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EXTENSION OF THE FUSION BY DIFFUSION MODEL FOR DESCRIPTION OF THE SYNTHESIS OF SUPERHEAVY NUCLEI IN (FUSION, xn) REACTIONS*

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We present a modified version of the Fusion by Diffusion model adapted to calculate cross sections and optimum bombarding energies for synthesis of superheavy nuclei in xn reactions. Model predictions of the cross sections for the synthesis of superheavy nuclei of the atomic numbers Z ranging from 104 to 113 in $1n$, $2n$ and $3n$ reactions with the use of lead and bismuth targets are compared with the experimental data from GSI Darmstadt, LBNL Berkeley and RIKEN Tokyo.

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1. Introduction

The Fusion by Diffusion (FBD) model was proposed by Świątecki *et al.* [1, 2] as a simple tool to calculate cross sections and optimum bombarding energies for a class of reactions leading to the synthesis of superheavy nuclei in the so-called cold fusion ($1n$) reactions, in which low excited compound nuclei emit only one neutron. Recently we extended [3] this model by accounting for the angular momentum effects. This allowed us to apply the model for reactions at higher excitation energies. In Ref. [3] a complete set of experimental data on cold fusion reactions was analyzed and a reliable empirical information on the starting-point configuration of the fusion reactions was established.

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In the present article, we propose a further extension of the model enabling an easy method of calculating the statistical decay of the compound nucleus in xn reactions, without necessity of using the Monte Carlo method. To test the new method we analyzed reactions induced by various projectiles on ^{208}Pb and ^{209}Bi targets, in which excitation functions had been measured not only for cold fusion ($1n$) channel, but also for $2n$ and $3n$ channels. This allowed us to establish the energy dependence of the fusion starting-point configuration in a wider energy range and to apply the model for description of both cold fusion and hot fusion reactions.

2. Basic information on the extended version of the FBD model

In the modified, l-dependent version of the FBD model [3], the partial evaporation-residue cross section $\sigma_{\rm ER}(l)$ for production of a given final nucleus in its ground state is factorized as the product of the partial capture cross section $\sigma_{\rm cap}(l) = \pi\lambda^2(2l+1)T(l)$, the fusion probability $P_{\rm fus}(l)$ and the survival probability $P_{\rm surv}(l)$

$$\sigma_{\rm ER} = \pi\lambda^2 \sum_{l=0}^{\infty}(2l+1)T(l)\,P_{\rm fus}(l)\,P_{\rm surv}(l)\,. \tag{1}$$

The capture transmission coefficients, $T(l)$, are calculated in a simple sharp cut-off approximation, where the upper limit $l_{\rm max}$ of full transmission, $T(l) = 1$, is determined by the capture cross section, $\sigma_{\rm cap}$, known from the systematics described in Refs. [3, 4]. Here λ is the wave length, $\lambda^2 = \hbar^2/2\mu E_{\rm c.m.}$, and μ is the reduced mass of the colliding system. The fusion probability, $P_{\rm fus}(l)$, is the probability that the colliding system, after reaching the capture configuration (sticking), will eventually overcome the saddle point and fuse, thus avoiding reseparation. The last factor in Eq. (1), the survival probability $P_{\rm surv}(l)$, is the probability for the compound nucleus to decay to the ground state of the final residual nucleus via evaporation of light particles (neutrons in our case) and γ rays, thus surviving fission.

The fusion probability, $P_{\rm fus}$, is a key factor in all models pretending to describe fusion of superheavy systems. It is responsible for dramatic hindrance of the fusion cross sections due to the fact that the saddle configuration of the heaviest compound nuclei is much more compact than the configuration of two colliding nuclei at sticking. It is assumed in the FBD model that after the contact of the two nuclei, a neck between them grows rapidly at an approximately fixed mass asymmetry and constant length of the system bringing it to the "injection point" at the bottom of the asymmetric fission valley. From this injection point the system starts its climb uphill over the saddle in the process of thermal fluctuations in the shape degrees of freedom.

The location of the injection point, $s_{\rm inj}$, is the only adjustable parameter of the FBD model. It was shown in Ref. [1] that by solving the Smoluchowski diffusion equation, the probability that the system injected on the outside of the saddle point at an energy H below the top will achieve fusion is

$$P_{\rm fus} = \tfrac{1}{2}\left(1 - {\rm erf}\sqrt{H/T}\right) , \tag{2}$$

where T is the temperature of the fusing system. The energy threshold H opposing fusion in the diffusion process is calculated using algebraic expressions given in [3] which approximate the potential energy surface.

As regards the survival probability $P_{\rm surv}$, the standard statistical-model calculation were done by applying the Weisskopf formula for the particle (neutron) emission width Γ_n, and the conventional expression of the transition-state theory for the fission width $\Gamma_{\rm f}$. The level density parameters a_n and $a_{\rm f}$ for neutron evaporation and fission channels were calculated as proposed by Reisdorf [5] with shell effects accounted for by the Ignatyuk formula [6]. All details can be found in Ref. [3].

The Monte Carlo method is the most suitable though very time consuming way of calculating survival probability in multiple evaporation (xn) channels. To avoid Monte Carlo calculations we used a simplified algorithm: We assume the competition of fission only with neutron emission, so the total decay width $\Gamma_{\rm tot} \approx \Gamma_n + \Gamma_{\rm f}$. In one-neutron-out reactions the survival probability is then given by $P_{\rm surv}^{(1n)} = (\Gamma_n/\Gamma_{\rm tot})P_<$, where $\Gamma_n/\Gamma_{\rm tot}$ is the probability of neutron emission and $P_<$ denotes the probability that the excitation energy of the residual nucleus, after the neutron emission, is less than the threshold energy for second-chance fission or emission of another neutron (whichever is lower). For exact formula for $P_<$ see Eq. (32) in [3]. Consequently, $1 - P_<$ denotes the probability that the emission of the next neutron is energetically possible. The above reasoning can be extended to emission of x neutrons. Thus the formula for the survival probability in xn channel can be written as

$$P_{\rm surv}^{(xn)} = \frac{\Gamma_n^{(1)}}{\Gamma_{\rm tot}^{(1)}}\left(1 - P_<^{(1)}\right)\frac{\Gamma_n^{(2)}}{\Gamma_{\rm tot}^{(2)}}\left(1 - P_<^{(2)}\right)\dots\frac{\Gamma_n^{(x)}}{\Gamma_{\rm tot}^{(x)}}P_<^{(x)} , \tag{3}$$

where the upper indexes in brackets numerate a step of the deexcitation cascade. At each evaporation step the initial excitation energy of the system is reduced by the binding energy of the emitted neutron and an average kinetic energy of the neutron. The average neutron kinetic energy is calculated using an exact expression for the nuclear level density function in an energy range from zero to the threshold energy [3].

3. Results of calculations

In this work we analyzed the following reactions studied in experiments at GSI Darmstadt: ^{208}Pb(^{50}Ti,xn)$^{258-x}$Rf [7, 8], ^{209}Bi(^{50}Ti,xn)$^{259-x}$Db [9] and ^{208}Pb(^{54}Cr,xn)$^{262-x}$Sg [7], where ($x = 2, 3$), and ^{209}Bi(^{51}V,$2n$)^{258}Sg [8], ^{209}Bi(^{54}Cr,$2n$)^{261}Bh [10], ^{208}Pb(^{58}Fe,$2n$)^{264}Hs [7]. Also we analyzed the following $2n$ reactions studied in experiments at LBNL Berkeley: ^{208}Pb(^{50}Ti,$2n$)^{256}Rf [11], ^{208}Pb(^{48}Ti,$2n$)^{254}Rf [11], ^{209}Bi(^{50}Ti,$2n$)^{257}Db [12], ^{208}Pb(^{51}V,$2n$)^{257}Db [12], ^{208}Pb(^{52}Cr,$2n$)^{258}Sg [13], ^{209}Bi(^{54}Cr,$2n$)^{261}Bh [14] and ^{208}Pb(^{55}Mn,$2n$)^{261}Bh [14].

The FBD model contains only one adjustable parameter, the injection-point distance $s_{\rm inj}$, which is defined as the excess of length of the deformed system at the injection-point configuration (in the asymmetric fission valley) over the sum of the projectile and target diameters. In Ref. [3] we presented a phenomenological systematics of this crucial parameter, based on an analysis of 27 cold fusion reactions. The parameterization of $s_{\rm inj}$ was limited, however, to rather narrow range of sub-Coulomb-barrier energies characteristic for cold fusion reactions. With the use of the present version of the model, adapted to calculate cross sections for xn reactions, we were able to determine location of the injection-point distance in a much wider energy range. A value of $s_{\rm inj}$ was determined individually for each reaction ($1n$, $2n$ or $3n$) by fitting the height of the theoretical excitation function (averaged over the target thickness) to the maximum of the experimental excitation function.

Fig. 1 combines all deduced values of $s_{\rm inj}$ obtained from analysis of cold fusion reactions in Ref. [3] and $2n$ and $3n$ reactions in this work. The values of $s_{\rm inj}$ are plotted as a function of the excess of available energy above the Coulomb barrier, $E_{\rm c.m.} - B_0$. The solid line represents the parametrization of $s_{\rm inj}$ from Ref. [3]. The injection-point distances deduced from $2n$ reactions are approximately as much dispersed, with respect to the assumed linear dependence on ($E_{\rm c.m.} - B_0$), as those deduced from cold fusion ($1n$) reactions. This considerable dispersion of the $s_{\rm inj}$ distances (much exceeding the error bars corresponding to uncertainties of the cross sections) can be linked perhaps to inaccurate estimates of the theoretical fission barriers (see Ref. [3]) used in the present version of the FBD model. It should be also noted that the $s_{\rm inj}$ distances deduced from $3n$ reactions fit quite well the extrapolation of $s_{\rm inj}$ values for $1n$ reactions. A weak trend of decreasing $s_{\rm inj}$ values with the increasing energy is now clearly established on the basis of data in a wide, nearly 30 MeV energy range. As the $s_{\rm inj}$ parameterization deduced from $1n$ reactions [3] is appropriate also for $2n$ and $3n$ reactions, we keep it unchanged for the whole set of reactions analyzed in this work.

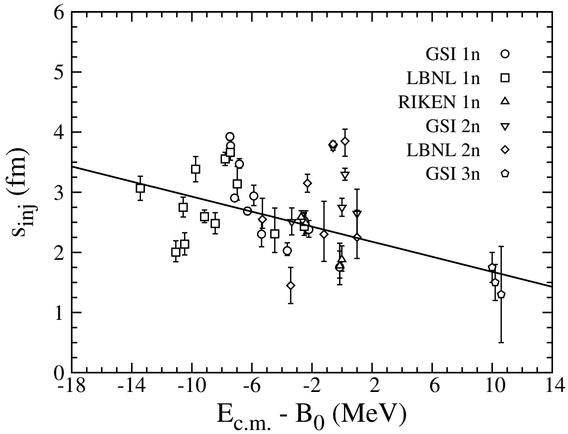

Fig. 1. The injection point distances, s_{inj}, deduced from the experimental data (see the text), plotted as a function of the excess of available energy above the Coulomb barrier, $E_{\mathrm{c.m.}} - B_0$. A linear parameterization of the injection point distance, $s_{\mathrm{inj}} \approx 2.30$ fm $- 0.062(E_{\mathrm{c.m.}} - B_0)$ fm/MeV, is shown by solid line.

Fig. 2 shows a set of experimental cross sections (at the maximum of the excitation function) for synthesis of superheavy nuclei of $Z = 104$–113 in $1n$ (see Table I in Ref. [3] and references there), $2n$ and $3n$ reaction channels [7,8,9,10,11,12,13,14]. The cross sections are plotted as a function of the Coulomb interaction parameter $z = Z_{\mathrm{P}}Z_{\mathrm{T}}/(A_{\mathrm{P}}^{1/3} + A_{\mathrm{T}}^{1/3})$, where indexes P and T denote the projectile and target nuclei, respectively. The corresponding theoretical values, predicted with the FBD model with a linear parameterization of the injection point distance, $s_{\mathrm{inj}} \approx 2.30$ fm $- 0.062(E_{\mathrm{c.m.}} - B_0)$ fm/MeV, are also shown in Fig. 2. They are connected by straight dotted lines. It is seen that the model predictions are in a quite good agreement with experimental data for all three reaction channels. Note that structure effects characterizing individual combinations of the target and projectile, which are clearly seen in z-dependence of the theoretical cross section, seem to be correlated with the experimental cross sections.

In conclusion, the FBD model was successfully extended to calculate multiple evaporation (xn) channels by using a simplified algorithm avoiding necessity of the Monte Carlo calculations. In the present version, our model satisfactorily describes all the data on synthesis of superheavy elements of $Z = 104$–113 in reactions on ^{207,208}Pb and ^{209}Bi targets. The extended version of the FBD model can be used now for calculating synthesis cross sections and optimum bombarding energies for both cold fusion and hot fusion reactions in a wide range of superheavy fusing systems.

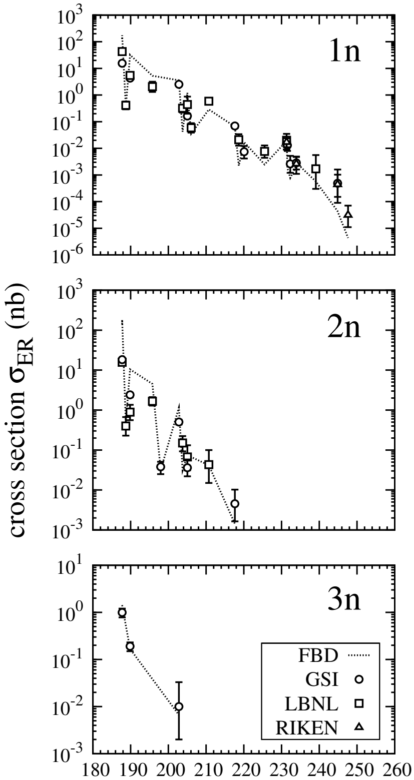

Fig. 2. Set of measured cross sections (at maximum of the excitation function) for synthesis of superheavy nuclei of $Z = 104$–113 in cold fusion ($1n$) reactions (upper panel), $2n$ reactions (middle panel) and $3n$ reactions (lower panel). Data for reactions induced by various projectiles on ^{207,208}Pb and ^{209}Bi targets, studied in experiments at GSI Darmstadt, LBNL Berkeley and RIKEN Tokyo, are shown. (For a list of $2n$ and $3n$ reactions with respective references see the text; All $1n$ reactions with references are listed in Table I of Ref. [3].) The cross sections are plotted as a function of the Coulomb interaction parameter z. Results of the FBD model calculations are shown by dotted lines. Each particular reaction in the plots can be identified according to its value of the Coulomb parameter z.

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