)$	2.328	2.477	2.466[8]	—
$\Sigma_c^{*0}(ddc)$	$1\left(\frac{3}{2}^+\right)$	2.456	2.544	2.533[8]	—
$\Xi_c^{+}(usc)$	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	2.467	2.53	2.468[12]	2.467[38]
$\Xi_c^{*+}(usc)$	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	2.577	2.603	2.650[12]	2.646[38]
$\Xi_c^{0}(dsc)$	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	2.453	2.548	2.536[8]	2.471[38]
$\Xi_c^{*0}(dsc)$	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	2.582	2.623	2.611[8]	2.646[38]
$\Omega_c^{0}(ssc)$	$0\left(\frac{1}{2}^+\right)$	2.587	2.620	2.696[11]	2.699[18]
$\Omega_c^{*0}(ssc)$	$0\left(\frac{3}{2}^+\right)$	2.716	2.704	2.757[11]	—

Baryon masses containing double and triple charm (beauty) quarks are shown in Table VI (VII).

Finally, in Table VIII, we presented baryon masses containing beauty and charm quarks. As we see from Tables I–VIII, our calculations are very close to the ones obtained in experiments or in the other works [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 38, 39].

TABLE V

Single beauty baryon masses (in GeV).

Baryon	$I\,(j^p)$	Present work	[11]	[16]	Exp.
$\Sigma_b^+(uub)$	$1\left(\frac{1}{2}^+\right)$	5.700	5.801	5.824	5.807[15]
$\Sigma_b^{*+}(uub)$	$1\left(\frac{3}{2}^+\right)$	5.826	5.823	5.840	5.829[15]
$\Lambda_b^0(udb)$	$0\left(\frac{1}{2}^+\right)$	5.683	5.629	—	5.624[38]
$\Sigma_b^-(ddb)$	$1\left(\frac{1}{2}^+\right)$	5.708	5.821	—	5.815[15]
$\Sigma_b^{*-}(ddb)$	$1\left(\frac{3}{2}^+\right)$	5.836	5.844	—	5.836[15]
$\Xi_b^0(usb)$	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	5.828	5.872	5.805	5.792[15]
$\Xi_b^{*0}(usb)$	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	5.957	5.936	5.996	—
$\Xi_b^-(dsb)$	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	5.833	5.887	—	—
$\Xi_b^{*-}(dsb)$	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	5.962	5.943	—	—
$\Omega_b^-(ssb)$	$0\left(\frac{1}{2}^+\right)$	5.967	6.005	6.068	—
$\Omega_b^{*-}(ssb)$	$0\left(\frac{3}{2}^+\right)$	6.096	6.065	6.083	—

TABLE VI

Double and triple charm baryon masses (in GeV).

Baryon	$I\,(j^p)$	Present work	[9]	[14]	Exp.
$\Xi_{cc}^{++}(ucc)$	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	3.579	3.730	3.480	3.519[38]
$\Xi_{cc}^{*++}(ucc)$	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	3.708	3.800	3.610	—
$\Xi_{cc}^+(dcc)$	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	3.584	3.755	3.480	—
$\Xi_{cc}^{*+}(dcc)$	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	3.713	3.828	3.610	—
$\Omega_{cc}^+(scc)$	$0\left(\frac{1}{2}^+\right)$	3.718	3.857	3.590	—
$\Omega_{cc}^{*+}(scc)$	$0\left(\frac{3}{2}^+\right)$	3.847	3.944	3.690	—
$\Omega_{ccc}^{*++}(ccc)$	$0\left(\frac{3}{2}^+\right)$	4.978	—	4.736[10]	—

TABLE VII

Double and triple beauty baryon masses (in GeV).

Baryon	$I\,(j^p)$	Present work	[11]	Others
$\Xi^0_{bb}(ubb)$	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	10.339	10.114	10.093[19]
$\Xi^{*0}_{bb}(ubb)$	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	10.468	10.165	10.330[17]
$\Xi^-_{bb}(dbb)$	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	10.344	10.117	—
$\Xi^{*-}_{bb}(dbb)$	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	10.473	10.170	—
$\Omega^-_{bb}(sbb)$	$0\left(\frac{1}{2}^+\right)$	10.478	10.164	10.340[24]
$\Omega^{*-}_{bb}(usb)$	$0\left(\frac{3}{2}^+\right)$	10.607	10.236	—
$\Omega^{*-}_{bbb}(bbb)$	$0\left(\frac{3}{2}^+\right)$	15.118	—	14.444[10]

TABLE VIII

Beauty and charm baryons masses (in GeV).

Baryon	$I\,(j^p)$	Present work	Others
$\Omega^+_{cb}(ucb)$	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	6.959	6.950[7]
$\Omega^0_{cb}(scb)$	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	7.098	7.050[7]
$\Omega^+_{ccb}(ccb)$	$\frac{1}{2}\left(\frac{1}{2}^+\right)$	8.229	8.089[10]
$\Omega^{*+}_{ccb}(ccb)$	$\frac{1}{2}\left(\frac{3}{2}^+\right)$	8.358	8.099[10]
$\Omega^0_{bbc}(cbb)$	$0\left(\frac{1}{2}^+\right)$	11.609	11.354[10]
$\Omega^{*0}_{bbc}(cbb)$	$0\left(\frac{3}{2}^+\right)$	11.738	11.394[10]

5. Conclusions

In this paper, we employed the hyperspherical formalism with potential of the coulomb plus power potential to study the masses of baryons containing heavy flavor quarks in the ground state. We solved the Schrödinger equation by the variational method. We have shown that it is possible to find baryon masses by a suitable confining and non-confining interaction potentials. Using the theory of time-independent perturbation and the wave function, we get the effects of spin and isospin potentials in the shift of baryon energy.

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