TWISTED ACCELERATION-ENLARGED NEWTON–HOOKE SPACE-TIMES AND BREAKING CLASSICAL SYMMETRY PHENOMENA

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(Received December 19, 2012)

We find the subgroup of classical acceleration-enlarged Newton–Hooke Hopf algebra which acts covariantly on the twisted acceleration-enlarged Newton–Hooke space-times. The case of classical acceleration-enlarged Galilei quantum group is considered as well.

DOI:10.5506/APhysPolB.43.51
PACS numbers: 02.20.Uw, 02.40.Gh

1. Introduction

Presently, physicists expect that there exist aberrations from relativistic kinematics in high energy (transplanckian) regime. Such a suggestion follows from many theoretical [1, 2] as well as experimental (see *e.g.* [3]) investigations performed in the last time.

Generally, there exist two approaches to describe the particle kinematics in ultra-high energy regime. First of them assumes that relativistic symmetry becomes broken at Planck's scale to the proper subgroup of Poincaré algebra [4, 5]. The second approach is more sofisticated, *i.e.* it assumes that relativistic symmetry is still present in high energy regime, but it becomes deformed [6].

The first treatment has been proposed in [4, 5] where authors assumed that the whole Lorentz algebra is broken to the four subgroups: T(2), E(2), HOM(2) and SIM(2) identified with four versions of so-called Very Special Relativity. The second treatment arises from Quantum Group Theory [7, 8] which, in accordance with Hopf-algebraic classification of all relativistic and nonrelativistic deformations [9, 10], provides three types of quantum spaces.

First of them corresponds to the well-known canonical type of noncommutativity

$$[\,\hat{x}_\mu, \hat{x}_\nu\,] = i\theta_{\mu\nu}\,, \tag{1}$$

with antisymmetric constant tensor $\theta^{\mu\nu}$. Its relativistic and nonrelativistic Hopf-algebraic counterparts have been proposed in [11] and [12] respectively.

The second kind of mentioned deformations introduces the Lie-algebraic type of space-time noncommutativity

$$[\,\hat{x}_\mu, \hat{x}_\nu\,] = i\theta^{\rho}_{\mu\nu}\hat{x}_\rho\,, \tag{2}$$

with particularly chosen coefficients $\theta^{\rho}_{\mu\nu}$ being constants. The corresponding Poincaré quantum groups have been introduced in [13, 14, 15], while the suitable Galilei algebras — in [16] and [12].

The last kind of quantum space, so-called quadratic type of noncommutativity

$$[\,\hat{x}_\mu, \hat{x}_\nu\,] = i\theta^{\rho\tau}_{\mu\nu}\hat{x}_\rho\hat{x}_\tau\,; \qquad \theta^{\rho\tau}_{\mu\nu} = \text{const.} \tag{3}$$

has been proposed in [17, 18] and [15] at relativistic and in [19] at nonrelativistic level.

The links between both (mentioned above) approaches have been investigated recently in articles [20] and [21]. Preciously, it has been demonstrated that the very particular realizations of canonical, Lie-algebraic and quadratic space-time noncommutativity are covariant with respect the action of undeformed T(2), E(2) and HOM(2) subgroups respectively. Such a result seems to be quite interesting because it connects two different approaches to the same problem — to the form of Poincaré algebra at Planck's scale. It also confirms expectation that relativistic symmetry in high energy regime should be modified, while the realizations of such an idea by breaking or deforming of Poincaré algebra plays only the secondary role.

In this article we extend described above investigations to the case of classical acceleration-enlarged Newton–Hooke Hopf algebras $\mathcal{U}_0(\widehat{NH}_\pm)$ [22, 23]. Particularly, we find their subgroups which act covariantly on the following (provided in [24] and [25]) twist-deformed acceleration-enlarged Newton–Hooke space-times[1,2]

$$[\,t, x_i\,] = 0\,, \qquad [\,x_i, x_j\,] = i f_\pm\left(\frac{t}{\tau}\right)\,, \tag{4}$$

[1] $x_0 = ct$.

[2] It should be noted that symbol τ plays the role of time scale parameter (cosmological constant), which is responsible for oscillation or expansion of space-time noncommutativity (4). For τ approaching infinity we reproduce the canonical (1), Lie-algebraic (2) and quadratic (3) type of space-time noncommutativity.

with

$$f_+\left(\frac{t}{\tau}\right) = f\left(\sinh\left(\frac{t}{\tau}\right), \cosh\left(\frac{t}{\tau}\right)\right), \qquad f_-\left(\frac{t}{\tau}\right) = f\left(\sin\left(\frac{t}{\tau}\right), \cos\left(\frac{t}{\tau}\right)\right).$$

Further, by contraction limit of obtained results ($\tau \to \infty$), we analyze the case of so-called classical acceleration-enlarged Galilei Hopf algebra $\mathcal{U}_0(\widehat{G})$ proposed in [26].

The paper is organized as follows. In second section we describe the general algorithm used in present article. Sections 3 and 4 are devoted respectively to the subgroups of classical acceleration-enlarged Newton–Hooke as well as classical acceleration-enlarged Galilei Hopf symmetries acting covariantly on the proper (acceleration-enlarged) twist-deformed space-times (4). Final remarks are presented in the last section.

2. General prescription

In this section we describe the general algorithm which can be applied to the arbitrary twist deformation of space-time symmetries algebra $\mathcal{A}$.

First of all, we recall basic facts related with the twist-deformed quantum group $\mathcal{U}_{\mathcal{F}}(\mathcal{A})$ and with the corresponding quantum space-time. In accordance with general twist procedure [27], the algebraic sector of Hopf structure $\mathcal{U}_{\mathcal{F}}(\mathcal{A})$ remains undeformed, while the coproducts and antipodes transform as follows

$$\Delta_0(a) \to \Delta_{\mathcal{F}}(a) = \mathcal{F} \circ \Delta_0(a) \circ \mathcal{F}^{-1}, \qquad S_{\mathcal{F}}(a) = u_{\mathcal{F}}\, S_0(a)\, u_{\mathcal{F}}^{-1}, \tag{5}$$

with $\Delta_0(a) = a \otimes 1 + 1 \otimes a$, $S_0(a) = -a$ and $u_{\mathcal{F}} = \sum f_{(1)} S_0(f_{(2)})$ (we use Sweedler's notation $\mathcal{F} = \sum f_{(1)} \otimes f_{(2)}$). Present in the above formula twist element $\mathcal{F} \in \mathcal{U}_{\mathcal{F}}(\mathcal{A}) \otimes \mathcal{U}_{\mathcal{F}}(\mathcal{A})$ satisfies the classical cocycle condition

$$\mathcal{F}_{12} \cdot (\Delta_0 \otimes 1)\, \mathcal{F} = \mathcal{F}_{23} \cdot (1 \otimes \Delta_0)\, \mathcal{F}, \tag{6}$$

and the normalization condition

$$(\epsilon \otimes 1)\, \mathcal{F} = (1 \otimes \epsilon)\, \mathcal{F} = 1, \tag{7}$$

with $\mathcal{F}_{12} = \mathcal{F} \otimes 1$, $\mathcal{F}_{23} = 1 \otimes \mathcal{F}$ and

$$\Delta_0(a) = a \otimes 1 + 1 \otimes a. \tag{8}$$

The corresponding to the above Hopf structure space-time is defined as quantum representation space (Hopf module) with action of the symmetry generators satisfying suitably deformed Leibnitz rules [28, 11]

$$h \triangleright \omega_{\mathcal{F}}\left(f(x) \otimes g(x)\right) = \omega_{\mathcal{F}}\left(\Delta_{\mathcal{F}}(h) \triangleright f(x) \otimes g(x)\right), \tag{9}$$

for $h \in \mathcal{U}_{\mathcal{F}}(\mathcal{A})$ or $\mathcal{U}_{\mathcal{F}}(\mathcal{A})$ and

$$\omega_{\mathcal{F}}\left(f(x)\otimes g(x)\right) = \omega\circ\left(\mathcal{F}^{-1}\rhd f(x)\otimes g(x)\right); \qquad \omega\circ(a\otimes b) = a\cdot b\,. \tag{10}$$

The action of $\mathcal{U}_{\mathcal{F}}(\mathcal{A})$ algebra on its Hopf module of functions depending on space-time coordinates x_μ is given by

$$h\rhd f(x) = h\left(x_\mu,\partial_\mu\right) f(x)\,, \tag{11}$$

while the $\star_{\mathcal{F}}$-multiplication of arbitrary two functions is defined as follows

$$f(x)\star_{\mathcal{F}} g(x) := \omega\circ\left(\mathcal{F}^{-1}\rhd f(x)\otimes g(x)\right). \tag{12}$$

It should be also noted that the commutation relations

$$[\, x_\mu, x_\nu\,]_{\star_{\mathcal{F}}} = x_\mu \star_{\mathcal{F}} x_\nu - x_\nu \star_{\mathcal{F}} x_\mu \tag{13}$$

are covariant (by definition) with respect to the action of Hopf algebra generators (see deformed Leibnitz rules (9)).

In this article we consider the action of undeformed acceleration-enlarged Newton–Hooke as well as classical acceleration-enlarged Galilei Hopf algebras on the commutation relations (13) ($\mathcal{A} = \widehat{NH}_\pm$ or $\widehat{G}$). It is given by the particular realizations of differential representation (16) and new classical Leibnitz rules

$$h\rhd\omega_{\mathcal{F}}\left(f(x)\otimes g(x)\right) = \omega_{\mathcal{F}}\left(\Delta_0(h)\rhd f(x)\otimes g(x)\right) \tag{14}$$

associated with coproduct (8). Further, we demonstrate that in such a case the relations (13) are not invariant with respect to the action of the whole algebras $\mathcal{U}_0(\widehat{NH}_\pm)$ and $\mathcal{U}_0(\widehat{G})$, but only with respect to their proper subgroups. Such an effect can be identified with the breaking classical symmetry phenomena associated with twist-deformed space-times (13).

3. Breaking of classical acceleration-enlarged Newton–Hooke symmetry

In this section we turn to the case of undeformed acceleration-enlarged Newton–Hooke Hopf algebra $\mathcal{U}_0(\widehat{NH}_\pm)$ defined by the following algebraic sector[3]

[3] The both Hopf structures $\mathcal{U}_0(\widehat{NH}_\pm)$ contain, apart from rotation (M_{ij}), boost (K_i) and space-time translation (P_i, H) generators, the additional ones denoted by F_i, responsible for constant acceleration.

$$
\begin{aligned}
&[M_{ij},M_{kl}]=i\left(\delta_{il}\,M_{jk}-\delta_{jl}M_{ik}+\delta_{jk}M_{il}-\delta_{ik}M_{jl}\right)\,, \quad [H,P_i]=\pm\frac{i}{\tau^2}K_i\,,\\
&[M_{ij},K_k]=i\left(\delta_{jk}\,K_i-\delta_{ik}K_j\right)\,, \qquad [M_{ij},P_k]=i\left(\delta_{jk}\,P_i-\delta_{ik}P_j\right)\,,\\
&[M_{ij},H]=[K_i,K_j]=[K_i,P_j]=0\,, \qquad [K_i,H]=-iP_i\,, \qquad [P_i,P_j]=0\,,\\
&[F_i,F_j]=[F_i,P_j]=[F_i,K_j]=0\,, \qquad [M_{ij},F_k]=i\left(\delta_{jk}\,F_i-\delta_{ik}\,F_j\right)\,,\\
&[H,F_i]=2iK_i\,,
\end{aligned}
\tag{15}
$$

and classical coproduct (8). One can check that the above structure is represented on Hopf module of functions as follows (see formula (11))

$$
H\rhd f(t,\overline{x})=i\partial_t f(t,\overline{x})\,, \qquad P_i\rhd f(t,\overline{x})=iC_{\pm}\left(\frac{t}{\tau}\right)\partial_i f(t,\overline{x})\,, \tag{16}
$$

$$
M_{ij}\rhd f(t,\overline{x})=i\left(x_i\partial_j-x_j\partial_i\right)f(t,\overline{x})\,, \qquad K_i\rhd f(t,\overline{x})=i\tau\,S_{\pm}\left(\frac{t}{\tau}\right)\partial_i\,f(t,\overline{x})\,, \tag{17}
$$

and

$$
F_i\rhd f(t,\overline{x})=\pm 2i\tau^2\left(C_{\pm}\left(\frac{t}{\tau}\right)-1\right)\partial_i f(t,\overline{x})\,, \tag{18}
$$

with

$$
C_{+/-}\left(\frac{t}{\tau}\right)=\cosh/\cos\left(\frac{t}{\tau}\right) \qquad \text{and} \qquad S_{+/-}\left(\frac{t}{\tau}\right)=\sinh/\sin\left(\frac{t}{\tau}\right)\,.
$$

As it was already mentioned in Introduction the twist deformations of quantum group $\mathcal{U}_0(\widehat{NH}_{\pm})$ have been provided in [24]. Here, we take under consideration the twisted acceleration-enlarged Newton–Hooke space-times defined by the following twist factors

$$
\mathcal{F} = \mathcal{F}_{\alpha_1}=\exp\left[\frac{i}{4}\sum_{k,l=1}^{2}\alpha_1^{kl}P_k\wedge P_l\right] \qquad \left[\alpha_1^{kl}=-\alpha_1^{lk}=\alpha_1\right]\,, \tag{19}
$$

$$
\mathcal{F} = \mathcal{F}_{\alpha_2}=\exp\left[\frac{i}{4}\sum_{k,l=1}^{2}\alpha_2^{kl}K_k\wedge P_l\right] \qquad \left[\alpha_2^{kl}=-\alpha_2^{lk}=\alpha_2\right]\,, \tag{20}
$$

$$
\mathcal{F} = \mathcal{F}_{\alpha_3}=\exp\left[\frac{i}{4}\sum_{k,l=1}^{2}\alpha_3^{kl}K_k\wedge K_l\right] \qquad \left[\alpha_3^{kl}=-\alpha_3^{lk}=\alpha_3\right]\,, \tag{21}
$$

$$
\mathcal{F} = \mathcal{F}_{\alpha_4}=\exp\left[\frac{i}{4}\sum_{k,l=1}^{2}\alpha_4^{kl}F_k\wedge F_l\right] \qquad \left[\alpha_4^{kl}=-\alpha_4^{lk}=\alpha_4\right]\,, \tag{22}
$$

$$\mathcal{F} = \mathcal{F}_{\alpha_5} = \exp\left[\frac{i}{4}\sum_{k,l=1}^{2}\alpha_5^{kl}F_k\wedge P_l\right] \qquad \left[\alpha_5^{kl} = -\alpha_5^{lk} = \alpha_5\right], \quad (23)$$

$$\mathcal{F} = \mathcal{F}_{\alpha_6} = \exp\left[\frac{i}{4}\sum_{k,l=1}^{2}\alpha_6^{kl}K_k\wedge F_l\right] \qquad \left[\alpha_6^{kl} = -\alpha_6^{lk} = \alpha_6\right]. \quad (24)$$

In other words, we consider spaces of the form

$$[\, t, \hat{x}_i \,]_{\star_{\mathcal{F}}} = [\, \hat{x}_1, \hat{x}_3 \,]_{\star_{\mathcal{F}}} = [\, \hat{x}_2, \hat{x}_3 \,]_{\star_{\mathcal{F}}} = 0\,, \quad [\, \hat{x}_1, \hat{x}_2 \,]_{\star_{\mathcal{F}}} = if(t)\,; \quad i = 1,2,3\,, \quad (25)$$

with function $f(t)$ given by

$$f(t) = f_{\kappa_1}(t) = f_{\pm,\kappa_1}\left(\frac{t}{\tau}\right) = \kappa_1\, C_\pm^2\left(\frac{t}{\tau}\right), \quad (26)$$

$$f(t) = f_{\kappa_2}(t) = f_{\pm,\kappa_2}\left(\frac{t}{\tau}\right) = \kappa_2\tau\, C_\pm\left(\frac{t}{\tau}\right) S_\pm\left(\frac{t}{\tau}\right), \quad (27)$$

$$f(t) = f_{\kappa_3}(t) = f_{\pm,\kappa_3}\left(\frac{t}{\tau}\right) = \kappa_3\tau^2\, S_\pm^2\left(\frac{t}{\tau}\right), \quad (28)$$

$$f(t) = f_{\kappa_4}(t) = f_{\pm,\kappa_4}\left(\frac{t}{\tau}\right) = 4\kappa_4\tau^4\left(C_\pm\left(\frac{t}{\tau}\right) - 1\right)^2, \quad (29)$$

$$f(t) = f_{\kappa_5}(t) = f_{\pm,\kappa_5}\left(\frac{t}{\tau}\right) = \pm\kappa_5\tau^2\left(C_\pm\left(\frac{t}{\tau}\right) - 1\right) C_\pm\left(\frac{t}{\tau}\right), \quad (30)$$

$$f(t) = f_{\kappa_6}(t) = f_{\pm,\kappa_6}\left(\frac{t}{\tau}\right) = \pm\kappa_6\tau^3\left(C_\pm\left(\frac{t}{\tau}\right) - 1\right) S_\pm\left(\frac{t}{\tau}\right). \quad (31)$$

Of course, for all parameters κ_a running to zero the above space-times become commutative.

Let us now turn to the covariance of relations (26)–(31) with respect to the action of undeformed Hopf algebra $\mathcal{U}_0(\widehat{NH}_\pm)$. Using differential representation (16)–(18), classical Leibnitz rules (8) and twist factors (19)–(24), one finds (see prescription (14))

$$G_k \rhd [t, x_i]_{\star_{\mathcal{F}}} = 0\,, \quad (32)$$

$$G_k \rhd \left[[x_i, x_j]_{\star_{\mathcal{F}}} - if(t)(\delta_{1i}\delta_{2j} - \delta_{1j}\delta_{2i})\right] = 0\,; \quad G_k = P_k,\ K_k,\ F_k\,, \quad (33)$$

$$M_{kl} \rhd [t, x_i]_{\star_{\mathcal{F}}} = 0\,, \quad M_{12} \rhd \left[[x_i, x_j]_{\star_{\mathcal{F}}} - if(t)(\delta_{1i}\delta_{2j} - \delta_{1j}\delta_{2i})\right] = 0\,, \quad (34)$$

$$M_{13} \rhd \left[[x_i, x_j]_{\star_{\mathcal{F}}} - if(t)(\delta_{1i}\delta_{2j} - \delta_{1j}\delta_{2i})\right] = f(t)(\delta_{2i}\delta_{3j} - \delta_{2j}\delta_{3i})\,, \quad (35)$$

$$M_{23} \;\triangleright\; \left[[x_i, x_j]_{\star_{\mathcal{F}}} - if(t)(\delta_{1i}\delta_{2j} - \delta_{1j}\delta_{2i})\right] = -f(t)(\delta_{1i}\delta_{3j} - \delta_{1j}\delta_{3i})\,, \tag{36}$$

$$H \;\triangleright\; \left[[x_i, x_j]_{\star_{\mathcal{F}}} - if(t)(\delta_{1i}\delta_{2j} - \delta_{1j}\delta_{2i})\right] = h(t)(\delta_{1i}\delta_{2j} - \delta_{1j}\delta_{2i})\,, \tag{37}$$

$$H \;\triangleright\; [t, x_i]_{\star_{\mathcal{F}}} = 0\,, \tag{38}$$

with $h(t) = \frac{df(t)}{dt}$, *i.e.*

$$h(t) \;=\; h_{\kappa_1}(t) = h_{\pm,\kappa_1}\left(\frac{t}{\tau}\right) = \pm\frac{\kappa_1}{\tau}\, S_\pm\left(\frac{2t}{\tau}\right)\,, \tag{39}$$

$$h(t) \;=\; h_{\kappa_2}(t) = h_{\pm,\kappa_2}\left(\frac{t}{\tau}\right) = \kappa_2\, C_\pm\left(\frac{2t}{\tau}\right)\,, \tag{40}$$

$$h(t) \;=\; h_{\kappa_3}(t) = h_{\pm,\kappa_3}\left(\frac{t}{\tau}\right) = \kappa_3\tau\, S_\pm\left(\frac{2t}{\tau}\right)\,, \tag{41}$$

$$h(t) \;=\; h_{\kappa_4}(t) = h_{\pm,\kappa_4}\left(\frac{t}{\tau}\right) = \pm 8\kappa_4\tau^3 S_\pm\left(\frac{t}{\tau}\right)\left(C_\pm\left(\frac{t}{\tau}\right) - 1\right)\,, \tag{42}$$

$$h(t) \;=\; h_{\kappa_5}(t) = h_{\pm,\kappa_5}\left(\frac{t}{\tau}\right) = \kappa_5\tau\left(S_\pm\left(\frac{2t}{\tau}\right) - S_\pm\left(\frac{t}{\tau}\right)\right)\,, \tag{43}$$

$$h(t) \;=\; h_{\kappa_6}(t) = h_{\pm,\kappa_6}\left(\frac{t}{\tau}\right) = 2\kappa_6\tau^2\left(2C_\pm\left(\frac{t}{\tau}\right) + 1\right) S_\pm^2\left(\frac{t}{2\tau}\right)\,. \tag{44}$$

The above result means that the commutation relations (26)–(31) remain invariant with respect to the action of P_i, K_i, F_i and M_{12} generators. Hence, the "isometry" condition for considered (twisted) spaces breaks the whole $\mathcal{U}_0(\widehat{NH}_\pm)$ quantum group into its subalgebra generated by spatial translations, boosts, constant acceleration generators and rotation in (x_1, x_2)-plane.

Finally, it should be noted that one can easily extend the above algorithm to the case of usual Newton–Hooke Hopf structure $\mathcal{U}_0(NH_\pm)$ by putting acceleration generators F_i equal zero.

4. The case of acceleration-enlarged Galilei Hopf algebra analyzed in the contraction limit ($\tau \to \infty$) of $\mathcal{U}_0(\widehat{NH}_\pm)$ Hopf structure

Let us now turn to the classical acceleration-enlarged Galilei Hopf algebra $\mathcal{U}_0(\widehat{G})$ given by the following algebraic sector

$$\begin{aligned}
&[\,M_{ij}, M_{kl}\,] = i\,(\delta_{il}\,M_{jk} - \delta_{jl}\,M_{ik} + \delta_{jk}M_{il} - \delta_{ik}M_{jl})\;, \qquad [\,H, P_i\,] = 0\,,\\
&[\,M_{ij}, K_k\,] = i\,(\delta_{jk}\,K_i - \delta_{ik}\,K_j)\;, \qquad [\,M_{ij}, P_k\,] = i\,(\delta_{jk}\,P_i - \delta_{ik}\,P_j)\;,\\
&[\,M_{ij}, H\,] = [\,K_i, K_j\,] = [\,K_i, P_j\,] = 0\,, \quad [\,K_i, H\,] = -iP_i\,, \quad [\,P_i, P_j\,] = 0\,,\\
&[\,F_i, F_j\,] = [\,F_i, P_j\,] = [\,F_i, K_j\,] = 0\,, \qquad [\,M_{ij}, F_k\,] = i\,(\delta_{jk}\,F_i - \delta_{ik}\,F_j)\;,\\
&[\,H, F_i\,] = 2iK_i\,,
\end{aligned} \tag{45}$$

and trivial coproduct (8). It is well-known that the above Hopf structure can be gotten by the contraction limit ($\tau \to \infty$) of discussed in pervious section quantum group $\mathcal{U}_0(\widehat{NH}_\pm)$.

The noncommutative space-times associated with twist deformations of Hopf algebra $\mathcal{U}_0(\widehat{G})$ can be provided by the contraction procedure of spaces (26)–(31); they take the form

$$[t, \hat{x}_i]_{\star_\mathcal{F}} = [\hat{x}_1, \hat{x}_3]_{\star_\mathcal{F}} = [\hat{x}_2, \hat{x}_3]_{\star_\mathcal{F}} = 0\,, \qquad [\hat{x}_1, \hat{x}_2]_{\star_\mathcal{F}} = iw(t)\,; \qquad i = 1, 2, 3\,, \tag{46}$$

with $(w_{\kappa_i}(t) = \lim_{\tau\to\infty} f_{\kappa_i}(t))$

$$w(t) = w_{\kappa_1}(t) = \kappa_1\,, \tag{47}$$
$$w(t) = w_{\kappa_2}(t) = \kappa_2\,t\,, \tag{48}$$
$$w(t) = w_{\kappa_3}(t) = \kappa_3\,t^2\,, \tag{49}$$
$$w(t) = w_{\kappa_4}(t) = \kappa_4\,t^4\,, \tag{50}$$
$$w(t) = w_{\kappa_5}(t) = \tfrac{1}{2}\kappa_5\,t^2\,, \tag{51}$$
$$w(t) = w_{\kappa_6}(t) = \tfrac{1}{2}\kappa_6\,t^3\,. \tag{52}$$

It should be also noted, that the Galilean counterpart of covariance conditions (32)–(38) in $\tau \to \infty$ limit looks as follows

$$G_k \rhd [t, x_i]_{\star_\mathcal{F}} = 0\,, \tag{53}$$
$$G_k \rhd \Big[[x_i, x_j]_{\star_\mathcal{F}} - iw(t)(\delta_{1i}\delta_{2j} - \delta_{1j}\delta_{2i})\Big] = 0\,; \quad G_k = P_k,\ K_k,\ F_k\,, \tag{54}$$
$$M_{kl} \rhd [t, x_i]_{\star_\mathcal{F}} = 0\,, \quad M_{12} \rhd \Big[[x_i, x_j]_{\star_\mathcal{F}} - iw(t)(\delta_{1i}\delta_{2j} - \delta_{1j}\delta_{2i})\Big] = 0\,, \tag{55}$$
$$M_{13} \rhd \Big[[x_i, x_j]_{\star_\mathcal{F}} - iw(t)(\delta_{1i}\delta_{2j} - \delta_{1j}\delta_{2i})\Big] = w(t)(\delta_{2i}\delta_{3j} - \delta_{2j}\delta_{3i})\,, \tag{56}$$
$$M_{23} \rhd \Big[[x_i, x_j]_{\star_\mathcal{F}} - iw(t)(\delta_{1i}\delta_{2j} - \delta_{1j}\delta_{2i})\Big] = -w(t)(\delta_{1i}\delta_{3j} - \delta_{1j}\delta_{3i})\,, \tag{57}$$
$$H \rhd \Big[[x_i, x_j]_{\star_\mathcal{F}} - iw(t)(\delta_{1i}\delta_{2j} - \delta_{1j}\delta_{2i})\Big] = g(t)(\delta_{1i}\delta_{2j} - \delta_{1j}\delta_{2i})\,, \tag{58}$$
$$H \rhd [t, x_i]_{\star_\mathcal{F}} = 0\,, \tag{59}$$

where ($g_{\kappa_i}(t) = \lim_{\tau\to\infty} h_{\kappa_i}(t)$)

$$g(t) = g_{\kappa_1}(t) = 0\,, \tag{60}$$
$$g(t) = g_{\kappa_2}(t) = \kappa_2\,, \tag{61}$$
$$g(t) = g_{\kappa_3}(t) = 2\kappa_3 t\,, \tag{62}$$
$$g(t) = g_{\kappa_4}(t) = 4\kappa_4 t^3\,, \tag{63}$$
$$g(t) = g_{\kappa_5}(t) = \kappa_5 t\,, \tag{64}$$
$$g(t) = g_{\kappa_6}(t) = \tfrac{3}{2}\kappa_6 t^2\,. \tag{65}$$

The above result means that the commutations relations (46) remain invariant with respect to the action of P_i, K_i, F_i, M_{12} and H generators in the case of deformation (47) as well as P_i, K_i, F_i and M_{12} for space-times (48)–(52).

Finally, let us observe that the above considerations can be applied to the case of classical Galilei quantum group $\mathcal{U}_0(G)$ by neglecting operators F_i.

5. Final remarks

In this article we provide the subgroups of classical acceleration-enlarged Newton–Hooke $\mathcal{U}_0(\widehat{NH}_\pm)$ as well as classical acceleration-enlarged Galilei $\mathcal{U}_0(\widehat{G})$ Hopf structures, which play the role of "isometry" groups for twist-deformed space-times (25) and (46). In such a way, by analogy to the investigations performed in [20,21], we get the link between twisted quantum spaces and the proper undeformed Hopf subalgebras. Consequently, the obtained results admit to analyze the twist-deformed dynamical models [29,30,31,32] in terms of the corresponding classical quantum subgroups of the whole non-relativistic symmetries. The works in this direction have already started and are in progress.

The author would like to thank J. Lukierski for valuable discussions.

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