THE PHASE STRUCTURE OF STRONGLY INTERACTING MATTER*

Helmut Satz

Faculty of Physics, Bielefeld University
Universitätsstrasse 25, 33615 Bielefeld, Germany

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With increasing temperature and density, strongly interacting matter will undergo two transitions: deconfinement and chiral symmetry restoration. While at low baryon density the two coincide, at high baryon density chiral symmetry can remain broken in a deconfined state. This leads to a phase diagram of three basic states: hadronic matter, a plasma of massive colored quarks, and a quark-gluon plasma.

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The essential features of hadrons are color confinement and spontaneous chiral symmetry breaking. The former binds colored quarks interacting through colored gluons to color-neutral hadrons. The latter brings in pions as Goldstone bosons and gives the essentially massless quarks in the QCD Lagrangian a dynamically generated effective mass. Both features will come to an end in hadronic matter at sufficiently high temperatures and/or baryon densities, though not *a priori* simultaneously. However, rather basic arguments suggest that chiral symmetry restoration occurs either together with or after color deconfinement [1].

It thus appears conceivable that QCD could lead to a three-state phase structure as function of the temperature T and the baryochemical potential μ, as shown in Fig. 1. In such a scenario, color deconfinement would result in a plasma of massive "dressed" quarks; the only role of gluons in this state would be to dynamically generate the effective quark mass, maintaining spontaneous chiral symmetry breaking. At still higher T and/or μ, this gluonic dressing of the quarks would then "evaporate" or "melt", leading to a plasma of deconfined massless quarks and gluons: the conventional

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QGP, with restored chiral symmetry. Evidently, this view of things ignores the possibility of bosonic diquark binding and condensation leading to color superconductivity; we shall return briefly to this aspect later on.

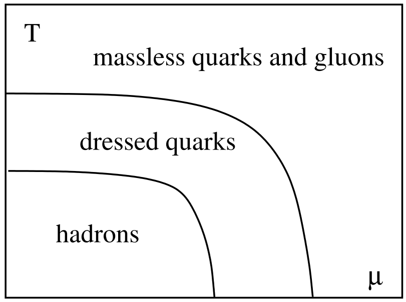

Fig. 1. Three-state scenario of QCD matter.

The basic idea in our considerations will be that a medium of constituents endowed with an intrinsic spatial scale L can only exist as long as its density remains below $1/L^3$. In strong interaction physics, this had first led to the prediction of an upper density limit for mesonic matter [2]. In the past decades it has found a quantitative formal basis in percolation theory [3]. The natural starting point is thus the determination of the intrinsic scales for the given medium.

Hadronic interactions lead to two intrinsic scales, one on the hadronic and the other on the quark level. Both can be expressed in different ways, and we shall elaborate on this. However, since we want to resort to geometric arguments, size parameters are the most useful. One scale then is given by the confinement radius $R_h \simeq 1$ fm, defining the range of the strong force and thus also the size of hadrons. As long as the density n of the medium remains below $n_{\rm c}^h \simeq 1/V_h$, with $V_h = 4\pi\, R_h^3/3$, it is expected to be of hadronic nature. Inside a hadron, the valence quark constituents acquire a dynamically generated effective mass M_q and size R_q. We shall show in the next section that both theoretical and experimental studies indicate $M_q \simeq$ 0.3–0.4 GeV and $R_q \simeq 0.3$ fm. In the density region $n_{\rm c}^h < n < n_{\rm c}^q \simeq 1/V_q$, with $V_q = 4\pi\, R_q^3/3$, we then expect a plasma of deconfined massive quarks, while for $n > n_{\rm c}^q$, the medium will become the conventional quark-gluon plasma, with pointlike massless quarks and gluons as constituents. If both the parameters R_h and R_q were independent of temperature and baryon density, the critical points in the phase diagram of Fig. 1 could thus be easily defined in terms of corresponding percolation points.

However, it is well-known that the latter premise is not generally correct. Lattice studies at or near $\mu = 0$ have shown that here color deconfinement and chiral symmetry restoration coincide [4]. Hence, in a medium of vanishing baryon density, the mass of the constituent quark vanishes at the deconfinement point T_{c}. This is in accord with the formation of the quark dressing through a polarization cloud in the surrounding gluonic medium; as the temperature approaches T_{c}, the screening radius defining cloud size decreases rapidly, and at T_{c}, the cloud has "evaporated", leaving pointlike quarks and gluons. Thus, there remains only one scale, which defines the simultaneous onset of color deconfinement and chiral symmetry restoration. Since the physical reason for this is apparently the presence of a hot gluonic environment, there is little reason to expect a similar behavior at low T and large μ. The aim of our study is therefore to introduce in this region an intermediate plasma of massive quarks, separating hadronic matter and QGP by a state of quark deconfinement and broken chiral symmetry.

There are two different regimes for the quark infrastructure of hadrons, depending how we probe. Relatively hard probes, such as deep inelastic lepton–hadron scattering or hadron–hadron interactions at large momentum transfer, lead to massless pointlike quarks and gluons. In this regime, the parton model with hadronic quark and gluon distribution functions provides a suitable description. On the other hand, soft interactions, as seen in minimum bias proton–proton or pion–proton interactions, suggest that mesons/nucleons consist of two/three "constituent" quarks having a size of about 0.3 fm and a mass of about 0.3–0.4 GeV. Here many features are well accounted for by the additive quark model [5]. We can thus imagine that inside a hadron, a quark polarizes the gluon medium in which it is held through color confinement, and the resulting gluon cloud forms the constituent quark mass M_q [6, 7].

This picture is today found to be quite compatible with heavy quark correlation studies in finite temperature lattice QCD at vanishing baryon density. By evaluating Polyakov loop correlations in a QCD medium of two or three light quark flavors below deconfinement ($T < T_{\mathrm{c}}$), one obtains the free energy $F(r,T)$ as function of the quark separation distance r. In the low temperature limit, $F(r, T = 0)$ saturates beyond a separation of $r \simeq 1.5$ fm, converging to the value $F(\infty, T = 0) \simeq 1.2 \pm 0.1$ GeV [8]. This result is quite universal; it is obtained by separating a heavy quark–antiquark pair, where the separation requires the formation of a light $q\bar{q}$ pair to assure color neutrality. It is obtained as well even if we separate a heavy quark–quark pair, where the formation of a light antiquark–antiquark pair is necessary [9]. Moreover, it is reached for any color channel (singlet or octet for $Q\bar{Q}$, antitriplet or sextet for QQ). The large r behavior of all cases coincides; any uncertainty in the numerical value of $F(\infty, T = 0)$ is due to the necessity to extrapolate to $T = 0$ and to uncertainties in the normalization.

To create an isolated heavy-light quark system, we thus need a gluonic energy input F_g, with

$$F(\infty, T=0) = 2\,F_g \simeq 1.2 \text{ GeV}\,. \tag{1}$$

What is the meaning of F_g? One possible and rather widely accepted interpretation is that it is the "mass" or the energy content of the gluonic string connecting quark and antiquark. With

$$F_g \simeq \sigma\, R_h \simeq 0.6 - 0.8 \text{ GeV} \tag{2}$$

and using $\sqrt{\sigma} = 0.4$ GeV and $R_h = 0.8$–1.0 fm, this does lead to the correct value of F_g, at least in the case of mesons; baryons are not so easily dealt with. We want to consider here instead a scenario in which F_g is the sum of the gluonic dressing masses of two constituent quarks. Then both mesons and baryons can be treated on equal footing, giving us

$$M_q = \frac{F_g}{2} + m_q \simeq 0.3 - 0.4 \text{ GeV}\,, \tag{3}$$

where m_q denotes the bare quark mass; the last term thus corresponds to the light quark limit. We emphasize that the constituent quarks retain their intrinsic quantum numbers; the gluon cloud thus is color-neutral and without any spin.

Such an interpretation is, as already mentioned, supported by the additive quark model [5]. In a collision energy range of about $\sqrt{s} \simeq 5$–20 GeV, in which hard processes do not yet play a significant role, the total cross sections for proton–proton and pion–proton collisions are given as

$$\sigma_{pp} = 3 \times 3\sigma_{qq} \simeq 38 \text{ mb}\,, \qquad \sigma_{\pi p} = 2 \times 3\sigma_{qq} \simeq 25 \text{ mb}\,. \tag{4}$$

The predicted ratio $3/2$ between pion and proton projectiles is seen to be in accord with the data; moreover, $\sigma_{pp} = \pi\, R_h^2$ leads to $R_h \simeq 0.9$ fm for the hadronic radius. Using Eq. (4), we obtain

$$\sigma_{qq} \simeq 3.3 \text{ mb} \;\rightarrow\; R_q \simeq 0.33 \text{ fm} \tag{5}$$

for the corresponding constituent quark sizes in the case of light bare quarks; we return to the more general case of $m_q \gg 0$ shortly. A similar constituent quark radius was also obtained through partonic arguments [7].

As mentioned, we consider the constituent quark to be made up of the bare quark and the gluonic polarization cloud surrounding it. This means that as we move a distance r away from the pointlike quark, we find an effective quark mass $M_q^{\text{eff}}(r)$, depending on how much of the cloud we include at a given r. Screening in the non-Abelian gluon medium limits the size of the cloud, so that beyond $r_0 \simeq 0.3$ fm, the cloud mass saturates, with the constituent quark mass M_q as saturation limit. The resulting behavior [10] is illustrated in Fig. 2, with $R_h \simeq 1$ fm denoting the radius of color confinement. The conceptual scenario just discussed is corroborated by perturbative QCD studies [11], for more detail, see Ref. [12].

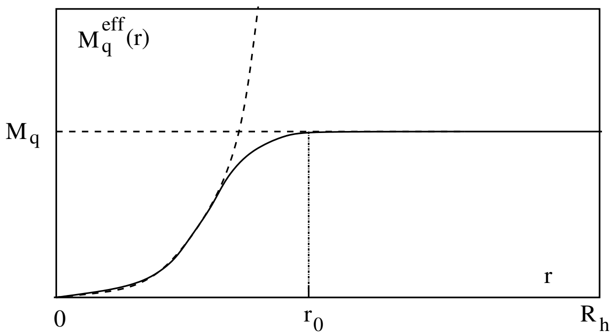

Fig. 2. Effective quark mass $M_q^{\rm eff}(r)$ as seen from a distance r [10].

The conceptual scenario just discussed is supported by perturbative QCD estimates [11], providing an analytic form for the approach of the effective quark mass to a value determined by the non-perturbative chiral condensate $\langle\bar\psi\psi\rangle$; for more detail, see Ref. [12].

The effective constituent quark mass in vacuum is thus determined by the size and energy density of the gluon cloud, or equivalently, by the chiral condensate value in the non-perturbative region. How do these quantities change with temperature in a hadronic medium at vanishing baryon density?

This can again be deduced from heavy quark correlation studies as function of the temperature of the medium. They show that the effective mass of the gluon cloud of an isolated static color charge (obtained by separating a static $Q\bar{Q}$ pair), starting from confinement values around 300 MeV, drops sharply at $T \simeq T_{\rm c}$ [4]. This is accompanied by a corresponding drop of the screening radius. We thus expect the effective quark mass to show the temperature dependence illustrated in Fig. 3.

Complementary to this, the temperature dependence of the chiral condensate is determined directly in finite temperature lattice QCD. Its behavior is also shown in Fig. 3; it is seen that at the deconfinement point, the chiral condensate vanishes as well [4]. This is in accord with the idea that at this point, the gluon cloud around the quark has essentially evaporated.

These considerations show that there are two distinct ways to reach chiral symmetry restoration. On the one hand, even for an interquark distance $2R_h$ well above $2R_q$, a sufficiently hot medium will through gluon screening cause the effective quark mass to vanish, as shown in Fig. 3. On the other hand, when a cold medium becomes so dense that the average interquark distance is $2\,R_q$ or less, the quarks form a connected cluster containing pointlike bare quarks.

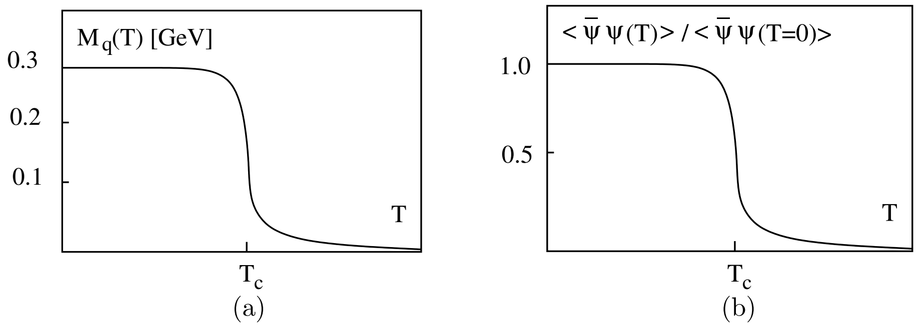

Fig. 3. Constituent quark mass $M_q(T)$ (a) and chiral condensate $\langle\bar{\psi}\psi(T)\rangle$ (b) as function of temperature T.

We note in passing that the two scales, R_h and R_q, have also been considered as the quark and gluon confinement scales, respectively. This implies that color-neutral hadrons have size R_h, whereas color-neutral glueballs have the much smaller intrinsic size R_q, and the spatial ground state glueball size is indeed in most calculations found to be about $R_q \simeq 0.3$ fm.

We want to argue in the following section that at $\mu = 0$, color deconfinement sets in at relatively large interquark separation, but in a hot gluonic environment. As a consequence, the gluon cloud giving rise to the effective quark mass has evaporated, causing deconfinement and chiral symmetry restoration to coincide. For the other extreme, for $T \simeq 0$, at the color deconfinement point the interquark distance is also still well above R_q. But now there is no hot gluonic medium to melt the gluon cloud, so that the cold deconfined medium will be a plasma of massive quarks. Only when its density is increased much more, to the percolation point of the massive constituent quarks, will the medium effectively consist of massless pointlike quarks.

Hadrons are color-singlet quark–antiquark or three-quark states having a characteristic spatial extension of about one fermi. In a hadronic medium, the quark constituents are restricted to the corresponding volume by color confinement, making it impossible for a given quark to separate more than about a fermi from its partner(s) in a color-singlet. When the density of a gas of hadrons is increased sufficiently, by raising either the temperature or the baryon density, a quark constituent in a given hadron will eventually find quarks and antiquarks from other hadrons as close by as its original partner(s). At this point, one can no longer define specific hadrons and hence the concept of hadronic matter becomes meaningless; the hadron gas has become a plasma of deconfined colored quarks and antiquarks. As mentioned, such a density limit to hadronic matter was first suggested by Pomeranchuk [2], well before the advent of the quark model of hadrons.

In the chiral limit, both mesons and baryons have an intrinsic spatial size of about 1 fm. Hence in both cases, the formation of percolating clusters provides a natural limit to the hadronic form of strongly interacting matter [13, 14, 15, 16]. We note that for spin dynamics, the resulting percolation theory can be rigorously formulated [17, 18]; for QCD, however, there were only some first approximative attempts [19], and a definitive theory is still lacking. In particular, both the definition of clusters (using bond weights) and the relevant thermal distribution law (with more than next neighbors) are not yet specified. Our argumentation thus has to remain on a qualitative level.

For mesons, increasing the density eventually leads to a medium in which the distance between a quark constituent from one hadron and an antiquark from another is equal to or less than the typical hadronic size, so that defining specific quark–antiquark pairs as hadrons ceases to make any sense. The percolating medium becomes a plasma of deconfined quarks, antiquarks and gluons. In the baryon-rich region, the increase of density of hard-core nucleons leads to "jamming", *i.e.*, a restriction in the mobility of the nucleons [20, 21]. But here as well we reach eventually the formation of a percolating medium [22], in which the overall quark density is too high to define individual nucleons. We thus consider the limit of confinement in the T–μ diagram of strongly interacting matter to be universally defined by percolation of the relevant hadron species in the given region [16]. It should be emphasized that such a percolation limit is a well-defined geometric form of critical behavior, specified by critical exponents and leading to universality classes, just as found for thermal critical behavior [3]. The essential difference is that geometric singularities (formation of infinite clusters) do not necessarily imply non-analytic behavior for the partition function.

At $\mu = 0$, the percolation density for permeable hadrons (mesons and low density baryons) of size $V = 4\pi R_h^3/3$ is found to be [16]

$$n_M \simeq \frac{1.2}{V_h} \simeq 0.6 \text{ fm}^{-3}\,, \tag{6}$$

with $R_h \simeq 0.8$ fm for the hadron radius. This is the density of the percolating cluster at the onset of percolation, or, equivalently, the density at which the medium no longer allows a spanning vacuum. The corresponding temperature can be obtained once the hadronic medium is specified. For $\mu \simeq 0$, this medium shows abundant resonance formation, and such interactions can be taken into account [23, 24] by replacing the interacting medium of pions and nucleons by an ideal gas of all observed resonance states. For such an ideal gas one finds as deconfinement temperature [16]

$$T_{\rm dec} \simeq 180 \text{ MeV}\,, \tag{7}$$

which agrees well with the value presently obtained in finite temperature lattice QCD.

The resulting medium at this point is, however, strongly non-perturbative. In fact, if we consider the onset of perturbative behavior to be given by $\alpha_{\rm s} \simeq 1/2$, the non-perturbative regime extends up to $T \simeq 4\ T_{\rm c}$. This is again in accord with lattice results, showing that above 3–4 $T_{\rm c}$ the interaction measure $(\epsilon - 3\,P)/T^4$ reaches perturbative behavior [25].

Assuming mesons to be the dominant constituents for $\mu = 0$, the density (6) implies for the average separation between quarks and/or antiquarks at the deconfinement point

$$d_q^M \simeq \frac{1}{n_M^{1/3}} \simeq 1.2 \text{ fm}\,. \tag{8}$$

At the other extreme, for $T = 0$, we have to consider the percolation of nucleons with a hard core [16,22]. Assuming a hard core radius $R_{\rm hc} = R_h/2$, one obtains for the critical density

$$n_B \simeq \frac{2}{V_h} \simeq 0.9 \text{ fm}^{-3}\,, \tag{9}$$

a value about 30% higher than that for permeable hadrons, as consequence of the baryon repulsion. This value can be used to obtain the percolation value of the baryochemical potential, using a van der Waals approach to account for the repulsion in the determination of the density as function of μ [16]. As result, one obtains

$$\mu_{\rm dec} \simeq 1.1 \text{ GeV} \tag{10}$$

as deconfinement point for μ. The separation between quarks at this point becomes

$$d_q^B \simeq \frac{1}{n_B^{1/3}} \simeq 1.0 \text{ fm}\,, \tag{11}$$

slightly less than at $T = 0$, due to the higher density. We note that since $\mu \geq M$, where M denotes the nucleon mass, this leaves as function of μ a rather small window

$$M \leq \mu \leq 1.2\ M \tag{12}$$

for the range of confined baryonic matter at $T = 0$. This window contains essentially all strongly interacting matter in the real world, from nuclei to neutron stars. The corresponding density range runs from $n_0 \simeq 0.17\ \mathrm{fm}^{-3}$ as standard nuclear matter density to the deconfinement value (9) of about $5\ n_0$.

We thus confirm through percolation arguments that color deconfinement is expected to set in at hadron densities for which, in general, the quark constituents are separated by about 1 fm. At this density, any partitioning into hadrons becomes meaningless, and we have a medium of deconfined quarks of mass $M_q \simeq 0.4$ GeV and size $r_0 \simeq 0.3$ fm, separated by a distance $r \simeq 1\ \mathrm{fm} > r_0$. Hence in the density range corresponding to $r_0 \leq r \leq 1$ fm, the quarks can retain their effective constituent mass, so that the deconfined medium now is a plasma of quarks of finite mass and spatial extent, with continued chiral symmetry breaking. A sufficient further increase in density will eventually lead to overlap and percolation of the constituent quarks. We assume that beyond this percolation point, chiral symmetry is effectively restored. Let us see what density the above obtained value of R_q leads to.

At $T = 0$, the density for a system of quarks of mass M_q is given by

$$n_q(\mu_q) = \frac{2}{\pi^2}\left(\mu_q^2 - M_q^2\right)^{3/2}, \tag{13}$$

with $\mu_q = \mu/3$ for the quark chemical potential. The percolation condition for quarks of radius R_q (see Eq. (6))

$$n_q^{\mathrm{ch}} = \frac{1.2}{(4\pi\, R_q^3/3)} \simeq \frac{0.29}{R_q^3} \tag{14}$$

then defines the onset of chiral symmetry restoration. With $R_q \simeq 0.3$ fm, we obtain

$$n_B^{\mathrm{ch}} \simeq 3.5\,\mathrm{fm}^{-3} \simeq 3.9\, n_B^{\mathrm{dec}}\,, \tag{15}$$

indicating that the baryon density threshold for chiral symmetry restoration is about four times higher than that for color deconfinement. The corresponding value for the baryochemical potential is found to be $\mu_B^{\mathrm{ch}} \simeq 2.2$ GeV, to be compared to $\mu_B^{\mathrm{dec}} \simeq 1.1$ GeV. Using the μ counterpart of the two-loop form, the strong coupling α_s has now dropped to the value $\alpha_s(\mu_c^{\mathrm{ch}}) \simeq 0.5$. The resulting phase structure is illustrated in Fig. 4.

Our considerations thus suggest the existence of a plasma of massive deconfined quarks between the hadronic matter state and the quark-gluon plasma. In this state, quarks are deconfined; gluons, however, are "bound" into the constituent quark mass and thus remain in a sense confined. The quark dressing is made up of gluons which form a color-neutral cloud, so that the massive quarks retain their fundamental color state as well as their other intrinsic quantum numbers. The effective degrees of freedom in the resulting quark plasma thus are just those of massive quarks. Its lower

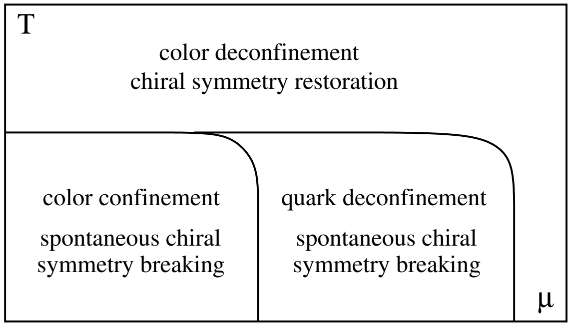

Fig. 4. Phase structure of strongly interacting matter.

limit in baryon density is defined by the onset of vacuum formation, forcing the quarks to bind into color neutral nucleons. The corresponding high density limit is given by the percolation point of the (spatially extended) quarks, beyond which there is a connected medium containing bare quarks and gluons. Finally, increasing the temperature at fixed μ leads here, just as in the hadronic phase, to an evaporation of the gluonic dressing of the quarks and thus to a restoration of chiral symmetry.

The constituent quarks in the deconfined medium will, in general, be interacting with each other. Of particular interest here is the presence of a qq attraction, which at sufficiently low temperatures could lead to the formation of bound colored bosonic qq states ("diquarks"). Baryons have, in fact, often been considered in terms of two quarks bound in a color antitriplet state, which then in turn binds with the remaining quark to form a color singlet. For $T \simeq 0$, such diquarks could lead to Bose-condensation and hence to a color superconductor [26].

To reach baryon densities beyond the percolation limit, the nucleons evidently have to break up into constituents which are subject to less or no hard-core repulsion, and a break-up into constituent quarks and bosonic diquarks as their excited states fills that requirement. Since at low T there is little or no thermal agitation, both could survive for some range of temperature and baryochemical potential. The resulting picture of the new medium thus parallels somewhat that of hadronic matter at $\mu \simeq 0$, where resonance interactions lead to a gas of basic hadrons (pions and nucleons) plus their resonance excitations. Here we have instead a gas of basic constituent quarks, together with the diquark excitations formed through their interaction. The difference is that the basic "particles" now are massive colored fermions, which can exist only in the colored background field provided by a sufficiently dense strongly interacting medium.

In summary, we have shown that chiral symmetry restoration can occur in two ways,

- by evaporation of the gluonic quark dressing in a hot environment, or
- by quark percolation in a cold environment, leading to cluster fusion of the gluon clouds making up the effective quark mass.

At vanishing baryon density, the coincidence of color deconfinement and chiral symmetry restoration arises through the first of these two mechanisms. For low temperature and large baryochemical potential, an evaporation of the gluon dressing does not appear likely, and hence the effective quark mass will survive quark deconfinement. We thus obtain a three-phase structure of matter in QCD, apart from a possible color superconductor. The phase of massive quarks is limited in temperature by evaporation, *i.e.*, also by the first of the above alternatives, and in baryon density by the second, *i.e.*, by quark percolation.

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