TSALLIS FITS TO $p_{\rm T}$ SPECTRA FOR pp COLLISIONS AT THE LHC

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Phenomenological Tsallis fits to the CMS and ATLAS transverse spectra of charged particles were found to extend for $p_{\rm T}$ from 0.5 to 181 GeV in pp collisions at the LHC at $\sqrt{s} = 7$ TeV, and for $p_{\rm T}$ from 0.5 to 31 GeV at $\sqrt{s} = 0.9$ TeV. The simplicity of the Tsallis parametrization and the large range of the fitting transverse momentum raise questions on the physical meaning of the degrees of freedom that enter into the Tsallis distribution or q-statistics.

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Recently, there is a lot of interest in the Tsallis fit to the transverse momentum data of charged particles measured at very high energies of the RHIC and LHC experiments [1–6]. By this, one understands that the use of the Tsallis distribution with a normalization constant C_q

$$h_q\left(p_{\rm T}\right) = C_q \left[1 - (1-q)\frac{p_{\rm T}}{T}\right]^{\frac{1}{1-q}} \tag{1}$$

describes the experimental transverse momentum distribution data. The Tsallis distribution can be regarded as a generalization of the usual exponential (Boltzmann–Gibbs) distribution, and converges to it when the parameter q tends to unity [7]

$$h_q\left(p_T\right) \stackrel{q \to 1}{\Longrightarrow} C_1 \exp\left(-\frac{p_{\mathrm{T}}}{T}\right) . \tag{2}$$

This approach is a nonextensive generalization of the usual statistical mechanics (characterized by a new parameter q, the nonextensivity parameter) and has been very successful in describing very different physical systems in terms of statistical approach, including multiparticle production processes at lower energies[1].

On the other hand, long time ago Hagedorn proposed the *QCD inspired* empirical formula describing the data of the invariant cross section of hadrons as a function of p_{T} over a wide range [11]

$$E\frac{d^3\sigma}{d^3p} = C\left(1+\frac{p_{\mathrm{T}}}{p_0}\right)^{-n} \longrightarrow \begin{cases} \exp\left(-\frac{np_{\mathrm{T}}}{p_0}\right) & \text{for } p_{\mathrm{T}} \to 0\,, \\ \left(\frac{p_0}{p_{\mathrm{T}}}\right)^n & \text{for } p_{\mathrm{T}} \to \infty\,, \end{cases} \tag{3}$$

where C, p_0, and n are fitting parameters. This becomes a purely exponential function for small p_{T} and a purely power law function for large p_{T} values[2]. It coincides with Eq. (1) for

$$n = \frac{1}{q-1} \quad \text{and} \quad p_0 = \frac{T}{q-1} . \tag{4}$$

Usually, both formulas are treated as equivalent from the point of view of phenomenological fits and are often used interchangeably [1–6]. We follow this attitude for a while and shall discuss the possible physical implications later.

For phenomenological as well as theoretical interests, it is useful to explore where the Tsallis fit begins to fail at higher and higher p_{T}. Here we concentrate only on the recent high-p_{T} data of CMS [5] and ATLAS [3] collaborations at the LHC. Excellent fit to the p_{T} spectra was obtained there with the Tsallis and/or Hagedorn distributions for p_{T} from 0.5 GeV up to 6 GeV, in pp collisions at $\sqrt{s} = 7$ TeV. However, CMS data extend to much higher range of p_{T}, up to ~ 200 GeV/c. It will be interesting to know whether the good Tsallis fit continues to higher transverse momenta and how it would relate to results of fits to data at the lower energy of $\sqrt{s} = 0.9$ TeV.

[1] For a summary of earlier attempts of using Tsallis fits and detailed explanations of the possible meaning of the q parameter, together with up-to-date literature on this subject, see [8–10].

[2] Actually, this QCD inspired formula was proposed earlier in [12, 13].

The CMS [5] and ATLAS [3] $\langle Ed^3N_{\rm ch}/dp^3\rangle_\eta$ data shown in Fig. 1 are taken essentially at the same kinematical windows, namely they correspond to an average over the data from $\eta = -\eta_0$ to $\eta = +\eta_0$ with $\eta_0 = 2.4$ for the CMS measurements, and 2.5 for the ATLAS measurements[3]. To get the theoretical results from the distribution for comparison with experimental data we, therefore, calculate numerically

$$\left\langle E\frac{d^3N_{\rm ch}}{dp^3}\right\rangle_\eta = \frac{1}{2\eta_0}\int\limits_{-\eta_0}^{\eta_0} d\eta \frac{dy}{d\eta}\left(E\frac{d^3N_{\rm ch}}{dp^3}\right)\,. \tag{5}$$

Here

$$\frac{dy}{d\eta}(\eta, p_{\rm T}) = \sqrt{1-\frac{m^2}{m_{\rm T}^2\cosh^2 y}} \tag{6}$$

which is from Eq. (2.31) of [14], and y is a function of η and $p_{\rm T}$ (Eq. 2.29) of [14]

$$y = \frac{1}{2}\ln\left[\frac{\sqrt{p_{\rm T}^2\cosh^2\eta + m^2}+p_{\rm T}\sinh\eta}{\sqrt{p_{\rm T}^2\cosh^2\eta + m^2}-p_{\rm T}\sinh\eta}\right]\,. \tag{7}$$

To provide a theoretical fit to the experimental data, we follow the CMS Collaboration [4] and consider the differential cross section with a transverse Tsallis distribution [7, 15] in the form

$$E\frac{d^3N_{\rm ch}}{dp^3} = C\frac{dN_{\rm ch}}{dy}\left(1+\frac{E_{\rm T}}{nT}\right)^{-n}\,, \tag{8}$$

where

$$E_{\rm T} = \sqrt{m^2+p_{\rm T}^2} - m\,, \tag{9}$$

and we assume $m = m_\pi = 0.14$ GeV. If we assume now a rapidity plateau structure with a constant $CdN_{\rm ch}/dy$, then the integral is

$$\left\langle E\frac{d^3N_{\rm ch}}{dp^3}\right\rangle_\eta = \frac{C}{2\eta_0}\frac{dN_{\rm ch}}{dy}\int\limits_{-\eta_0}^{\eta_0} d\eta\frac{dy}{d\eta}\left(1+\frac{E_{\rm T}}{nT}\right)^{-n}\,. \tag{10}$$

For each value of $p_{\rm T}$, the curves in Fig. 1 are obtained from such a numerical integration over η. The $p_{\rm T}$ spectrum is therefore described by an overall constant $A = CdN_{\rm ch}/dy$ and the parameters n and T.

[3] We do not include here ALICE data [6] because they are for a smaller window, $-0.8 < \eta < 0.8$, and the data points are slightly higher for large $p_{\rm T}$ because of the slight η dependence of the spectra [6].

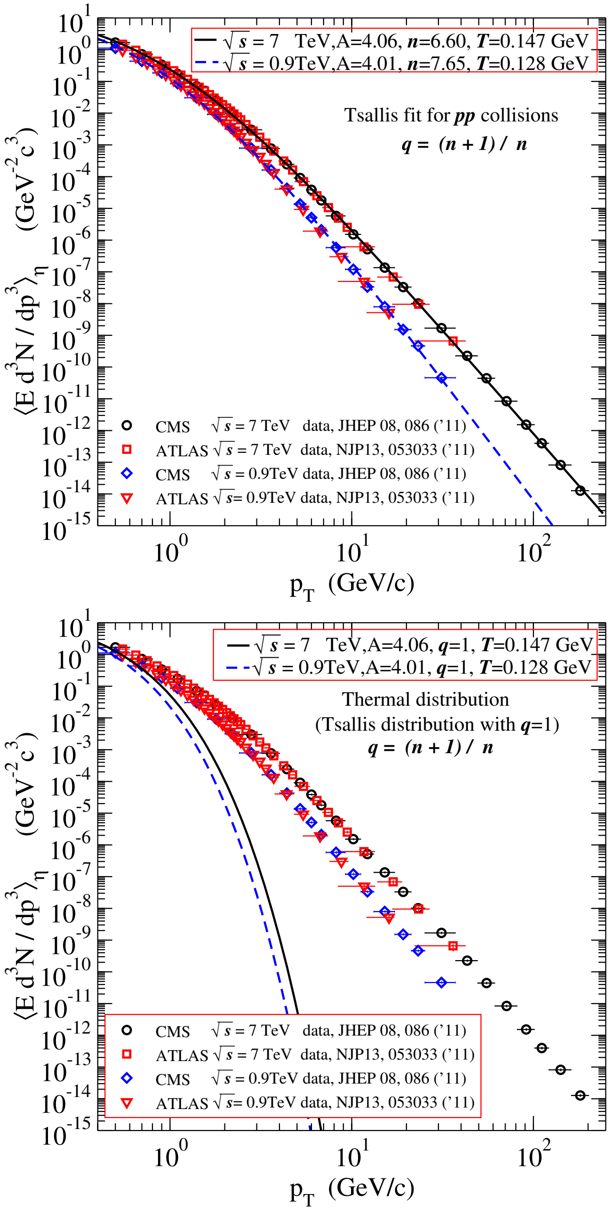

Fig. 1. (Color online) Upper panel: Tsallis fits (10) to the CMS [5] and ATLAS [3] collaborations data for pp at 7 and 0.9 TeV [5]. Lower panel: the same data compared with the corresponding $q = 1$ (or $n \to \infty$) curves.

Previously, excellent Tsallis fit to the CMS p_{T} spectra at $\sqrt{s} = 7$ TeV was obtained for $0.5 < p_T < 6$ GeV/c by using $n = 6.6$ and $T = 0.145$ GeV [4]. By using this set of parameters as initial guess, we search for the fits to the spectrum from 0.5 GeV up to 181 GeV. We find that for the set including the data points at the higher p_{T} region, the best fit is obtained with $A = 4.06$, $n = 6.6$, and $T = 0.147$ GeV, which is essentially the same as the set obtained previously in [4] (Fig. 1, upper panel). What is surprising is that the Tsallis distribution that describes the data well at lower p_{T} can describe the CMS data at high p_{T} just as well. There is no departure of data from the Tsallis distribution from $p_{\mathrm{T}} = 0.5$ GeV up to $p_{\mathrm{T}} \sim 200$ GeV. For additional comparison, we include the ATLAS data in Fig. 1. The ATLAS measurement has a slightly larger η window, $|\eta|_{\mathrm{ATLAS}} \leq 2.5$, instead of CMS's $|\eta|_{\mathrm{CMS}} \leq 2.4$, and its p_{T} values extends from 0.5 GeV/c to 36 GeV/c for $\sqrt{s} = 7$ TeV. The ATLAS data at $\sqrt{s} = 7$ TeV are consistent with the CMS data and the Tsallis fit (Fig. 1, upper panel).

On the other hand, at the lower energy of $\sqrt{s} = 0.9$ TeV, we find that the *pp* data can be described well by the higher value of $n = 7.65$ (with $A = 4.01$ and $T = 0.128$ GeV) in Fig. 1. Again, the CMS data and the ATLAS data are consistent with each other. The Tsallis distribution gives a good description of the $\sqrt{s} = 0.9$ TeV data from $p_{\mathrm{T}} = 0.5$ GeV/c to $p_{\mathrm{T}} = 31$ GeV/c (Fig. 1, upper panel).

It is instructive to visualize the difference between the Tsallis distribution and the corresponding thermal distribution characterized by the same temperature T and by $q = 1$ (or $n \to \infty$). This is shown on the lower panel of Fig. 1. Changing only the temperature parameter T would not give a better fit to the experimental data, to do this it is necessary to allow q to vary and become $q > 1$.

The overall good agreement of the Tsallis parametrization and the experimental data is amazingly good. The absence of a departure of data from the Tsallis distribution from $p_{\mathrm{T}} = 0.5$ GeV to 181 GeV indicates that there are essentially only three degrees of freedom that count for the description of the p_{T} distribution: an overall magnitude, and two other degrees of freedom to describe the shape. This result should be confronted with the (apparently equally successful) many parameter fits also presented in [3–6] and using known Monte Carlo programs. It can be interpreted as indication that, whereas hadronizing system formed in the process of particle production is very complex, only few degrees of freedom are really important. At lower energies (and for lower values of p_{T}) this was regarded as indication that such system can be described by simple (with $q = 1$) or generalized (with $q > 1$) statistical models [8, 15]. The results presented here indicate that, either such models work also for such large p_{T} or there are some specific, dynamical rather than purely statistical, phenomena at work as advocated recently in [16]. In any case, we need to take the physics contents of the Tsallis model seriously.

At this point, let us come back to formula (3) (or (10)). It was proposed in [12] long time before Tsallis works [7] (and followed later by [13] and [11]) with a simple aim: to phenomenologically interpolate between the *soft region* of $p_{\rm T} \to 0$, characterized by exponential behavior of $p_{\rm T}$ distributions, and the *hard region* of $p_{\rm T} \gg 1$ GeV, believed to be properly described by QCD. However at that time, the exponent index obtained from fits to $p_{\rm T}$ data was equal to $n \sim 8$ (or bigger), far away from the expected point interaction value of $n \sim 4$ [17–19]. As one can see from our fits, this region of dominance of truly point-like hard interactions is still far away (albeit n diminishes noticeably between 900 GeV and 7 TeV). Actually, comparing our results with compilation of results at lower energies provided by [20] one observes that the naive counting rule result of $n = 4$–6 [19] seems to be out of reach. It means therefore, that even at the highest energies, we do not deal with the point-like objects expected from the naive field theory expectations and there is always an additional (to that resulting from a hard collision) $p_{\rm T}$ transfer, perhaps preceded by a kind of the multiple scattering process or constituent scattering which make the finally observed spectra softer than naively expected.

It is of interest to note in this connection that the power of $n \sim 6.6$–7.6 in the transverse momentum spectrum at these high energies may be related to the constituent interchange model of Blankenbecler and Brodsky and Gunion [17]. In the basic quark models diagrams, the power index in $p_{\rm T}$ dependence of the inclusive spectra can be inferred from the counting rule involving the collision of the active constituents [17, 18] (for a review, see [14]). If one assumes that the dominant basic high-$p_{\rm T}$ process in $pp \to \pi X$ comes from $qq \to qq$, then the counting rule gives a transverse momentum dependence of $1/(p_{\rm T}^2)^2$ with $n = 4$, which differ from the observed power index, as we mentioned earlier. On the other hand, if one assumes that the basic process is $q+$ meson $\to q+$ meson, then the counting rule gives $n = 8$ which is close to the observed value. The basic process of $q+$ meson $\to q+$ meson may appear dominant because of the strong quark-hadron coupling.

We close with the following observation. From what was shown here it seems that phenomenologically the two-parameter, QCD-inspired formula (8) is as good as the two-parameter Tsallis formula with $n \to 1/(q-1)$. The only difference is in the interpretation, *i.e.*, in the possible thermodynamical origin (among others) of the Tsallis formula [8–10]. In this case, Tsallis fits would cover the whole energy range of experiments uniformly interpreted in terms of thermal (extensive or nonextensive) model. That this view is reasonable was shown in papers explicitly demonstrating that nonextensive-thermodynamics satisfies all demands of the usual thermodynamics applied to systems that posses intrinsic fluctuations, memory effects, are limited and/or nonhomogeneous *etc.* [21–23]. Whether the recent CMS result presented here fits to this picture or rather calls for some novel explanation of Tsallis formula (and parameter q) remains, however, for a time being an open question.

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REFERENCES

[1] B.I. Abelev *et al.* [STAR Collaboration], *Phys. Rev.* **C75**, 064901 (2007).

[2] A. Adare *et al.* [PHENIX Collaboration], *Phys. Rev.* **D83**, 052004 (2011); *Phys. Rev.* **C83**, 064903 (2011).

[3] G. Aad *et al.* [ATLAS Collaboration], *New J. Phys.* **13**, 053033 (2011).

[4] V. Khachatryan *et al.* [CMS Collaboration], *J. High Energy Phys.* **02**, 041 (2010); *Phys. Rev. Lett.* **105**, 022002 (2010).

[5] S. Chatrchyan *et al.* [CMS Collaboration], *J. High Energy Phys.* **08**, 086 (2011).

[6] K. Aamodt *et al.* [ALICE Collaboration], *Phys. Lett.* **B693**, 53 (2010); *Eur. Phys. J.* **C71**, 1594 (2011); **C71**, 1655 (2011).

[7] C. Tsallis, *J. Stat. Phys.* **52**, 479 (1988); *Eur. Phys. J.* **A40**, 257 (2009); *cf.* also C. Tsallis, *Introduction to Nonextensive Statistical Mechanics*, Springer, Berlin 2009; for an updated bibliography on this subject, see `http://tsallis.cat.cbpf.br/biblio.htm`

[8] G. Wilk, Z. Włodarczyk, *Eur. Phys. J.* **A48**, 161 (2012) [`arXiv:1203.4452[hep-ph]`].

[9] K. Ürmösy, G.G. Barnaföldi, T.S. Biró, *Phys. Lett.* **B718**, 125 (2012).

[10] J. Cleymans, D. Worku, *Eur. Phys. J.* **A48**, 160 (2012).

[11] R. Hagedorn, *Riv. Nuovo Cimento* **6**, 1 (1984).

[12] C. Michael, L. Vanryckeghem, *J. Phys. G* **3**, L151 (1977); C. Michael, *Prog. Part. Nucl. Phys.* **2**, 1 (1979).

[13] G. Arnison *et al.* [UA1 Collaboration), *Phys. Lett.* **B118**, 167 (1982).

[14] C.Y. Wong, *Introduction to High-Energy Heavy-Ion Collisions*, World Scientific Publisher, 1994.

[15] G. Wilk, Z. Włodarczyk, *Eur. Phys. J.* **A40**, 299 (2009); *Cent. Eur. J. Phys.* **10**, 568 (2012).

[16] M. Rybczyński, Z. Włodarczyk, G. Wilk, *J. Phys. G* **39**, 095004 (2012).

[17] R. Blankenbecler, S.J. Brodsky, J. Gunion, *Phys. Rev.* **D12**, 3469 (1975).

[18] R. Blankenbecler, lectures presented at Tübingen University, Germany, SLAC-PUB-2077, June 1977.

[19] S.J. Brodsky, H.J. Pirner, J. Raufeisen, *Phys. Lett.* **B637**, 58 (2006).

[20] T. Wibig, *J. Phys. G* **37**, 115009 (2010).

[21] O.J.E. Maroney, *Phys. Rev.* **E80**, 061141 (2009).

[22] T.S. Biró, K. Ürmösy, Z. Schram, *J. Phys. G* **37**, 094027 (2010); T.S. Biró, P. Ván, *Phys. Rev.* **E83**, 061147 (2011); T.S. Biró, Z. Schram, *EPJ Web of Conferences* **13**, 05004 (2011); T.S. Biró, *Is There a Temperature? Conceptual Challenges at High Energy, Acceleration and Complexity*, Springer, 2011.

[23] J. Cleymans, D. Worku, *J. Phys. G* **39**, 025006 (2012).