

CROSS-CORRELATIONS BETWEEN WTI CRUDE OIL MARKET AND U.S. STOCK MARKET: A PERSPECTIVE FROM ECONOPHYSICS

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In this study, we take a fresh look at the cross-correlations between WTI crude oil market and U.S. stock market from the perspective of econophysics. We choose the three major U.S. stock indices (*i.e.*, DJIA, NASDAQ and S&P 500) as the research objects and select the sample data from Jan 2, 2002 to Jun 29, 2012. In the empirical process, first, using a statistical test in analogy to the Ljung-Box test, we find that there are cross-correlations between WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500 at the 5% significance level. Then, employing the multifractal detrended cross-correlation analysis (MF-DCCA) method, we find that the cross-correlated behavior between WTI crude oil market and U.S. stock market is nonlinear and multifractal. An interesting finding is that the cross-correlation exponent is smaller than the average scaling exponent when $q < 0$, and larger than the average scaling exponent when $q > 0$. Finally, using the rolling windows method, which can capture the dynamics of cross-correlations, we find that there are three special periods whose time-varying Hurst exponents are different from the others.

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1. Introduction

Financial markets are considered as complex dynamic systems [1, 2]. One of the important features of market dynamics is the presence of cross-correlations between financial variables [3]. Although the price changes of the crude oil market are usually acknowledged as an important incentive for the price fluctuations of the stock market, the economists do not reach a consensus on the cross-correlations between crude oil prices and stock prices [4]. For instance, Jones and Kaul [5] first revealed a stable negative

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cross-correlation between oil prices and stock prices. The negative cross-correlations were also found in Refs. [6–8]. On the contrary, Al Janabi *et al.* [9] examined whether the Gulf Cooperation Council (GCC) stock markets are informationally efficient with regard to oil prices during the period of 2006–2008, and found that the oil prices do not show the trend to affect the stock markets. The same conclusions were drawn by Cong *et al.* [10] who found that oil price fluctuations do not have impact on stock returns of most Chinese stock indices. Furthermore, Ciner [11] reported a statistically significant nonlinear relationship between oil prices and stock prices.

In this paper, we try to take a fresh look at the cross-correlations between West Texas Intermediate (WTI) crude oil market and U.S. stock market from the perspective of econophysics. The motivations can be summarized as follows: on the one hand, most of the previous works [4–11] derived by economists are based on the standard economic theory, such as the efficient market hypothesis (EMH) [12], in which the returns of financial entities follow a normal (Gaussian) distribution and prices obey a random walk. However, lots of empirical literatures provided evidence that the distribution of returns shows a fat tail instead of a normal distribution and the price fluctuations have persistence or anti-persistence behaviors over time [13]. That is to say, EMH has become disputed because a larger amount of inefficiencies are approved in some works [14, 15]. On the other hand, to explain or describe the aforementioned phenomenon that the traditional economic theory is inadequate or insufficient to address it, many econophysicists developed a variety of methods from the perspective of physics, such as the complex systems theory [1, 2], the correlation network theory [16–18], the random matrix theory [19–21], and the monofractal and multifractal analysis theory [22, 23]. Especially, the multifractal analysis is able to describe the scaling properties of financial markets because it can divide a complex financial system into varieties of regions according to their complexity, and becomes a useful analytical tool in financial markets [24]. In fact, many scholars confirmed that “*the existence of multifractality has been a ‘stylized fact’ in financial markets*”, such as in Refs. [24–29].

In previous works, various methods were developed to quantify the autocorrelation and cross-correlation behaviors of financial markets based on the monofractal and multifractal theory. For example, Peng *et al.* [30] proposed the detrended fluctuation analysis (DFA) to explore the long-range autocorrelations of a non-stationary time series and widely used in financial time series analysis. Then, DFA was extended into two important methods: one is the multifractal detrended fluctuation analysis (MF-DFA) proposed by Kantelhardt *et al.* [23], which is a powerful tool to investigate multifractality of the financial time series [25–29]; the other is the detrended cross-correlation analysis (DCCA) proposed by Podobnik and Stanley [31], which

can be used to quantify the cross-correlations between two non-stationary financial time series [3, 32, 33]. To examine the multifractal characteristics of two cross-correlated non-stationary time series, Zhou [34] proposed the multifractal detrended cross-correlation analysis (MF-DCCA) based on MF-DFA and DCCA. After that, MF-DCCA was widely used to investigate the cross-correlations in financial markets [24, 35–40]. For instance, based on MF-DCCA, Yuan *et al.* [24] examined the cross-correlations between stock price changes and trading volume changes in Chinese stock markets and found that multifractality existed. He and Chen [36] investigated the cross-correlations between the China's and U.S. agricultural futures markets via MF-DCCA. They found that multifractal cross-correlation behavior is significant in the two agricultural futures markets. Cao *et al.* [40] studied the cross-correlations between the Chinese foreign exchange market and stock market by MF-DCCA. Their results showed that multifractality exists in cross-correlations and cross-correlated behavior is persistent.

Therefore, in this study, we aim at investigating the cross-correlations between WTI crude oil market and U.S. stock market based on the multifractal analysis. The WTI crude oil market is one of the most important crude oil markets and is the underlying commodity of Chicago Mercantile Exchange's oil futures contracts. As for the U.S. stock market, we choose the three major U.S. stock indices: the Dow Jones Industrial Average (DJIA), the National Association of Securities Dealer Automated Quotation (NASDAQ) Composite, and the Standard & Poor 500 (S&P 500) as our research objects. In other words, in this paper, we examine the cross-correlations between WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500. In the empirical analysis, we first make a preliminary analysis of the four time series (*i.e.*, WTI, DJIA, NASDAQ and S&P 500) from Jan 2, 2002 to Jun 29, 2012. Next, we qualitatively analyze the three pairs of cross-correlations based on the cross-correlation statistics proposed by Podobnik *et al.* [41]. Then, we use MF-DCCA to investigate the existence of cross-correlations quantitatively. Finally, we employ the rolling windows method to capture the dynamics of cross-correlations.

The remainder of this paper is organized as follows. In Sec. 2, we describe the methodology of MF-DCCA. In Sec. 3, we present the data set and make a preliminary analysis. We show the main empirical results in Sec. 4. Finally, in Sec. 5 we draw some conclusions.

2. Methodology

Suppose that there are two time series $\{x(t)\}$ and $\{y(t)\}$ of the same length N , where $t = 1, 2, \dots, N$, then MF-DCCA method can be described as follows [34]

Step 1. Determine the “profile” as two new series

$$X(t) = \sum_{i=1}^t (x(i) - \langle x \rangle), \quad Y(t) = \sum_{i=1}^t (y(i) - \langle y \rangle), \quad t = 1, 2, \dots, N. \quad (1)$$

Step 2. Both of the profiles $\{X(t)\}$ and $\{Y(t)\}$ are divided into $N_s = \text{int}(N/s)$ non-overlapping segments of equal length s . Since N is often not a multiple of s , a short part at the end of profile may remain. To include this part of the series, we repeat the same procedure starting from the opposite end. Therefore, we obtain $2N_s$ segments. In this study, we set $10 \leq s \leq N/5$.

Step 3. We estimate the local trends for each of the $2N_s$ segments by a least-square fit of each series. Then determine the variance [36, 40]

$$F_v^2(s) = \frac{1}{s} \sum_{t=1}^s \left| X_{(v-1)s+t}(t) - \tilde{X}_v(t) \right| \left| Y_{(v-1)s+t}(t) - \tilde{Y}_v(t) \right| \quad (2)$$

for each segment v , $v = 1, 2, \dots, N_s$ and

$$F_v^2(s) = \frac{1}{s} \sum_{t=1}^s \left| X_{N-(v-N_s)s+t}(t) - \tilde{X}_v(t) \right| \left| Y_{N-(v-N_s)s+t}(t) - \tilde{Y}_v(t) \right| \quad (3)$$

for $v = N_s + 1, N_s + 2, \dots, 2N_s$. Here, $\tilde{X}_v(t)$ and $\tilde{Y}_v(t)$ are the fitting polynomials in segments v , respectively.

Step 4. We average over all segments to obtain the q th order cross-correlation fluctuation function

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F_v^2(s)]^{q/2} \right\}^{1/q} \quad (4)$$

for any $q \neq 0$ and

$$F_0(s) = \left\{ \frac{1}{4N_s} \sum_{v=1}^{2N_s} \ln [F_v^2(s)] \right\}. \quad (5)$$

Step 5. By observing the log-log plots $F_q(s)$ versus s for each value of q , we can determine the scaling behavior of the fluctuation function. If the original series $\{x(t)\}$ and $\{y(t)\}$ are power-law cross-correlated, then

$$F_q(s) \propto s^{h_{xy}(q)}, \quad (6)$$

where the cross-correlation scaling exponent $h_{xy}(q)$ can be obtained by the slope of log-log plot of $F_q(s)$ versus s via ordinary least squares (OLS) [39].

Especially, if the time series $\{x(t)\}$ is identical to $\{y(t)\}$, MF-DCCA is equivalent to MF-DFA; and when $q = 2$, MF-DCCA is just DCCA. $h_{xy}(q)$ is also known as a generalization of Hurst exponent H with the equivalence $H \equiv h_{xy}(2)$ [42]. If $h_{xy}(q) = H$ for all q , *i.e.*, $h_{xy}(q)$ is independent on q , then the cross-correlations between two time series are monofractal; otherwise they are multifractal.

Generally, there are three cases of $h_{xy}(q)$: (i) If $h_{xy}(q) < 0.5$, the cross-correlations between the two time series are anti-persistent (negative). This implies that one price is likely to increase following a decrease of the other price, and *vice versa* [35]. (ii) If $h_{xy}(q) > 0.5$, the cross-correlations between the two time series are persistent (positive). This means that an increase (a decrease) of one price is likely to be followed by an increase (a decrease) of the other price [35]. (iii) If $h_{xy}(q) = 0.5$, there are no cross-correlations between the two time series, and the change of one price cannot affect the behavior of the other price [35].

By analyzing the spectrum of the cross-correlations scaling exponent $h_{xy}(q)$, we can calculate the singularity strength α and the multifractal spectrum $f(\alpha)$ by [42]

$$\alpha = h_{xy}(q) + qh'_{xy}(q) \quad (7)$$

and

$$f(\alpha) = q[\alpha - h_{xy}(q)] + 1, \quad (8)$$

where $h'_{xy}(q)$ stands for the derivative of $h_{xy}(q)$ with respect to q . In this study, we set q ranging from -10 to 10 with a step of one.

The strength of multifractality can be defined by the width of the multifractal spectrum [36], which is presented as follows

$$\Delta\alpha = \alpha_{\max} - \alpha_{\min}. \quad (9)$$

3. Date and preliminary analysis

We choose the daily closing prices of WTI, DJIA, NASDAQ and S&P 500 from Jan 2, 2002 to Jun 29, 2012. The WTI crude oil spot prices are provided by U.S. Energy Information Administration (EIA) (<http://www.eia.gov/petroleum>). We obtain the daily closing prices of the three major U.S. stock indices (*i.e.*, DJIA, NASDAQ and S&P 500) from Yahoo Finance (<http://finance.yahoo.com>).

Let $P(t)$ denote the daily closing price on day t . The daily return, $r(t)$, is defined as the logarithmic difference of $P(t)$ and $P(t-1)$, *i.e.*, $r(t) = \ln(P(t)) - \ln(P(t-1))$. The volatility is defined by the absolute return $|r(t)|$. Figure 1 shows the graphical representation of the four returns (*i.e.*, WTI, DJIA, NASDAQ and S&P 500). We can find that the four returns

have a mutual zone of large fluctuations in Fig. 1 from Jan 2008 to Dec 2009, which may be the worst days of the global economy during the U.S. sub-prime crisis.

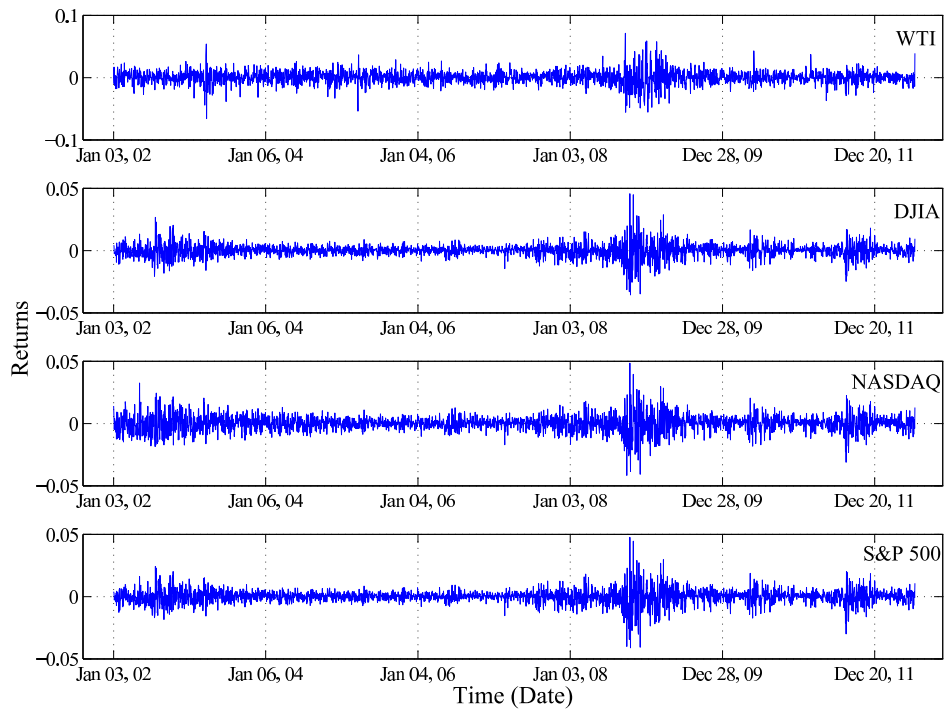


Fig. 1. Returns of the WTI, DJIA, NASDAQ and S&P 500.

The descriptive statistics of the four returns are organized in Table I. The mean values of the four returns are very close to zero, and quite small by comparison with the standard deviations. The Jarque–Bera statistics reject the null hypothesis of the normal distribution at the 1% significance level. This phenomenon is also accompanied by non-zero skewness and kurtosis larger than three, which indicates that the four returns are fat-tailed. The Ljung–Box statistics reject the null hypothesis of no auto-correlations up to the 20th order at the 1% significance level, which implies that the four returns present the auto-correlations.

TABLE I

Descriptive statistics of returns of WTI, DJIA, NASDAQ and S&P 500.

	WTI	DJIA	NASDAQ	S&P 500
Mean ($\times 10^{-4}$)	2.2430	0.3522	0.5788	0.2228
Maximum	0.0713	0.0456	0.0485	0.0476
Minimum	-0.0660	-0.0356	-0.0416	-0.0411
Standard deviation	0.0109	0.0055	0.0067	0.0059
Skewness	-0.0932	0.0360	-0.0707	-0.1935
Kurtosis	7.4528	11.1173	7.5445	11.2824
Jarque-Bera ($\times 10^3$)	2.1727***	7.2116***	2.2614***	7.5237***
$Q(20)$	65.4455***	97.6337***	57.2772***	98.1227***
Observations	2623	2623	2623	2623

Notes: The Jarque-Bera statistic tests for the null hypothesis of normality distribution. $Q(20)$ denotes the value of the Ljung-Box statistics of the return series for up to the 20th order serial correlation. *** Indicates rejection of the null hypothesis at the 1% significance level.

In order to further examine the fat-tailed distribution of the four returns, we use a novel method of power-law estimation proposed by Podobnik *et al.* [43]. They indicated that, on average, there is one volatility above threshold q after each time interval $\tau_{\text{ave}}(q)$, then

$$1/\tau_{\text{ave}}(q) \approx \int_q^{\infty} P(|x|)d|x| = P(|x| > q) \sim q^{-\beta}. \quad (10)$$

We calculate the average time interval $\tau_{\text{ave}}(q)$ for different values of q , and acquire the estimates for β by the relationship

$$\tau_{\text{ave}}(q) \propto q^{\beta}. \quad (11)$$

The log-log plots of $\tau_{\text{ave}}(q)$ versus threshold q are presented in Fig. 2. The thresholds q range from 2σ to 8σ with a fixed step of 0.25σ , where σ is the standard deviation of each absolute return. There is a power-law relationship with Podobnik's tail exponent $\beta = 3.2302$, $\beta = 3.0233$, $\beta = 3.4019$ and $\beta = 3.0293$ for WTI, DJIA, NASDAQ and S&P 500, respectively. One can find that the four Podobnik's tail exponents are close to three, which is consistent with the "inverse cubic power-law" and is found in many financial markets [28, 37, 43].

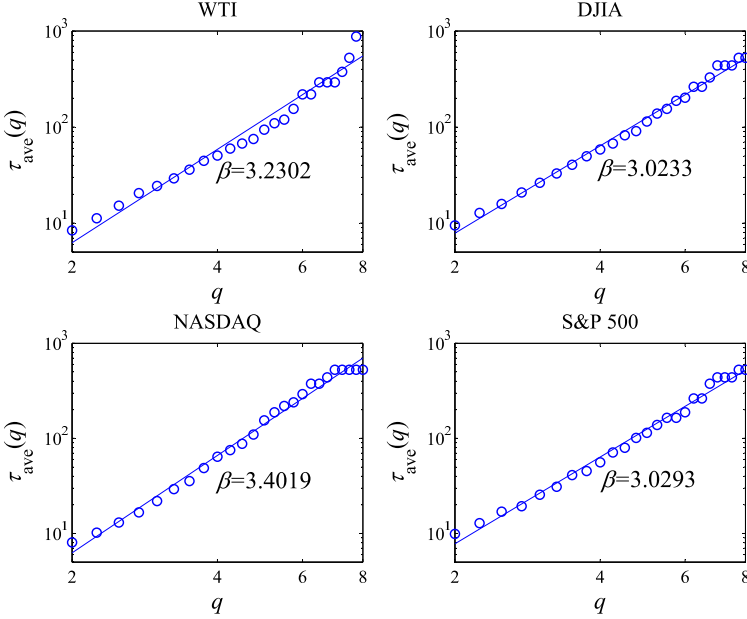


Fig. 2. Log-log plots of the average time interval $\tau_{\text{ave}}(q)$ versus threshold q (in units of σ).

4. Empirical results

4.1. Cross-correlation test

In this subsection, we employ a new cross-correlation test proposed by Podobnik *et al.* [41] to quantify the cross-correlations between WTI crude oil market and U.S. stock market (*i.e.*, the three pairs of cross-correlations: WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500). This test is analogous to the Ljung-Box test [44] and widely used to test the cross-correlations in the financial markets [24, 35, 37–40]. The cross-correlation statistic between two time series $\{x(t)|t = 1, 2, \dots, N\}$ and $\{y(t)|t = 1, 2, \dots, N\}$ is defined as

$$Q_{\text{cc}}(m) = N^2 \sum_{t=1}^m \frac{C^2(t)}{N-t}, \quad (12)$$

where the cross-correlation coefficient $C(t)$ is defined by

$$C(t) = \frac{\sum_{k=t+1}^N x(k)y(k-t)}{\sqrt{\sum_{k=1}^N x^2(k) \sum_{k=1}^N y^2(k)}}. \quad (13)$$

Podobnik *et al.* [41] proposed that, the cross-correlation statistic $Q_{cc}(m)$ is approximately $\chi^2(m)$ distributed with m degrees of freedom. It can be used to test the null hypothesis that none of the first m cross-correlation coefficients is different from zero [41].

We show the log-log plots of cross-correlation statistics $Q_{cc}(m)$ versus degrees of freedom m for WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500 in Fig. 3. The degrees of freedom, m , range from 10^0 to 10^3 . As a comparison, we also present the critical value for the $\chi^2(m)$ distribution at the 5% significance level in Fig. 3. For a broad range of m , all the test statistics $Q_{cc}(m) > \chi_{0.95}^2(m)$. Therefore, we can reject the null hypothesis of no cross-correlations. That is to say, cross-correlations evidently exist between WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500.

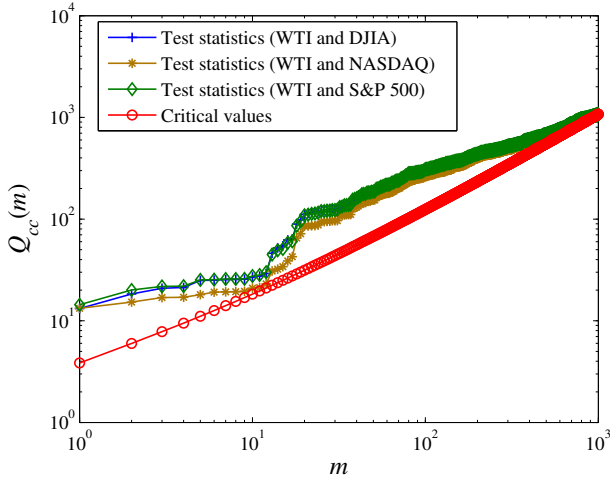


Fig. 3. Log-log plots of test statistics $Q_{cc}(m)$ versus degrees of freedom m .

4.2. Multifractal detrended cross-correlation analysis

Podobnik *et al.* [41] also proposed that the cross-correlation test based on the statistic $Q_{cc}(m)$ of Eq. (12) can only test the existence of cross-correlation *qualitatively*. Thus, in this subsection, we use MF-DCCA approach to investigate the cross-correlation *quantitatively* by estimating the cross-correlation scaling exponent.

In Figs. 4, 5 and 6, we display the relationship between cross-correlation scaling exponent $h_{xy}(q)$ and q (the curves with circle symbols) for WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500, respectively. Here, we denote the WTI returns as the $\{x(t)\}$ time series, and respectively denote the three returns of DJIA, NASDAQ and S&P 500 as the $\{y(t)\}$ time series.

As a comparison, we also estimate the scaling exponents $h_{xx}(q)$ and $h_{yy}(q)$ of WTI crude oil market and U.S. stock market (*i.e.*, DJIA, NASDAQ and S&P 500) by means of MF-DFA, respectively. In Figs. 4, 5 and 6 the curves with triangle symbols represent $h_{xx}(q)$ of WTI and the curves with square symbols stand for $h_{yy}(q)$ of DJIA, NASDAQ and S&P 500, respectively.

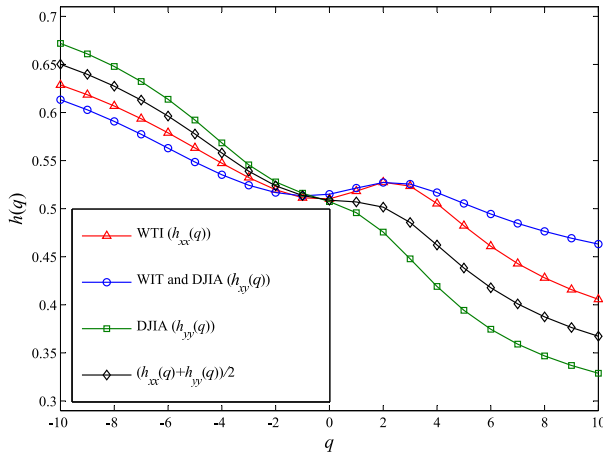


Fig. 4. The relationship between $h(q)$ and q for WTI and DJIA.

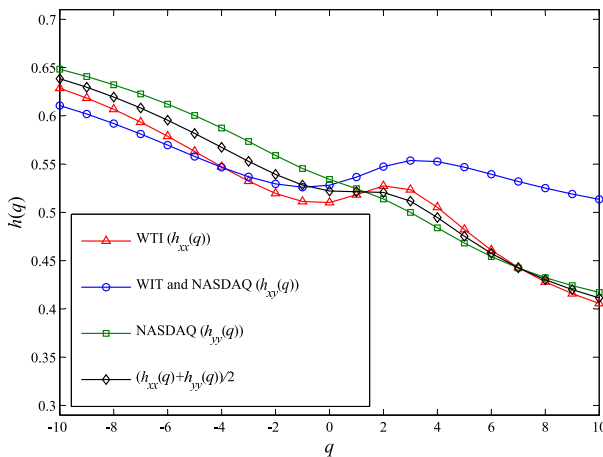


Fig. 5. The relationship between $h(q)$ and q for WTI and NASDAQ.

In general, if the scaling exponent $h(q)$ depends on the values of q , then the auto-correlations or cross-correlations are multifractal; otherwise, there are monofractal. From Figs. 4, 5 and 6, one can observe that, for different q , there is a different exponent $h_{xy}(q)$, which indicates that the cross-

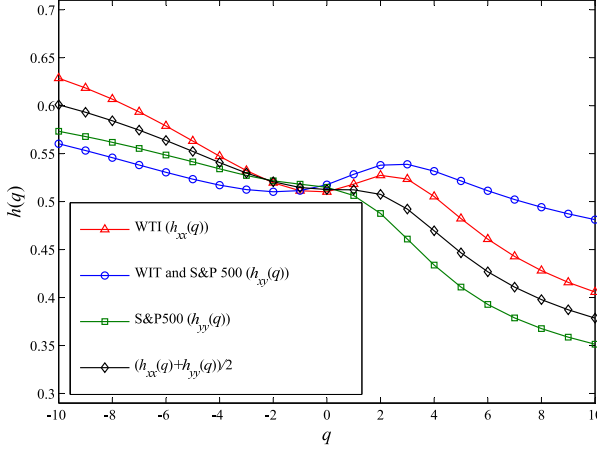


Fig. 6. The relationship between $h(q)$ and q for WTI and S&P 500.

correlations between WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500 have obvious multifractal natures. For the same reason, we can see that the multifractal features also exist in the individual market (*i.e.*, WTI, DJIA, NASDAQ and S&P 500) by observing the changes of $h_{xx}(q)$ or $h_{yy}(q)$. According to the previous work by Zhou [34], for two time series constructed by binomial measure from p -model, there is the following relationship among $h_{xy}(q)$, $h_{xx}(q)$ and $h_{yy}(q)$

$$h_{xy}(q) = (h_{xx}(q) + h_{yy}(q))/2, \quad (14)$$

where $(h_{xx}(q) + h_{yy}(q))/2$ is denoted as the average scaling exponent [31]. Nevertheless, He and Chen [38] proved that if the time scale $s \rightarrow \infty$, the relationship between bivariate cross-correlation exponent $h_{xy}(q)$ and the average scaling exponent $(h_{xx}(q) + h_{yy}(q))/2$ satisfies the following inequality

$$h_{xy}(q) \leq (h_{xx}(q) + h_{yy}(q))/2. \quad (15)$$

To evaluate the above-mentioned relationship, we calculate the average scaling exponents between WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500, and respectively draw the graphical representations in Figs. 4, 5 and 6 (the curves with diamond symbols). From Figs. 4, 5 and 6, an interesting finding is that, the cross-correlation exponent $h_{xy}(q)$ is smaller than the average scaling exponent $(h_{xx}(q) + h_{yy}(q))/2$ when $q < 0$, and larger than the average scaling exponent $(h_{xx}(q) + h_{yy}(q))/2$ when $q > 0$. This suggests that our results do not hold for Eq. (14) for all values of q and Eq. (15) when $q > 0$. In other words, Eqs. (14) and (15) are not verified by the empirical analysis between WTI crude oil market and U.S. stock market.

For better understanding of the nonlinear dependency, we further examine the multifractal strength via multifractal spectra. First, we calculate the multifractal spectra between WTI crude oil market and U.S. stock market by Eqs. (7) and (8), and plot the results in Fig. 7. If multifractal spectrum shows as a point, it is monofractal [40]. In Fig. 7, one can see that the multifractal spectra in the two markets are not a point, which indicates that multifractality exists separately in WTI crude oil market and U.S. stock market (*i.e.*, DJIA, NASDAQ and S&P 500) and in the cross-correlated markets (*i.e.*, WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500). Then, we estimate the multifractal degrees (*i.e.*, the widths of multifractal spectra) by means of Eq. (9), and list the numerical results in Table II. By comparing the results in Table II or Fig. 7, we can find that: (*i*) The multifractal degrees between the two markets are smaller than these of the individual market, *i.e.*, the strength of multifractal of WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500 is smaller than that of WTI,

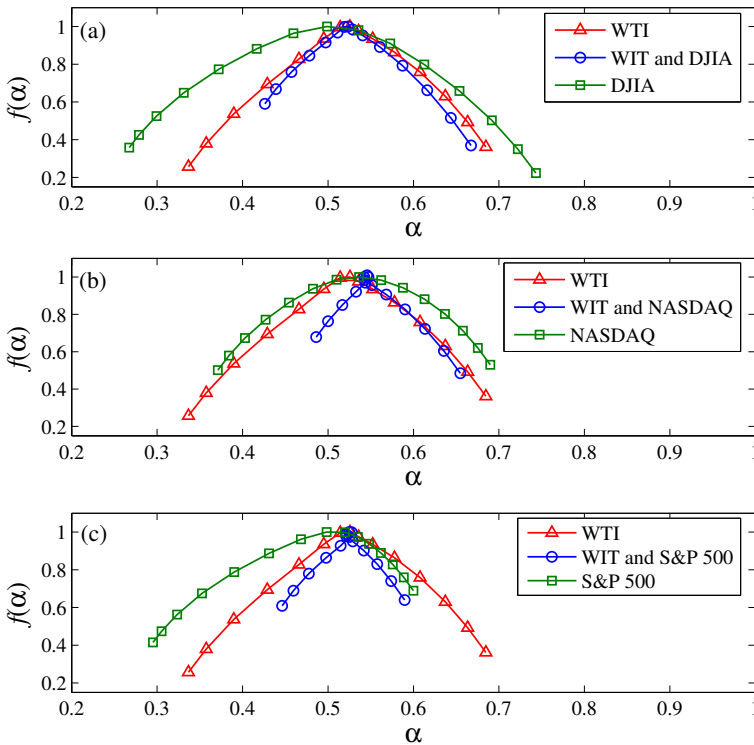


Fig. 7. Multifractal spectra between WTI crude oil market and U.S. stock market. Panels (a), (b) and (c) show the multifractal relationships between $f(\alpha)$ and α for WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500, respectively.

DJIA, NASDAQ and S&P 500, respectively. (ii) The DJIA stock index has the largest multifractal degree, and the smallest one is the cross-correlations between WTI and S&P 500. The cross-correlated behavior between WTI crude oil market and U.S. stock market is nonlinear and multifractal, which implies that traditional linear models (*e.g.*, vector auto-regression models (VAR)) in the standard economic theory could not be applied to describe the dynamics of the cross-correlations between the two markets.

TABLE II

Multifractality degrees $\Delta\alpha$.

	$\Delta\alpha$
WTI	0.3480
DJIA	0.4758
NASDAQ	0.3187
S&P 500	0.3047
WTI and DJIA	0.2413
WTI and NASDAQ	0.1687
WTI and S&P 500	0.1433

4.3. Rolling windows analysis

In this subsection, we use the rolling windows method to analyze the time-varying features of the cross-correlations between WTI crude oil market and U.S. stock market. The rolling windows method is used to investigate the temporal evolution of the Hurst exponent $h_{xy}(2)$ at different scales, which is also called as the local Hurst exponent [41], or a rolling test [45]. For details, see Refs. [41, 45].

In previous studies, many scholars employed the rolling windows method to analyze the long-range and short-term dynamics of financial markets [35, 40, 45], and discussed the choice of the window size [46, 47]. Grech and Mazur [46] indicated that the local Hurst exponent at a given time t depends on the window size. As for the selection of window size, Liu *et al.* [47] proposed that: to examine the general trend of long-range market dynamics (*e.g.*, market efficiency), one should select a large window size (*e.g.*, Ref. [48] set the window size to four years); on the contrary, to investigate the effects of exogenous events (*e.g.*, seasonal factors and financial crisis) on the short-term market dynamics, one should choose a small window size (*e.g.*, in Refs. [35, 47, 49], the window size was fixed to one year). In our work, like in Refs. [35, 47, 49], the window size is set to 250 trading days, which are roughly equal to one calendar year. The window step length is a

single day. We illustrate the graphical representations of time-varying Hurst exponents $h_{xy}(2)$ of WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500 in Fig. 8. The time x -axis represents the date of the beginning and the last day in each window.

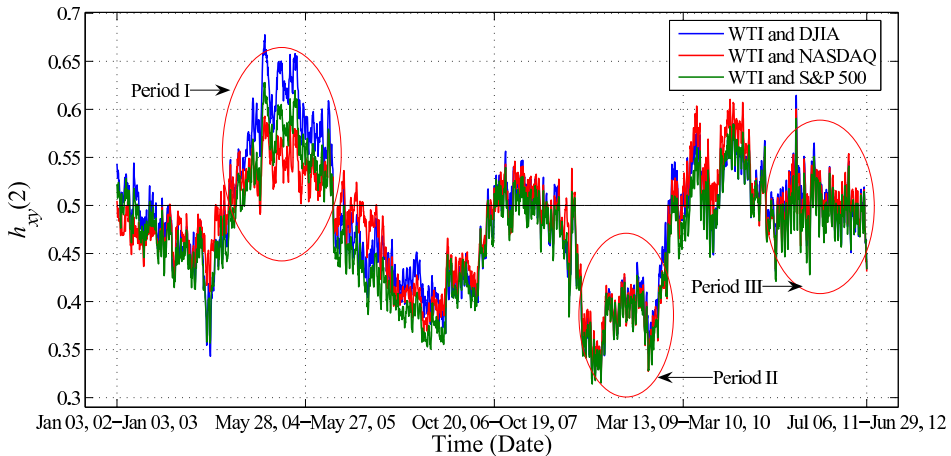


Fig. 8. (Color online) Time-varying Hurst exponents $h_{xy}(2)$ of WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500.

From Fig. 8, we can find that the time-varying Hurst exponents of WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500 have the same trends. The reason may be that the three major indices are all from the U.S. stock market and with the same market trends. There are three special periods in Fig. 8: the first period from Jan 2003 to Oct 2005 (denoted as Period I), the second period from Nov 2007 to Dec 2009 (denoted as Period II) and the third period from Mar 2010 to present (denoted as Period III). During Period I, there is a major event: the second war in Iraq, which caused disorder in oil and stock prices. The cross-correlations between WTI crude oil market and U.S. stock market are positive during Period I because the time-varying Hurst exponents are larger than 0.5. The main event in Period II is the U.S. sub-prime crisis. At this period, the pattern shows that most of the time-varying Hurst exponents are smaller than 0.4 (*i.e.*, the cross-correlations between the two markets are negative). As shown in Fig. 8, it is interesting to note that the tendency of time-varying Hurst exponents to fluctuate above and below 0.5 during Period III. The primary event at this period is the European debt crisis, which starts from Dec 2009. The tendency implies that the cross-correlations between the two markets are uncertain, and the tendency of global economy is instable at present.

5. Conclusions

In summary, we examine the cross-correlations between WTI crude oil market and U.S. stock market. We choose the three major U.S. stock indices (*i.e.*, DJIA, NASDAQ and S&P 500) as the research objects. Or rather, in this study, we investigate the cross-correlations between WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500. In the empirical process, we first use a statistical test proposed by Podobnik *et al.* [41] to test the presence of cross-correlations qualitatively and find that the cross-correlations significantly exist between WTI and DJIA, WTI and NASDAQ, and WTI and S&P 500. Then, we employ MF-DCCA method to examine the presence of cross-correlations quantitatively and find that the cross-correlated behaviors between crude oil market and U.S. stock market are nonlinear and multifractal. Finally, we use the rolling windows approach to capture the dynamics of cross-correlations and find that there are three special periods whose time-varying Hurst exponents are different from the others.

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