

WOUNDED CONSTITUENTS

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The concept of the “wounded” hadronic constituents is formulated. Preliminary estimates indicate that it may help to understand the transverse mass dependence of the particle production in hadron–nucleus and nucleus–nucleus collisions.

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1. Introduction

The concept of a “wounded” source of particles, formulated long time ago [1, 2], turned out useful in description of particle production from nuclear targets at low transverse momentum [3]. In this note, after recalling the physical origin of the idea, I discuss its possible extension which may lead to new applications¹.

A wounded source, by definition, emits a certain density of particles, independently of the number of collisions it underwent inside the nucleus. To explain the physical meaning of this concept, let us recall that the idea originated from the observation that the process of particle production is not instantaneous [5]. A simplified version of the argument [6] can be presented as follows.

Consider a particle created in a high-energy collision. In the reference frame, where the longitudinal momentum of this particle vanishes, the minimal time necessary for its creation is $t_0 \geq 1/m_\perp$, where $m_\perp = \sqrt{m^2 + p_\perp^2}$

¹ The history and recent developments in the subject were summarized briefly in [4].

is its energy. In the “laboratory” frame, where the target nucleus is at rest, the particle in question acquires some longitudinal momentum, the time is multiplied by Lorentz factor, and we have

$$t \geq \gamma t_0 = \frac{E}{m_{\perp}^2} = \frac{\cosh y_{\text{lab}}}{m_{\perp}}, \quad (1)$$

where E is the energy of the particle. Consequently, the uncertainty of the distance from the collision point to that at which the particle is created (*i.e.* the resolving power in the longitudinal distance) is

$$L = vt \geq \frac{\sinh y_{\text{lab}}}{m_t}. \quad (2)$$

When the rapidity of the produced particle is large enough so that $L > Z(b)$, where $Z(b)$ is the size of the nucleus at a given impact parameter, the particle cannot resolve separate collisions and therefore it is natural to suggest that its creation may be insensitive to the number of collisions of the source. This is the origin of the concept of a wounded source. One sees that it makes sense only for production of particles with the laboratory rapidity exceeding that determined by the condition $L > Z(b)$.

Applications of this idea to “minimum bias” events dominated, as is well-known, by production of pions at low transverse momentum [7] were met with a good deal of success [4]. A particularly good description is obtained, within the quark–diquark dominance picture [8], which may be considered as a modification of the dual parton model [9].

It is also well-known, however, that the model fails for production of heavy particles and/or particles having transverse momentum exceeding ~ 500 MeV [10, 11]. At low energies this may have been attributed to m_{\perp} in the denominator of (2), implying small L for larger m_{\perp} . But the data from RHIC proved without any doubt that even when the condition (2) is satisfied, production of particles at high m_{\perp} exceeds that predicted in [8] (for the review of data, see *e.g.* [11]).

In the present note, I explore the possibility that the transverse mass of a created particle, apart from defining the minimal time needed for its creation (*c.f.* (1)), is also related to the transverse size δ of the source from which it is emitted. The idea is based on the observation that the quantum nature of the emission process suggests the uncertainty relation

$$\langle \delta \rangle \langle p_{\perp} \rangle \simeq 1. \quad (3)$$

We shall investigate the consequences of this idea for the A -dependence of transverse momentum of produced particles.

In the next section we remind briefly the quark–diquark model. Generalization of the concept of wounded constituents implementing (3) is presented in Sec. 3. The formulae for the transverse mass spectra are derived in Secs. 4 and 5. The cross-sections of the wounded constituents are discussed in Sec. 6 together with some numerical exercises. Our conclusions are listed in the last section. Application to the Tsallis distribution is developed in the Appendix.

2. Wounded nucleons, quarks and diquarks

The beginning of the idea of wounded nucleons [1] was purely empirical. The first accelerator measurements of multiplicities in nucleon–nucleus collisions [12] have shown that the average multiplicity follows the simple rule

$$n_{HA} = \frac{1}{2}(\nu_A + 1)n_{HH}, \quad (4)$$

where

$$\nu_A = \frac{A\sigma_{HH}}{\sigma_{HA}} \quad (5)$$

is the average number of collisions of the projectile inside the nucleus.

This result came as a surprise because “everybody” was expecting the relation $n_{HA} = \nu_A n_{HH}$ which seemed much more natural, as it suggests that each collision contributes approximately the same amount to the observed particle multiplicity. The formula (4), on the other hand, can be easily understood if one accepts that each nucleon contributes the same amount, independently of the number of collisions it suffered in the process.

Although the idea worked reasonably well for total multiplicities, the understanding of the rapidity distributions came only 30 years later. To make the long story short², let me just say that, as far as I can judge, there were three essential steps:

- (i) the generalization of the concept of wounded nucleons to that of wounded constituents (originally: quarks [2, 13–16], see also [17]) which allowed to make the idea more flexible,
- (ii) abandoning the requirement of boost-invariance [18] and
- (iii) accepting that the contribution from a single wounded constituent is not restricted to one hemisphere [18] (see also [19, 20]).

Finally, a good description of (p_\perp integrated) RHIC data at 200 GeV was obtained assuming that nucleon contains two independent sources of particles: a constituent quarks and a constituent diquark [8]. Particle densities

² A brief history of some of these efforts can be found in [4].

produced by quark and by diquark were assumed identical and could be determined from data. They turn out strongly asymmetric and thus obviously violating boost-invariance. They are not restricted to one hemisphere but extend throughout almost full rapidity region, in conformity with the results of [18].

3. Generalization: wounded constituents

We have seen that the concept of wounded sources is well founded in the basic theory and — at the same time — it is an useful tool in description of data on particle production from various projectiles and targets. It is also clear, however, that definite predictions can only be obtained when the specific nature of these sources is precisely defined. Indeed, the results from the wounded nucleon model are substantially different from those of the wounded quark–diquark model and those differ, in turn, from the wounded quark model. In short, the concept of wounded sources must be supplemented by information on the nature of sources, about their numbers and their cross-sections. Only then the concept may be effectively used to uncover the hidden relations between various processes.

As already mentioned, the idea of wounded sources, as exploited till now [4], shows one serious disadvantage. While it describes reasonably well the physics at low p_\perp , it fails badly at p_\perp exceeding 500 MeV and for heavy particles [10, 11], where particle production increases with the size of the target faster than predicted by any wounded source model. At high energies (*e.g.* those of RHIC) this failure cannot be attributed to the violation of the coherence condition (2). One sees therefore that some element of the game is missing.

A hint can be obtained from comparison with data. It was recently shown that the wounded nucleon model works very well for distributions in the limit $p_\perp \rightarrow 0$ [21]. Furthermore, the data integrated over p_\perp (*i.e.* dominated by p_\perp below $\simeq 300$ MeV) can be described by the wounded quark–diquark model [8]. Moreover, it is well-established (*e.g.* from numerous experiments in deep inelastic lepton–hadron collisions) that the number, life times, energies and (transverse) sizes of the constituents in a hadron are by no means fixed but are distributed within a rather broad spectrum. Consequently, in a collision of two hadrons various constituents may interact and get “wounded”. Each wounded constituent emits secondary particles and the final result is a sum of contributions from all of them. The number of wounded constituents of a given type depends on how many are present in the colliding hadrons as well as on their corresponding cross-sections which in turn may depend on their characteristics (*e.g.* colour charge, interaction strength and transverse size).

In the present paper, we show that this new picture radically changes the predictions of the wounded constituent model for particle production at medium and large transverse momenta. Two effects are contributing to this result:

- (i) Constituents of various energies and (transverse) sizes are expected to emit particles (mostly gluons) with various distributions of transverse momenta and rapidities. Here we study the consequences of the simplest and natural choice, suggested by uncertainty principle

$$\frac{dn(p_{\perp}, \delta; y)}{d^2p_{\perp} dy} \equiv \rho(p_{\perp}, \delta; y) = I(\delta; y) e^{-p_{\perp}^2 \delta^2}, \quad (6)$$

where δ is the transverse size of the “source” (*i.e.* a constituent wounded in the collision). The intensity $I(\delta, y)$ is independent of p_{\perp} but may depend on some other relevant variables as *e.g.* mass of the emitted particle m and the energy of the collision.

- (ii) The cross-section $\sigma_{\delta H}$ for the collision of a constituent with a nucleon is also expected to be sensitive to constituent’s size. This in turn will influence the number of wounded constituents contributing to particle emission.

Using these ideas one can formulate the prediction of the model of wounded constituents for the observed distribution in the collision of the nuclei A and B as

$$\begin{aligned} \frac{dn_{AB}(p_{\perp}, y)}{d^2p_{\perp} dy} &\equiv \rho_{AB}(p_{\perp}, y) = \int dw_A(\delta, b; B) \rho(p_{\perp}, \delta; y) \\ &+ \int dw_B(\delta, b; A) \rho(p_{\perp}, \delta; -y), \end{aligned} \quad (7)$$

where $dw_A(\delta, b; B)$ is the number of constituents of size between δ and $\delta + d\delta$ wounded in nucleus A in the collision with the nucleus B at the impact parameter b .

To make use of this prediction it is necessary to recall the old formula for the number of wounded constituents in a collision of two composite objects [1, 2]. Consider a collision of two nuclei A and B . For the number dw_A of wounded constituents of size between δ and $\delta + d\delta$ in A we have

$$\begin{aligned} \sigma_{AB}(b) dw_A(b; \delta; B) &= \int dN_A(\delta; s) \sigma_{\delta B}(b - s) \\ &= AdN_H(\delta) \int d^2s D_A(s) \sigma_{\delta B}(b - s) \equiv AdN_H(\delta) \hat{\sigma}_{\delta B}(b) \end{aligned} \quad (8)$$

and an analogous formula for dw_B . Here $dN_H(\delta)$ is the number of constituents of size between δ and $\delta + d\delta$ in the nucleon, $dN_A(b; s) = AD_A(s)dN_H(\delta)$ is the number of constituents of size between δ and $\delta + d\delta$ in the nucleus at the impact parameter s , $D_A(s)$ is the (transverse) distribution of the nucleons in the nucleus A normalized to unity, $\sigma_{\delta B}(b)$ is the cross-section of one constituent of size δ on the nucleus B , and $\sigma_{AB}(b)$ is the total (inelastic) cross-section for the A - B collisions³.

4. Nucleon-nucleon collisions

The basic formula (7) contains a product of three, essentially unknown, functions: the intensity $I(\delta; y)$, the number $dN_H(\delta)$ of the constituents inside a nucleon and the cross-section $\sigma_{\delta B}(b)$. In this section, we show that this product can be derived from the existing data on transverse momentum distribution in nucleon-nucleon collisions.

For nucleon-nucleon collisions, Eq. (8) gives for the number of wounded constituents in one of them

$$dw_H(\delta) = dN_H(\delta) \frac{\sigma_{\delta H}}{\sigma_{HH}}. \quad (9)$$

When this is inserted into (7) we obtain for the observed transverse momentum distribution in nucleon-nucleon collisions

$$\begin{aligned} \rho_{HH}(p_\perp; y) &= \frac{1}{\sigma_{HH}} \int dN_H(\delta) \sigma_{\delta H} [I(\delta, y) + I(\delta, -y)] e^{-p_\perp^2 \delta^2} \\ &\equiv \int [G(\delta, y) + G(\delta, -y)] e^{-p_\perp^2 \delta^2} d\delta \\ &= \int_0^\infty \frac{[G(\delta, y) + G(\delta, -y)]}{2\delta} e^{-p_\perp^2 \delta^2} d\delta^2 \end{aligned} \quad (10)$$

with

$$G(\delta, y) \equiv \sigma_{\delta H} \frac{dN_H(\delta)}{d\delta} I(\delta, y). \quad (11)$$

Thus one sees that the function $[G(\delta, y) + G(\delta, -y)]/\delta$ can be obtained from the measured transverse momentum distribution by inverting the Laplace transform (10).

³ All cross-sections we discuss are understood as inelastic, non-diffractive.

In the present paper, we take advantage of the observation that, for p_\perp below 1–2 GeV, the measured distributions of transverse momenta are well described by the exponential⁴

$$\rho_{HH}(p_\perp, y) = [n(y) + n(-y)]e^{-\beta m_\perp}, \quad (12)$$

where β is a constant and $n(y)$ describes the rapidity dependence.

Combining (10) and (12) and using the identity [22]

$$e^{-\beta m_\perp} = \int_0^\infty du e^{-um_\perp^2} \frac{\beta}{2\sqrt{\pi}u^3} e^{-\beta^2/4u} = \frac{\beta}{\sqrt{\pi}} \int \frac{d\delta}{\delta^2} e^{-\beta^2/4\delta^2} e^{-m_\perp^2 \delta^2} \quad (13)$$

we obtain [23]

$$G(\delta; y) d\delta = n(y) \frac{\beta}{\delta^2 \sqrt{\pi}} e^{-m^2 \delta^2} e^{-\beta^2/4\delta^2} d\delta. \quad (14)$$

At this point a remark is necessary. For the emission of gluons ($m = 0$) formula (14) for $G(\delta; y)$ implies that the nucleon contains very large constituents. Indeed, for $\delta \rightarrow \infty$, G falls only as $1/\delta^2$, giving a really long tail, hardly acceptable. Therefore an additional cut-off is necessary. We shall take it in the form $e^{-\mu^2 \delta^2}$ with $\mu \approx 1/2r_H$, where r_H is the nucleon radius. This gives

$$G(\delta; y) d\delta = n(y) \frac{\beta}{\delta^2 \sqrt{\pi}} e^{-\delta^2/4r_H^2} e^{-\beta^2/4\delta^2} d\delta \quad (15)$$

and thus the transverse momentum distributions depend solely on

$$\mu_\perp \equiv \sqrt{p_\perp^2 + \frac{1}{4r_H^2}}. \quad (16)$$

5. Nuclear collisions

For the nucleon–nucleus collision at the impact parameter b one sees from (8) that the number of wounded constituents in the nucleon is

$$\sigma_{HA}(b) dw_A(b, \delta) = dN_H(\delta) \sigma_{\delta A}(b), \quad (17)$$

whereas the number of wounded constituents in the nucleus is

$$dw(b, \delta) = \nu_A(b) dw_H(\delta), \quad (18)$$

⁴ For discussion of larger transverse momenta, see the Appendix.

where $\nu_A(b)$ is the number of nucleon–nucleon collisions at the impact parameter b

$$\nu_A(b) = A \frac{\sigma_{HH} D_A(b)}{\sigma_{HA}(b)} \quad (19)$$

and $D_A(b)$ is the distribution of the nucleons in the nucleus A (normalized to 1). Consequently, for the distribution of transverse momenta we obtain from (7)

$$\begin{aligned} & \sigma_{HA}(b) \rho_{HA}(p_\perp; b; y) \\ &= \int dN_H(\delta) \sigma_{\delta A}(b) I(\delta; y) e^{-p_\perp^2 \delta^2} d\delta + A D_A(b) \int dN_H(\delta) \sigma_{\delta H} I(\delta; -y) e^{-p_\perp^2 \delta^2} d\delta \\ &= \sigma_{HH} A \int d\delta G(\delta, y) \frac{\sigma_{\delta A}(b)}{A \sigma_{\delta H}} e^{-p_\perp^2 \delta^2} + A \sigma_{HH} D_A(b) n(-y) e^{-\beta \mu_\perp}. \end{aligned} \quad (20)$$

Integration over impact parameter gives

$$\rho_{HA}(p_\perp; y) = \nu_A \left[\int d\delta G(\delta, y) \frac{\sigma_{\delta A}}{A \sigma_{\delta H}} e^{-p_\perp^2 \delta^2} + n(-y) e^{-\beta \mu_\perp} \right], \quad (21)$$

where $\nu_A = A \sigma_{HH} / \sigma_{HA}$ is the number of collisions averaged over all impact parameters.

For nucleus–nucleus (A – B) collisions, Eqs. (7) and (8) give

$$\sigma_{AB}(b) \rho_{AB}(p_\perp, b; y) = AB \sigma_{HH} \int \left\{ G(\delta, y) \frac{\sigma_{\delta A}(b)}{A \sigma_{\delta H}} + G(\delta, -y) \frac{\sigma_{\delta B}(b)}{B \sigma_{\delta H}} \right\} e^{-p_\perp^2 \delta^2} d\delta. \quad (22)$$

When integrated over impact parameters, this formula gives

$$\rho_{AB}(p_\perp; y) = \nu_{AB} \int \left\{ G(\delta, y) \frac{\sigma_{\delta A}}{A \sigma_{\delta H}} + G(\delta, -y) \frac{\sigma_{\delta B}}{B \sigma_{\delta H}} \right\} e^{-p_\perp^2 \delta^2} d\delta, \quad (23)$$

where $\nu_{AB} = AB \sigma_{HH} / \sigma_{AB}$ is the average number of nucleon–nucleon collisions.

Eqs. (20)–(23) give the distribution of transverse momentum of the observed particle. It is not difficult to see that the corresponding formulae for data integrated over some region of transverse momenta (from $p_\perp^{(\min)}$ to $p_\perp^{(\max)}$) are obtained by the simple substitution

$$G(\delta, y) \rightarrow \pi G(\delta, y) \frac{e^{-[p_\perp^{(\min)}]^2 \delta^2} - e^{-[p_\perp^{(\max)}]^2 \delta^2}}{\delta^2}. \quad (24)$$

This formula may be interesting for two reasons:

- (i) Very often data are taken within a limited p_{\perp} range [24];
- (ii) In nucleus–nucleus collisions substantial corrections to (22) and (23) are expected because the observed spectra are modified by the effects of the flow and of the “jet quenchin” [11]. These corrections are, however, much less important for data integrated over d^2p_{\perp} and thus such data may provide a more direct test of the model.

6. Cross-sections and a numerical exercise

One sees from the previous discussion that the only unknown in the problem is the cross-section $\sigma_{\delta H}$ from which the ratio $\sigma_{\delta A}/\sigma_{\delta H}$ can be evaluated by standard methods.

To have nuclear enhancement increasing with increasing transverse mass (as observed experimentally for $m_{\perp} \leq 2$ GeV, the region of interest here), the cross-section $\sigma_{\delta A}/\sigma_{\delta H}$ should be small at small δ . This can be naturally accommodated if the constituents we are dealing with are colour neutral and thus exhibiting the phenomenon of colour transparency. Accepting this point of view (a possible interpretation is discussed in the last section) we take, as a first choice, the form used in [25], *i.e.*

$$\sigma_{\delta H} = \sigma_0 \left[1 - e^{-\delta^2/R^2} \right], \quad (25)$$

where σ_0 and R are parameters. This formula implies that for large δ the cross-section saturates at the value σ_0 . In this limit $\delta \rightarrow \infty$ there is apparently only one constituent inside the nucleon, and therefore one may expect that the nuclear effects are identical to those of the wounded nucleon model. Therefore, as a first approximation, we take

$$\sigma_0 \equiv \sigma_{HH}. \quad (26)$$

Thus we are left with only one parameter, R , which determines how fast the constituent cross-section increases from zero to its limiting value⁵. It should be clear that, since $\sigma_{\delta H}$ is always smaller than σ_{HH} , particle production in the present model is always larger than in the wounded nucleon model.

For illustration, and to obtain a feeling how strong are the effects we are discussing and how sensitive are they to the value of R , we evaluated the nuclear enhancement ratios from the formulae given in Secs. 4 and 5. We have taken $r_H = 0.7$ fm, $\sigma_{HH} = 30$ mb, $\sigma_{HAu} = 1550$ mb, $\sigma_{AuAu} =$

⁵ For the physical interpretaion of R , see [25, 26]

2560 mb which seem appropriate at RHIC energy $\sqrt{s} = 200$ GeV. To avoid the problem of rapidity dependence we have considered $y = 0$, where $n(y) = n(-y)$ and thus $n(y)$ simply drops in the ratios.

In Fig. 1 the μ_\perp dependence of the ratios

$$R_{HA}(p_\perp) = \frac{\sigma_{HA}}{A\sigma_{HH}} \frac{\rho_{HA}(p_\perp)}{\rho_{HH}(p_\perp)} = \frac{1}{\nu_{HA}} \frac{\rho_{HA}(p_\perp)}{\rho_{HH}(p_\perp)} \quad (27)$$

and

$$R_{AA}(p_\perp) = \frac{\sigma_{AA}}{A^2\sigma_{HH}} \frac{\rho_{AA}(p_\perp)}{\rho_{HH}(p_\perp)} = \frac{1}{\nu_{AA}} \frac{\rho_{AA}(p_\perp)}{\rho_{HH}(p_\perp)} \quad (28)$$

evaluated from Eqs. (21) and (23) for p -Au and Au-Au collisions is plotted *versus* μ_\perp for various values of R , ranging from 0.125 to 0.5 fm. One sees a strong dependence on R . At the smallest value, $R = 0.125$ fm, the results are consistent with the wounded nucleon model. As R increases, one observes a clear increase of R_A with μ_\perp from the value close to that predicted by the

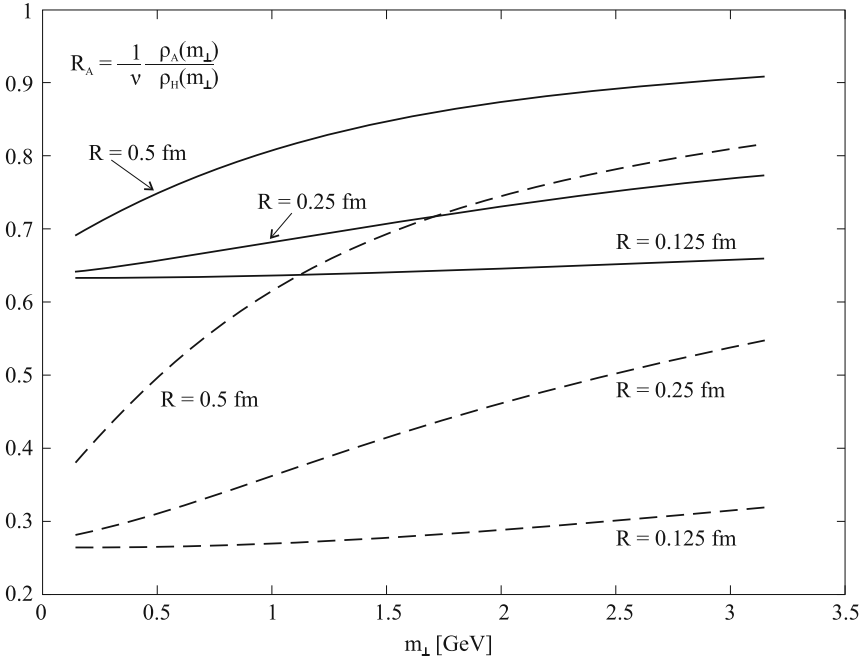


Fig. 1. The nuclear enhancement ratios $R_{HA}(m_\perp)$ and $R_{AA}(m_\perp)$ ((27),(28)), plotted *versus* μ_\perp for various values of R , as indicated in the figure. Full lines: p -Au; Dashed lines: Au-Au.

wounded nucleon model at small μ_\perp towards the asymptotic value⁶ $R_A = 1$ at larger μ_\perp .

In Fig. 2 the same ratios but for the spectra integrated over p_\perp (Eq. (21)) are plotted for pions, kaons, and nucleons⁷. One sees that, as expected, the nuclear effects increase with increasing mass of the particle.

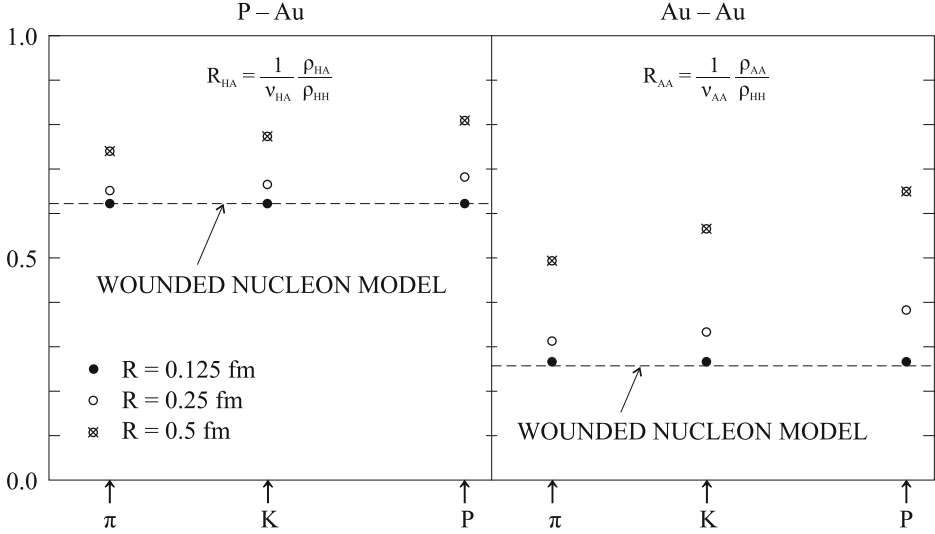


Fig. 2. The nuclear enhancement ratios R_{HA} and R_{AA} of yields for various particles, integrated over d^2p_\perp , plotted for various values of R , as indicated in the figure.

7. Conclusions and outlook

The main purpose of this paper is to point out that the composite nature of hadrons, as revealed in numerous experiments of deep inelastic scattering, does have important consequences for the process of particle production at high energies. Following the old argument [5, 6], based on the observation that particle production process is not instantaneous, the idea of the “wounded” constituents is formulated. Its simplest consequences for nucleon–nucleus and nucleus–nucleus collisions were studied. It turns out that this approach has a good chance to improve significantly the description of the main features of these processes, particularly at transverse momenta exceeding ~ 200 MeV.

⁶ Note that in this model $R_A \leq 1$ for any p_\perp . Thus the Cronin effect [27] cannot be described in this framework, unless the multiple scattering corrections are included [28].

⁷ Here, we assume that the m_\perp dependence of the observed hadrons spectra (but not their normalization, of course) are identical to the μ_\perp spectra of gluons.

In the present formulation, many potentially important effects were omitted either for the sake of clarity or because they require more work. Their (most likely incomplete) list is given below.

- (i) Although, as argued in Sec. 2, multiple scatterings inside the nucleus should not influence the rapidity distribution of particles emitted from a source, they are expected to change the transverse momentum of the source and thus, in consequence, also the transverse momentum distribution of the emitted particles with respect to the direction of the projectile. It was shown [28] that multiple scattering can be responsible for the Cronin effect [27] observed in the proton–nucleus scattering. Thus it should be included before serious comparison with data is done.
- (ii) In nucleus–nucleus collisions the collective phenomena are changing the observed spectra: the transverse flow modifies spectra at low transverse momenta while the effects of jet quenching influences the large transverse momentum tail. They have to be taken into account before the data are analyzed. These effects are much weaker (if present at all) in the nucleon–nucleus collisions which seem, therefore, a better place to test the soundness of the ideas presented here.
- (iii) The corrections listed in (i) and (ii) influence mostly the transverse momentum distributions. If one integrates over the transverse momenta they are largely removed and the result may be closer to reality. Such integration removes also, however, the most spectacular predictions of the present approach.
- (iv) The emission from the “wounded” source is, most likely, dominated by gluons and thus the argument presented in this paper refers, at least formally, solely to gluon distributions. To discuss the actual particle spectra, the hadronization with all its complications has to be included. Hopefully, these effects may, at least partly, cancel in the ratios of nuclear and nucleon particle yields.

Additional comments are in order.

- (i) It may be interesting to speculate about the nature of the constituents we are considering. Accepting that a high-energy hadron is built from quarks and gluons, it is natural to expect the effect of *colour screening*, *i.e.* formation of domains of various sizes in which the colour is locally compensated. These regions are of course fluctuating in size and in time but at very high energy they are frozen and can be treated as “constituents”. During the inelastic (*i.e.* colour-exchanging) collision

with the target such a “constituent” becomes coloured (*i.e.* “wounded”) and starts to radiate gluons⁸.

- (ii) A progress could be obtained if one knew the proper normalization of the Eq. (6) *i.e.* the intensity of the emission from a Gaussian source. This is, however, a difficult problem which requires a separate study.
- (iii) Our argument opens a way to more detailed investigation of the distribution and of the nature of the constituents forming the nucleon. For example, an attempt to derive (25) from the “elementary” cross-section of the two constituents could give some interesting clues about $dN_H(\delta)$ (*c.f.* (8), (9)). Also the relation between the constituent cross-section and the total nucleon–nucleon cross-section [30] can be used for this purpose. All these problems, although interesting, fall far beyond the subject of the present investigation.

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Appendix

Tsallis distribution

Extension of the analysis to larger transverse momenta demands more precise treatment of p_\perp distribution. The exponential (12) represents a reasonable approximation only at p_\perp below 1 GeV. At larger p_\perp the so-called Tsallis formula [31], which can be interpreted as a superposition of simple exponentials [32], is more adequate [33]

$$\frac{dn^{HH}}{d^2p_\perp} = \frac{n_0}{[1 + \beta m_\perp/k]^k} = n_0 \frac{k^k}{\Gamma(k)} \int_0^\infty t^{k-1} e^{-kt} e^{-\beta m_\perp t}, \quad (29)$$

where k and β are independent of m_\perp but may depend on other variables, *e.g.* the energy of the collision and rapidity. n_0 is the normalization factor, responsible for the total multiplicity.

⁸ It was pointed out already long time ago by Baym [29] that the very concept of a “wounded constituent” can be consistently defined only for colour-neutral objects.

Thus we have to find a function $G(\delta)$ satisfying the condition⁹

$$\int G(\delta) e^{-p_{\perp}^2 \delta^2} d\delta = n_0 [1 + \beta m_{\perp}/k]^{-k}. \quad (30)$$

This can be done in two steps. First we convert $e^{-m_{\perp}^2 \delta^2}$ into exponential, as shown in Sec. 4, and then use (29) to obtain

$$G(\delta) d\delta = \frac{k^k}{\Gamma(k)} \frac{\beta}{\sqrt{\pi}} \frac{n_0}{\delta^2} e^{-m^2 \delta^2} d\delta \int_0^{\infty} t^k dt e^{-kt} e^{-\beta^2 t^2 / 4\delta^2}. \quad (31)$$

In the limit of $k \rightarrow \infty$ the Tsallis formula goes into exponential and we recover the formula for $D(\delta)$ used in the main text.

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