EFFECT OF A FLUCTUATING ELECTRIC FIELD ON ELECTRON SPIN DEPHASING TIME IN III–V SEMICONDUCTORS*

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We investigate the electron spin dephasing in low n-doped GaAs semiconductor bulks driven by a correlated fluctuating electric field. The electron dynamics is simulated by a Monte Carlo procedure which keeps into account all the possible scattering phenomena of the hot electrons in the medium and includes the evolution of spin polarization. Spin relaxation times are computed through the D'yakonov–Perel process, which is the only relevant relaxation mechanism in zinc-blende semiconductors. The decay of initial spin polarization of conduction electrons is calculated for different values of field strength, noise intensity and noise correlation time. For values of noise correlation time comparable to the spin lifetime of the system, we find that spin relaxation times are significantly affected by the external noise. The effect increases with the noise amplitude. Moreover, for each value of the noise amplitude, a nonmonotonic behaviour of spin relaxation time as a function of the noise correlation time is found.

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1. Introduction

A great emerging interest within condensed matter physics is the use of electron spin in semiconductor-based devices. Employing spin as an additional degree of freedom is promising for boosting the efficiency of future low-power spintronic devices, with high potential for both memory and logic applications [1,2]. In particular, information is encoded in the electron spin

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state, transferred as attached to mobile carriers by applying an external electric field, and finally detected. Hence, in order to make spintronics a viable technology, sufficiently long spin lifetimes are required together with the possibility to manipulate, control and detect spin polarization [3, 4, 5, 6, 7, 8, 9]. However, a disadvantage of the use of spin polarization as information carrier is that each initial non-equilibrium orientation decays over time during the transport. Therefore, the spin dephasing time could not be sufficient to permit a reliable detection of information. A full understanding of the spin relaxation process is an important issue for the design of spintronic devices [5, 6, 7, 8, 9].

In electronics industry, and particularly in portable and mobile devices, there is a continuing tendency to reduce the supply voltage in order to save power and reduce unwanted emitted radiation. However, low voltages are more subjected to the background noise, and hence the understanding of the influence of fluctuations of voltages on the spin depolarization process is essential. The presence of noise is generally considered a disturbance that affects the performance of a device. Random fluctuations, for example, can reduce the storage time of the information in a memory cell and affect the correct performing of boolean operations. In quantum computation, the loss of coherence, related to destruction of entangled states of qubits by interaction with the environment, is a crucial problem [10]. On the other hand, under specific conditions, the noise can constructively interact with an intrinsically nonlinear system [11, 12, 13] giving rise to positive effects, such as noise enhanced stability (NES) [14, 15, 16, 17, 18, 19]. Recently, the effect of an external source of noise on the electron transport in GaAs crystals in the presence of static and/or periodic electric fields has been studied [18, 20, 21, 22]. In quantum wells and wires, Glazov *et al.* have demonstrated that the randomness in spin–orbit coupling is inevitable and can be attributed both to the electron–electron dynamic collisions and the static fluctuations in the density of the dopant ions [23, 24]. Nevertheless, to the best of our knowledge, the investigation of the role of the externally added noise on the electron spin dynamics in semiconductors is still lacking.

In this paper, we analyze the effect of a correlated fluctuating electric field on the spin lifetime in III–V semiconductors. The depolarization time is calculated for different values of field strength, noise amplitude and correlation time. Electron dynamics is simulated by a Monte Carlo procedure which keeps into account all the possible scattering phenomena of the hot electrons in the medium and includes the evolution of spin polarization vector [25, 26]. Spin lifetimes are computed through the D'yakonov–Perel process [27], which is the only relevant relaxation mechanism in zinc-blende semiconductors [9, 28]. We find that the presence of an external correlated noise source can strongly affect the value of the spin relaxation times. For electric field amplitudes smaller than the Gunn field, the dephasing time decreases with increasing noise intensity. However, for larger amplitudes of the electric field, critically dependent on the noise correlation time, external fluctuations can positively influence the lifetime of the spin. We find a nonmonotonic behaviour with a maximum of the spin depolarization time *versus* the noise correlation time. This maximum is obtained for values of the noise correlation time equal to the dephasing characteristic time of the system. Moreover, noise induced effects are slightly reduced by the inclusion of the electron–electron Coulomb interaction.

The paper is organized as follows. In Sec. 2 the model of the spin relaxation dynamics in the presence of an external correlated noise source is presented. In Sec. 3 the numerical results are given and discussed. Final comments and conclusions are drawn in Sec. 4.

2. Model

2.1. Spin relaxation process

The spin–orbit interaction couples the spin of conduction electrons to the electron momentum, which is randomized by scattering with phonons, impurities and other carriers. The spin–orbit coupling gives rise to a spin precession, while momentum scattering makes this precession randomly fluctuating, both in magnitude and orientation [29].

For delocalized electrons and under nondegenerate regime, the D'yakonov–Perel (DP) mechanism [27] is the only relevant relaxation process in n-type III–V semiconductors [9, 28, 30]. In a semiclassical formalism, the term of the single electron Hamiltonian which accounts for the spin–orbit interaction can be written as $H_{\mathrm{SO}} = \frac{\hbar}{2}\vec{\sigma} \cdot \vec{\Omega}$. It represents the energy of electron spins precessing around an effective magnetic field ($\vec{B} = \hbar\vec{\Omega}/\mu_B g$) with angular frequency $\vec{\Omega}$, which depends on the orientation of the electron momentum vector with respect to the crystal axes. Near the bottom of each valley, the precession vector can be written as

$$\vec{\Omega}_\Gamma = \frac{\beta_\Gamma}{\hbar}\left[k_x\left(k_y^2 - k_z^2\right)\hat{x} + k_y\left(k_z^2 - k_x^2\right)\hat{y} + k_z\left(k_x^2 - k_y^2\right)\hat{z}\right] \tag{1}$$

in the Γ-valley [31] and

$$\vec{\Omega}_L = \frac{\beta_L}{\sqrt{3}}\left[(k_y - k_z)\hat{x} + (k_z - k_x)\hat{y} + (k_x - k_y)\hat{z}\right] \tag{2}$$

in the L-valleys located along the [111] direction in the crystallographic axes [32]. In equations (1)–(2), k_i ($i = x, y, z$) are the components of the electron wave vector, β_Γ and β_L are the spin–orbit coupling coefficients.

Here, we assume $\beta_L = 0.26$ eV/Å·$2/\hbar$, as recently theoretically estimated in Ref. [33], while β_Γ is calculated as in Ref. [26]. Since the quantum-mechanical description of the electron spin evolution is equivalent to that of a classical momentum $\vec{S}$ experiencing the magnetic field $\vec{B}$, we describe the spin dynamics by the classical equation of precession motion $\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$.

The DP mechanism acts between two scattering events and reorients the direction of the precession axis and the effective magnetic field $\vec{B}$ in random and trajectory-dependent way. This effect leads the spin precession frequencies $\vec{\Omega}$ and their directions to vary from part to part within the electron spin ensemble. This spatial variation is called *inhomogeneous broadening* [34]. The inhomogeneous broadening, quantified by the average squared precession frequency $\langle | \vec{\Omega}(\vec{k}) |^2 \rangle$, together with the correlation time of the random angular diffusion of spin precession vector τ_c, are the relevant variables in the D'yakonov–Perel's formula [27]

$$\tau = \frac{1}{\langle | \vec{\Omega}(\vec{k}) |^2 \rangle \tau_c} \,. \tag{3}$$

The spin relaxation time τ is inversely proportional to both the correlation time of the fluctuating spin precession vector τ_c and the inhomogeneous broadening $\langle | \vec{\Omega}(\vec{k}) |^2 \rangle$.

2.2. Noise modeling and simulation details

In our simulations, the semiconductor crystal is driven by a fluctuating electric field $F(t) = F_0 + \eta(t)$, where F_0 is the amplitude of the deterministic part and $\eta(t)$ is the stochastic term due to an external noise source. Here, $\eta(t)$ is modeled as an Ornstein–Uhlenbeck (OU) process, which obeys the stochastic differential equation [35]

$$\frac{d\eta(t)}{dt} = -\frac{\eta(t)}{\tau_D} + \sqrt{\frac{2D}{\tau_D}}\xi(t) \,, \tag{4}$$

where τ_D and D are the correlation time and the intensity of the noise, respectively. The autocorrelation function of the OU process (4) is $\langle \eta(t)\eta(t') \rangle = D\exp(-|t-t'|/\tau_D)$. $\xi(t)$ is a Gaussian white noise with zero mean $\langle \xi(t) \rangle = 0$ and autocorrelation function $\langle \xi(t)\xi(t') \rangle = \delta(t-t')$. Within the framework of the Ito's calculus, the general solution of the Eq. (4) leads to the following expression for the stochastic evolution of the amplitude of the electric field

$$F(t) = F_0 + \eta(0)e^{-t/\tau_D} + \sqrt{\frac{2D}{\tau_D}} \int_0^t e^{-\frac{t-t'}{\tau_D}} dW(t') \,, \tag{5}$$

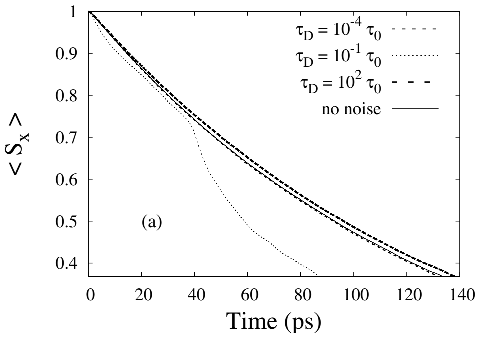
where the initial condition is $\eta(0) = 0$, and $W(t)$ is the Wiener process [35].

The Monte Carlo code used here follows the procedure described in Ref. [36,37]. The spin polarization vector has been incorporated into the algorithm as an additional quantity and calculated for each free carrier [25,26]. All simulations are performed in a GaAs bulk with a free electrons concentration equal to 10^{13} cm^{-3} and lattice temperature T_L equal to 77 K. Moreover, we assume that all donors are ionized and that the free electron concentration is equal to the doping density. The temporal step is 10 fs and an ensemble of 5×10^4 electrons is used to collect spin statistics. All physical quantities of interest are calculated after a transient time of typically 10^4 time steps, long enough to achieve the steady-state transport regime. The spin relaxation simulation starts with all electrons of the ensemble initially polarized ($S = 1$) along $\hat{x}$-axis of the crystal, at the injection plane ($x_0 = 0$) [5, 9]. The spin lifetime τ is calculated by extracting the time corresponding to a reduction of the initial spin polarization by a factor $1/e$.

3. Numerical results and discussion

In Fig. 1, we show the electron spin average polarization $\langle S_x \rangle$ as a function of the time, in the presence of a fluctuating field having deterministic amplitude F_0 and a random component with standard deviation $D^{1/2}$, for three different values of noise correlation time τ_D and in absence of noise; τ_0 is the spin relaxation time obtained with the same value of F_0, in the absence of noise. In panel (a): $F_0 = 1$ kV/cm, $D^{1/2} = 0.4$ kV/cm and $\tau_0 = 0.133$ ns; in panel (b): $F_0 = 6$ kV/cm, $D^{1/2} = 2.4$ kV/cm and $\tau_0 = 1.13$ ps. Both panels show that, when $\tau_D \ll \tau_0$ ($\tau_D = 10^{-4}\tau_0$), the spin dephasing process is not affected by the fluctuations of the electric field, which have a negligible memory (τ_D) with respect to characteristic time τ_0 of the system. The behaviours of $\langle S_x \rangle$ *versus* time in Figs. 1 (a) and 1 (b) coincide with the deterministic one. The spin relaxation process is significantly influenced by the fluctuating field only for values of noise correlation time comparable with τ_0 ($\tau_D = 10^{-1}\tau_0$, $\tau_D = \tau_0$), while the process becomes quasi-deterministic when $\tau_D \gg \tau_0$ ($\tau_D = 10^2\tau_0$).

With the aim to investigate the effects of the correlated noise source on the spin depolarization process, we performed 100 different realizations and evaluated both average values and error bars of the extracted spin lifetimes. In panel (a) of Fig. 2, we show the spin lifetime τ as a function of the ratio between the noise correlation time τ_D and τ_0, for different values of noise intensity D and with $F_0 = 1$ kV/cm. The addition of a correlated noise source, characterized by values of the correlation time τ_D in the range $10^{-2}\tau_0 \div \tau_0$, reduces the values of the spin depolarization time τ up to 0.105 ns, that is with a reduction of $\approx 20\%$. For both $\tau_D \ll \tau_0$ and $\tau_D \gg \tau_0$, the values of τ coincide with τ_0. In panel (b) of Fig. 2, we show the spin lifetime τ as a func-



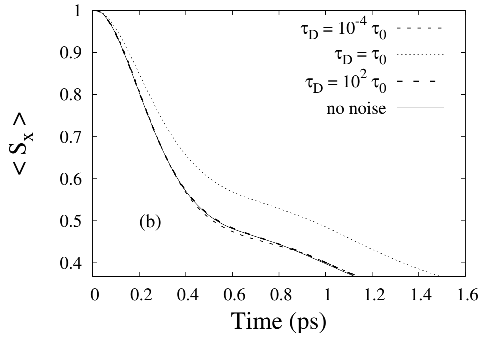


Fig. 1. Spin average polarization $\langle S_x \rangle$ as a function of time obtained by applying a fluctuating field, for different values of the noise correlation time τ_D. (a) $F_0 = 1$ kV/cm, $D^{1/2} = 0.4$ kV/cm and $\tau_0 = 0.133$ ns; (b) $F_0 = 6$ kV/cm, $D^{1/2} = 2.4$ kV/cm and $\tau_0 = 1.13$ ps.

tion of the ratio between the noise amplitude $D^{1/2}$ and F_0 for $\tau_D = 0.1\ \tau_0$. We see that τ decreases with increasing noise intensity. The nonmonotonic behaviour of τ as a function of τ_D/τ_0 (see Fig. 2 (a)) is characterized by a wide minimum centered in $\tau_D/\tau_0 \approx 0.1$. This nonmonotonic behaviour can be explained by analyzing the temporal evolution of the quantities related to both the electron transport and the spin relaxation dynamics. In panels (c) and (d) of Fig. 2, we show the $\hat{x}$-component of the electron momentum k_x and the squared precession frequency $\mid \vec{\Omega}(\vec{k}) \mid^2$ as a function of time for different values of τ_D, namely $\tau_D = 10^{-4}\ \tau_0$, $10^{-1}\ \tau_0$, $10^2\ \tau_0$. In this realization, $F_0 = 1$ kV/cm and $D^{1/2} = 0.4$ kV/cm. For very low values of τ_D, k_x symmetrically fluctuates around its average value, corresponding to that obtained in absence of noise. By increasing the value of τ_D, the temporal evolution of k_x, within a time window comparable with the spin relaxation time, shows an evident asymmetry. The same asymmetry is observed in the temporal evolution of $\mid \vec{\Omega}(\vec{k}) \mid^2$. Because of the proportionality between the electron momentum k_x and the electric field $F(t)$, the Eq. (1) leads to a quadratic relation between $\mid \vec{\Omega}(\vec{k}) \mid^2$ and $F(t)$ on the $k_x^2(k_y^2 - k_z^2)^2$ term and at fourth power on the other two terms. Hence, the positive fluctuations of values of F (k_x) give rise to values of $\mid \vec{\Omega}(\vec{k}) \mid^2$ much greater than those obtained for negative fluctuations of F. So, in accordance with Eq. (3), the asymmetry of $\mid \vec{\Omega}(\vec{k}) \mid^2$ is responsible for the observed reduction of spin lifetime. By further increasing the value of τ_D, the random fluctuating term $\eta(t)$ of the electric field tends to its initial value $\eta(0) = 0$ (see Eq. (4)), and $F(t) \to F_0$. Therefore, the behaviour of k_x and $\mid \vec{\Omega}(\vec{k}) \mid^2$ becomes quasi-deterministic and the spin dephasing time τ approaches its deterministic value τ_0.

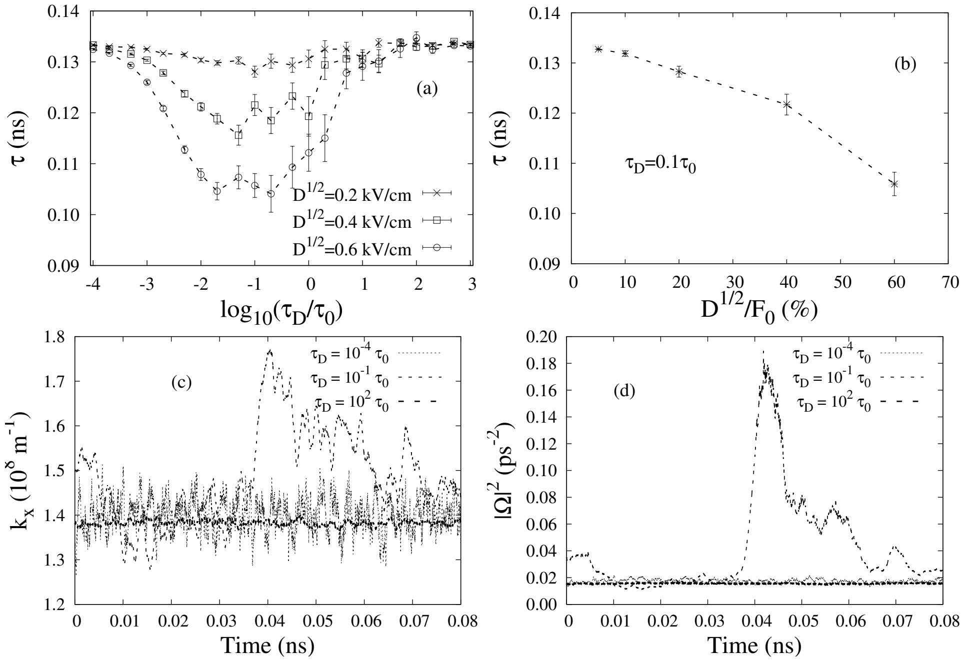


Fig. 2. (a) Spin lifetime τ as a function of the ratio between the noise correlation time τ_D and the spin relaxation time in absence of noise $\tau_0 = 0.133$ ns, at different values of noise intensity D; (b) spin lifetime τ as a function of the ratio between the noise amplitude $D^{1/2}$ and $F_0 = 1$ kV/cm for $\tau_D = 0.1\ \tau_0$; (c) $\hat{x}$-component of the electron momentum k_x and (d) squared precession frequency $\mid \vec{\Omega}(\vec{k}) \mid^2$ as a function of time at different values of τ_D, namely $10^{-4}\ \tau_0$, $10^{-1}\ \tau_0$, $10^2\ \tau_0$, obtained with $F_0 = 1$ kV/cm and $D^{1/2} = 0.4$ kV/cm.

In Fig. 3 (a), the spin lifetime τ as a function of τ_D/τ_0, for different values of noise intensity D is shown. In Fig. 3 (b), the spin lifetime τ as a function of the ratio between the noise amplitude $D^{1/2}$ and F_0, for noise correlation time τ_D equal to τ_0, is shown. In this case, the amplitude of the deterministic component of the electric field F_0 is 6 kV/cm and $\tau_0 = 1.13$ ps. In the presence of a driving electric field greater than the necessary static field to allow the electrons to move towards the upper energy valleys, *i.e.* the Gunn field $E_G = 3.25$ kV/cm, we find a positive effect of the field fluctuations. In fact, our results show that the addition of a correlated noise source, characterized by values of the correlation time τ_D in the range $10^{-1}\ \tau_0 \div 10\ \tau_0$, can increase the value of the spin relaxation time τ up to ≈ 1.35 ps, that is of $\approx 25\%$. This effect increases with the value of noise intensity D and is characterized by a maximum for $\tau_D/\tau_0 \approx 1$. As seen and discussed in the low-field case (see Fig. 2), for both very low and very high values of

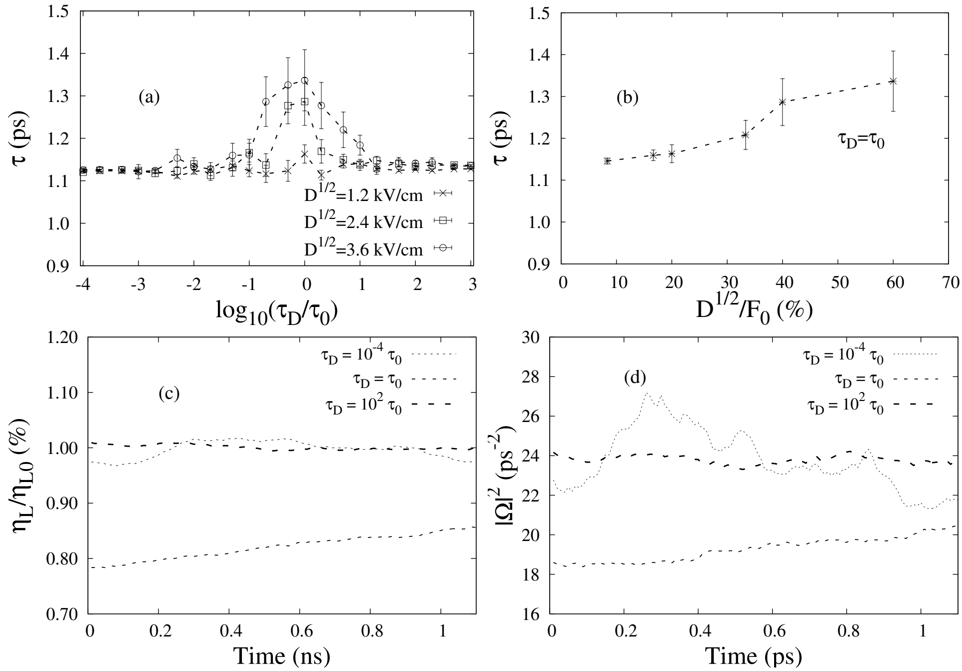


Fig. 3. (a) Spin lifetime τ as a function of the ratio between the noise correlation time τ_D and the spin relaxation time in absence of noise $\tau_0 = 1.13$ ps, for different values of noise intensity D; (b) spin lifetime τ as a function of the ratio between the noise amplitude $D^{1/2}$ and $F_0 = 6$ kV/cm for $\tau_D = \tau_0$. (c) Ratio between electron occupation percentage in L-valleys η_L and electron occupation percentage η_{L0}, obtained in the absence of noise; (d) squared precession frequency $\mid \vec{\Omega}(\vec{k}) \mid^2$ as a function of time for different values of τ_D, namely 10^{-4} τ_0, τ_0, 10^2 τ_0, obtained with $F_0 = 6$ kV/cm and $D^{1/2} = 2.4$ kV/cm.

noise correlation time τ_D, the value of τ approaches τ_0. The presence of a positive effect of noise can be ascribed to the reduction of the electron occupation percentage in L-valleys. By analyzing the temporal evolution of the intervalley transitions it is possible to explain this finding, which can be considered as a further example of *noise enhanced stability* (NES) [18, 19]. In Fig. 3 (c), we show the ratio between the electron occupation percentage in L-valleys η_L and the electron occupation percentage η_{L0}, obtained in the absence of noise. In Fig. 3 (d), the squared precession frequency $\mid \vec{\Omega}(\vec{k}) \mid^2$ as a function of time, for different values of τ_D, namely $\tau_D = 10^{-4}$ τ_0, τ_0, 10^2 τ_0, is shown. In this realization, $F_0 = 6$ kV/cm and $D^{1/2} = 2.4$ kV/cm. For both the lowest and the highest values of τ_D, η_L fluctuates around η_{L0}, that is the presence of noise does not significantly affect the electron L-valleys occupation percentage. Also $\mid \vec{\Omega}(\vec{k}) \mid^2$ fluctuates around the average value observed in the absence of noise, which is equal to 23.9 ps^{-2}. When the noise correlation time τ_D is equal to τ_0, the number of electrons experiencing the spin–orbit coupling in L-valleys is reduced. A similar reduction of the $\mid \vec{\Omega}(\vec{k}) \mid^2$ is found (see panel (d)). Actually the average reduction of the L-valleys occupation is $\approx 2\%$. However, because the spin–orbit coupling in L-valleys is ≈ 16 times stronger than that present in Γ-valley [26], in accordance with Eq. (3), this circumstance causes a decrease of efficacy of the DP dephasing mechanism, leading to the observed increase of the spin lifetime.

Recently, it has been shown that the electron–electron (e–e) scattering inclusion leads to a strong increase of the spin lifetime in III–V semiconductors bulks and heterojunctions [28, 38]. We have verified that the inclusion of the Coulomb e–e scattering mechanism only slightly reduces the effect of the fluctuations on the spin depolarization process [39]. This finding could be ascribed to the fact that the frequent momentum redistribution, experienced from the electrons ensemble [9], gives rise to an enhanced intrinsic noise, which interplays in a different way with the added external noise.

4. Conclusions

In this work, we have investigated the noise influence on the electron spin relaxation process in slightly n-doped GaAs semiconductor bulks. The findings show that a fluctuating electric field, obtained by adding a source of correlated noise to a static field, can modify the spin dephasing time. For electric fields lower than the Gunn field and values of the noise correlation time τ_D comparable with the value of the spin lifetime τ_0, obtained in absence of noise, a reduction of the spin depolarization time, up to $\approx 20\%$, has been observed. This behaviour, strongly dependent on the noise amplitude, can be explained by the time evolution of the squared precession frequency of the electron ensemble, within a time window comparable with τ_0. On the contrary, in the high electric field regime, for $\tau_D = \tau_0$, we find an enhancement of the spin relaxation time up to $\approx 25\%$. This positive effect is ascribed to the decrease of the occupation of the L-valleys, where the strength of spin–orbit coupling felt by electrons is at least one order of magnitude greater than that present in Γ-valley. This finding represents an example of NES in the electron spin relaxation process in III–V semiconductors.

To conclude, our results show that the presence of fluctuations in the applied voltage changes the maintenance of long spin lifetimes in a way strongly dependent on both the strength of the applied electric field and the noise correlation time. This fact could be used as a control mechanism of information storage and processing.

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