# FLUCTUATION OF GOLD PRICE: A MULTIFRACTAL APPROACH

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The time series for gold price has been investigated in past and it was observed that the time series has multifractal properties. The gold price data from 1973 to the present time has been divided into sets of five years each and the variation of degree of multifractality with time is investigated. An attempt has been made to physically interpret the results and to make future predictions of variation in price.

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## 1. Introduction

Gold is a very precious metal and it is extremely popular as an investment. It has been used as money and as a relative standard for currency throughout history. It has since become a flight to safety investment for many big funds and investors. Gold tends to rise in periods of market instability, whether it be due to an economic, political or currency crisis. The gold market is also subject to speculation just like other markets. It is now believed that it is not a commodity anymore, but rather a currency due to the way the price has moved during the financial crisis in the late 2000s.

Commodities tend to move with inflation, but in recent years there has been a distinct correlation between gold and the U.S. dollar. As the U.S. dollar index drops, gold rises and *vice versa*. In periods of global economic uncertainty, gold also rises as investors are unsure of what the market will do.

Fig. 1 depicts the price of gold from 1973 to the present time. It can be observed that though there have been occasional downfalls, the price of gold has a steadily increased decade. Since 2000, gold has gone from around \$270 an ounce to over \$1900 an ounce. The most recent fluctuations in gold price occurred in 2008 and now in 2011 due to the current global debt crisis. Gold jumped to over \$1000 an ounce in March of 2008, but then dropped back down to around \$700 per ounce in November. In this latest economic crisis, gold has exploded. In late August, gold reached a record high of \$1,908 per ounce. The price of gold has come back somewhat and has been trading in the range of \$1,600 and \$1,700 in late September. Gold will continue to be a safe haven investment for big investors as long as they still believe that the current global debt crisis will continue. So it becomes very interesting to study the fluctuation of gold price and to investigate whether any predictions can be made on the market.

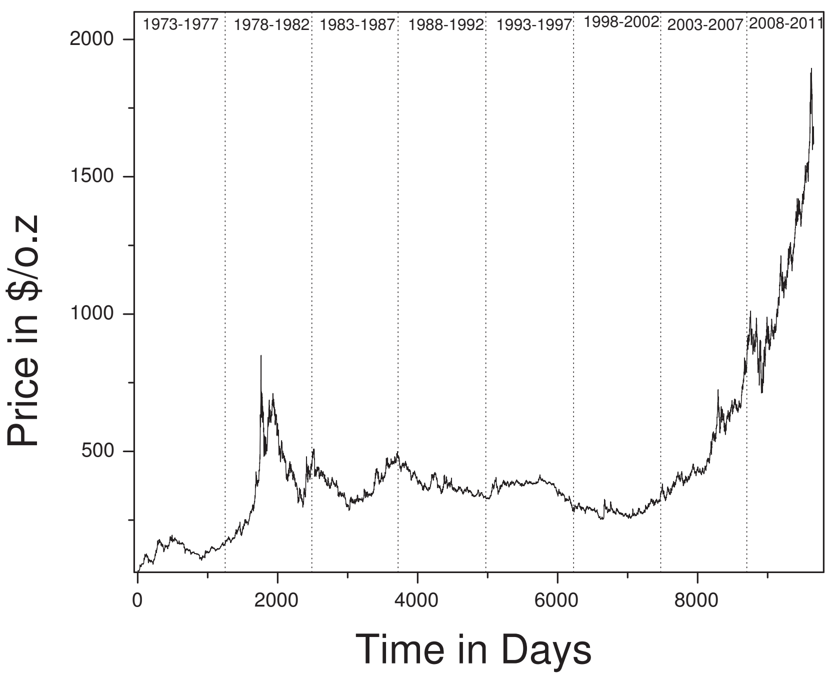


Fig. 1. Plot of values of gold price in \$/oz in the period Jan 1973–Oct 2011.

In economy, multifractality is one of the well known stylized facts which characterize non-trivial properties of financial time series. Bolgorian *et al.* [1] have studied the time series of gold price fluctuation. They found the time series to exhibit multifractal properties. The origin of multifractal properties was predominantly due to temporal correlations.

Fractal is a geometrical pattern that is iterated at even smaller or larger scales to produce self similar irregular shapes or surfaces that cannot be represented by Euclidian geometry. Relative to science, fractals are essentially geometrical shapes or forms that are represented in natural objects, from a fern leaf or tree, through a spider web or snowflake, to larger phenomena such as clouds, hurricanes or even galaxies in space. One amazing characteristic of fractals is that they are infinite. Fractals extend to infeasible large values of their coordinates, outwards in all direction from the centre. Fractals also have infinite details, in that one can zoom in or out without limit to show ever increasing detail within the image. Another distinguishing feature of fractals is its property of self-similarity, an arbitrary region of a fractal looks very similar, but not necessarily identical, to the entire fractal. Just as DNA stores all the information for each of us, all the information for the fractal is contained in its parent image. Fractal geometry mathematically characterizes systems that are basically irregular at all scales. The word 'fractal' was first introduced by Mandelbrot [2] from the Latin adjective fractus (the corresponding Latin verb frangere means to create irregular fragments). Fractals can be classified into two categories: monofractals and multifractals. Multifractals are a type of fractal but they stand in contrast to the monofractals in that multifractals scale with multiple scaling rules. Multifractals are more complicated self-similar objects that consist of differently weighted fractals with different non-integer dimensions. Thus the fundamental characteristic of multifractality is the scaling properties may be different in different regions of the systems. Monofractals are those whose scaling properties are the same in different regions of the system.

DFA is an algorithm based on the statistical theory of random walk. According to the theory, a walker from an initial point in space and making one step at a time towards any direction will cover a distance depending on time and the correlation between the individual steps. If the direction of each step is decided by a random process, for instance the throw of a dice, the walker will cover a distance of $D(s) = cs^{0.5}$, where $D =$ distance, $c$ a proportionality constant, $s$ the number of steps representing time and 0.5 an exponent corresponding to the random correlations in the direction of steps.

The detrended fluctuation analysis (DFA) methodology has been applied to diverse fields, *e.g.* Geology, DNA sequences, neuron spiking, and heart rate dynamics economic time series, and also to weather related and earthquake signals [3,4,5,6,7,8,9,10,11]. It is observed that many geophysical signals as well as medical patterns do not represent simple monofractal behaviour which can be accounted for by a single scaling exponent [12,13]. Different scaling exponents are required for different parts of the series [14]. So in general, a multifractal analysis of the data should be performed. The multifractal detrended fluctuation analysis (MFDFA) was first conceived by Kantelhardt *et al.* [15]. MFDFA is capable of determining multifractal scaling behaviour of non-stationary time series. It has been applied successfully to study various non-stationary time series [16,17,18,19,20,21,22,23]. MFDFA is a standard methodology in econophysics [24,25,26,27,28,29,30,31,32].

In this paper, we have adopted MFDFA methodology to study the variation of degree of multifractality with time. The gold price data from 1973 to the present time has been divided into sets of five years each and MFDFA is applied to each. An attempt has been made to predict future variations in gold price from the results obtained. The results are very interesting and discussed in detail.

## 2. Method of analysis

We have performed a multifractal analysis of the non-stationary time series of fluctuation of gold price following the prescription of Kantelhardt *et al.* [15].

Let us suppose $x(i)$ for $i = 1, 2, \ldots, N$, be a non-stationary time series of length $N$. The mean of the above series is given by

$$x_{\text{avg}} = \frac{1}{N}\sum_{i=1}^{N} x(i)\,. \tag{1}$$

If $x(i)$ is assumed to be the increments of random walk process around the average, the trajectory can be obtained by integration of the signal

$$Y(i) = \sum_{k=1}^{i}[x(k) - x_{\text{avg}}] \tag{2}$$

for $i = 1, 2, \ldots, N$.

The integration also reduces the level of measurement noise present in experimental records and finite data. Then we have divided the integrated time series to $N_s$ non-overlapping bins, where $N_s = \text{int}(N/s)$, and $s$ is the length of the bin. Now, since $N$ is not a multiple of $s$, a short part of the series is left at the end. So in order to include this part of the series, we have repeated the entire process starting from the opposite end thus leaving a short part at the beginning. Thus we have obtained $2N_s$ bins. For each bin we perform least square fit of the series and then determine the variance

$$F^2(s,\nu) = \frac{1}{s}\sum_{i=1}^{s}[Y[(\nu-1)s+i] - y_\nu(i)]^2$$

for each bin $\nu$, $\nu = 1, 2, 3, \ldots N_s$ and

$$F^2(s,\nu) = \frac{1}{s}\sum_{i=1}^{s}[Y[N-(\nu-N_s)s+i] - y_\nu(i)]^2$$

for $\nu = N_s+1,\ N_s+2,\ N_s+3\ldots 2N_s$, where $y_\nu(i)$ is the least square fitted value in the bin $\nu$ . We have performed a least square linear fit (MF-DFA 1). The study can also be extended to higher orders by fitting quadratic, cubic or higher order polynomials. A comparison of results for different orders gives an indication of the type of polynomial trend in given time series.

The $q$th order fluctuation function $F_q(s)$ is obtained after averaging over $2N_s$ bins

$$F_q(s) = \left[\frac{1}{2N_s}\sum_{\nu=1}^{2N_s}\left\{F^2(s,\nu)\right\}^{\frac{q}{2}}\right]^{\frac{1}{q}}, \tag{3}$$

where $q$ is an index which can take all possible values except zero because in that case the factor $1/q$ blows up. The procedure can be repeated by varying the value of $s$. $F_q(s)$ increases with the increase in value of $s$. If the series is long range power correlated, then $F_q(s)$ will show power law behaviour

$$F_q(s) \propto s^{h(q)}\,.$$

If such a scaling exists, $\ln F_q$ will depend linearly on $\ln s$, with $h(q)$ as the slope. In general, the exponent $h(q)$ depends on $q$. For stationary time series $h(2)$ is identical with the Hurst exponent $H$. $h(q)$ is said to be the generalized Hurst exponent. We cannot obtain the value of $h(0)$ directly because $F_q$ blows up at $q=0$. $F_q$ cannot be obtained by the normal averaging procedure; instead a logarithmic averaging procedure is applied

$$F_0(s) \equiv \exp\left[\frac{1}{4N_s}\sum_{\nu=1}^{2N_s}\ln\left\{F^2(s,\nu)\right\}\right] \sim s^{h(0)}\,. \tag{4}$$

A monofractal time series is characterized by unique $h(q)$ for all values of $q$. If the small and large fluctuations scale differently, then $h(q)$ will depend on $q$ or, in other words, the time series is multifractal. Kantelhardt *et al.* [15] have explained that the values of $h(q)$ for $q < 0$ will be larger than that for $q > 0$.

The generalized Hurst exponent $h(q)$ of MF-DFA is related to the classical scaling exponent $\tau(q)$ by the relation

$$\tau(q) = qh(q) - 1\,. \tag{5}$$

A monofractal series, with long range correlation, is characterized by linearly dependent $q$ order exponent $\tau(q)$ with a single Hurst exponent $H$. Multifractal signal have multiple Hurst exponent and $\tau(q)$ depends non-linearly on $q$ [33]. The singularity spectrum $f(\alpha)$ is related to $\tau(q)$ by a Legendre transform [34]

$$\alpha = \frac{d\tau}{dq}$$

and

$$f(\alpha) = q\alpha - \tau(q)\,,$$

where $\alpha$ is the singularity strength or Hölder exponent and $f(\alpha)$ specifies the dimension of subset series that is characterized by $\alpha$. Using Eq. (5) we can write $\alpha$ and $f(\alpha)$ in terms of $h(q)$

$$\alpha \;=\; h(q) + qh'(q)\,, \tag{6}$$

$$f(\alpha) \;=\; q[\alpha - h(q)] + 1\,. \tag{7}$$

The singularity spectrum, in general, quantifies the long range correlation property of a time series [35]. The multifractal spectrum is capable of providing information about relative importance of various fractal exponents in the series *e.g.* the width of the spectrum denotes range of exponents. We can make a quantitative characterization of the spectra by least square, fitting it to a quadratic function [36] around the position of maximum $\alpha_0$

$$f(\alpha) = A(\alpha - \alpha_0)^2 + B(\alpha - \alpha_0) + C\,, \tag{8}$$

where $C$ is an additive constant $C = f(\alpha_0) = 1$ and $B$ indicates the asymmetry of the spectrum. It is zero for a symmetric spectrum. The width of the spectrum can be obtained by extrapolating the fitted curve to zero. Width $W$ is defined as $W = \alpha_1 - \alpha_2$ with $f(\alpha_1) = f(\alpha_2) = 0$. It has been proposed by some workers [37] that the width of the multifractal spectra is a measure of degree of multifractality. Singularity strength or Hölder exponent $\alpha$ and the dimension of subset series $f(\alpha)$ can be obtained from relation (6) and (7). For a monofractal series, $h(q)$ is independent of $q$. Hence from relation (6) and (7) it is evident that there will be a unique value of $\alpha$ and $f(\alpha)$, the value of $\alpha$ being the generalized Hurst exponent $H$ and the value of $f(\alpha)$ being 1. Hence the width of the spectrum will be zero for a monofractal series. The more the width, the more multifractal is the spectrum.

One can also ascertain the origin of multifractality. Two different types of multifractality may be present in a time series:

*(i)* due to broad probability density function for the values of time series,

*(ii)* due to different long range correlation for small and large fluctuation.

The easiest way to distinguish between the two is to analyze the corresponding randomly shuffled series. In the shuffling procedure, the values are put into random order and hence all correlations are destroyed. Hence, if the multifractality is due to long range correlation, then, the shuffled series will exhibit non-multifractal scaling. On the other hand, if the multifractality is due to broad probability density, then, the original $h(q)$ dependence is not changed, $h(q) = h_{\text{shuf}}(q)$. However, if both kinds of multifractality are present in a given series, the shuffled series will show weaker multifractality than the original one. The autocorrelation exponent $\gamma$ can be estimated from the relation given below [12, 38]

$$\gamma = 2 - 2h(q = 2)\,. \tag{9}$$

For uncorrelated or short range correlated data, $h(2)$ is expected to have a value 0.5 while a value $> 0.5$ is expected for long range correlation. Therefore for uncorrelated data, $\gamma$ has a value 1 and the lower the value of more correlated is the data.

## 3. Results and discussions

The non-stationary times series of gold price for the period January 1973–October 2011 are analyzed following the method described above. As mentioned above, Bolgorian *et al.* [1] have studied the time series of gold price fluctuation and found the time series to exhibit multifractal properties originating primarily from temporal correlations. In this paper, we have divided the time series into subsets: *(i)* Jan 1973–Dec 1977, *(ii)* Jan 1978–Dec 1982, *(iii)* Jan 1983–Dec1987, *(iv)* Jan 1988–Dec 1992, *(v)* Jan 1993–Dec 1997, *(vi)* Jan 1998–Dec 2002, *(vi)* Jan 2003–Dec 2007, and finally *(viii)* Jan 2008–Oct 2011.

Multifractal analysis was employed for each subset. The data was transformed to obtain the integrated signal. This process is effective in reducing noise in the data. The integrated time series was divided to $N_s$ bins, where $N_s = \text{int}(N/s)$, $N$ is the length of the series. The $q$th order fluctuation function $F_q(s)$ for $q = -10$ to $+10$ in steps of 1 was determined. Linear dependence of $\ln F_q$ on $\ln s$ was observed suggesting scaling behaviour. The slope of linear fit to $\ln F_q(s)$ *versus* $\ln s$ plots gives the values of $h(q)$. The values of $\tau(q)$ were also determined. As we have mentioned earlier, nonlinear dependence of on $\tau(q)$ on $q$ suggests multifractality, whereas for a monofractal series $\tau(q)$ depends linearly on $q$. The values of $h(q)$ and $\tau(q)$ are depicted in Fig. 2 (left) and Fig. 2 (right) respectively.

The nonlinear dependence of $\tau(q)$ on $q$ and the dependence of $h(q)$ on $q$ give evidence for the multifractality of the gold price in the above mentioned periods. Fig. 2 (left) also depicts that the degree of dependence of $h(q)$ on $q$, or in other words, the degree of multifractality is different in different cases. We can also make a quantitative determination of the degree of multifractality from the multifractal spectrum. Ashkenazy *et al.* [36] have associated the width of the multifractal spectrum ($f(\alpha)$ *versus* $\alpha$) with the degree of multifractality. Fig. 3 shows the multifractal spectrum for different sets. The values of A, B, C and the width of the spectrum $W$ obtained by fitting the multifractal spectrums to Eq. (8) are listed in Table I. Values of autocorrelation $\gamma$ are also presented in Table I.

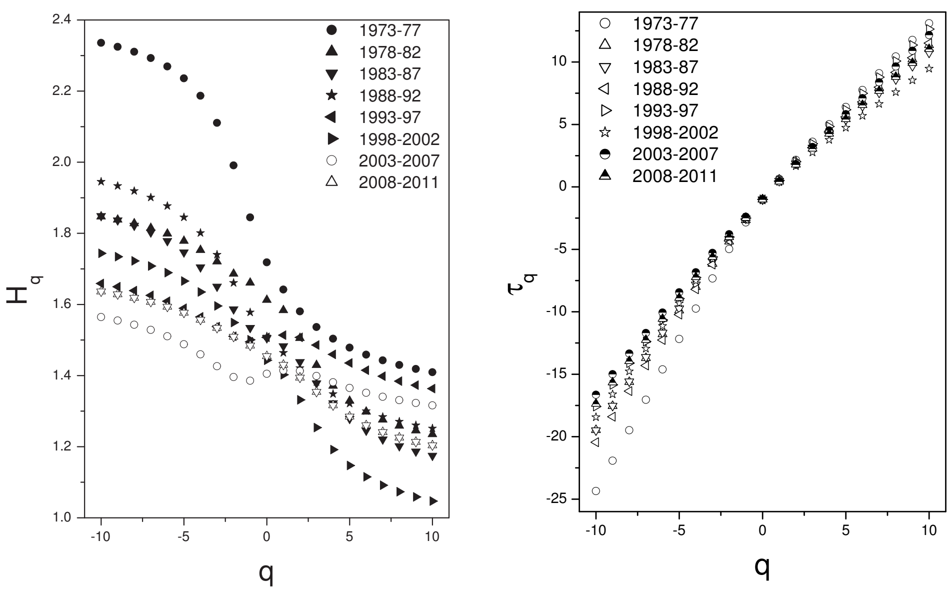


Fig. 2. Left: Plot of $h(q)$ *versus* $q$. Right: Plot of $\tau(q)$ *versus* $q$.

To ascertain the origin of multifractality, the corresponding randomly shuffled series was analyzed for each set. Figs. 4 and 5 represent the values of $h(q)$ *versus* $q$, $\tau(q)$ *versus* $q$ and $f(\alpha)$ *versus* $\alpha$, respectively for the original series and the corresponding randomly shuffled series for the period 1983–87. The multifractal width and the values of autocorrelation for the corresponding randomly shuffled series are also depicted in Table I. It is observed that all the values of $\gamma_{\text{shuff}}$ are quite close to 1 as expected since the correlations

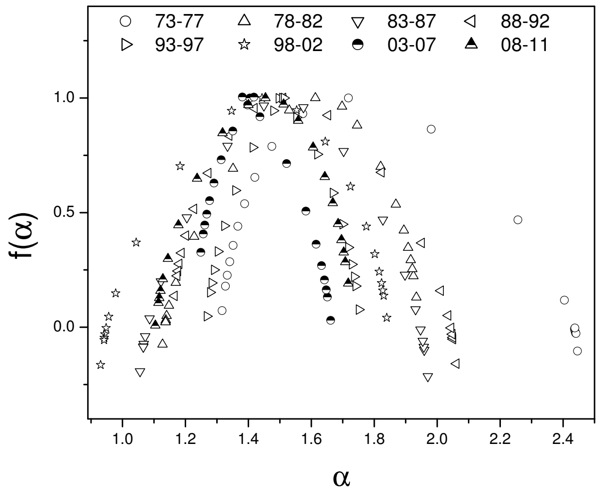


Fig. 3. Multifractal spectrum $f(\alpha)$ *versus* $\alpha$.

TABLE I

Values of parameters $A$, $B$, $C$, $W$, $W_{\text{shuff}}$, $\gamma$ and $\gamma_{\text{shuff}}$.

| Set | $A$ | $B$ | $C$ | $W$ | $W_{\text{shuff}}$ | $\gamma$ | $\gamma_{\text{shuff}}$ |
|---|---|---|---|---|---|---|---|
| 1973–1977 | −3.04 ± 0.19 | 0.73 ± 0.08 | 1.02 ± 0.04 | 1.18 ± 0.06 | 0.31 ± 0.02 | −1.16 ± 0.02 | 1.0 ± 0.006 |
| 1978–1982 | −5.65 ± 0.20 | −0.61 ± 0.05 | 1.00 ± 0.02 | 0.85 ± 0.03 | 0.32 ± 0.007 | −1.01 ± 0.02 | 0.94 ± 0.01 |
| 1983–1987 | −5.39 ± 0.08 | 0.07 ± 0.02 | 0.99 ± 0.01 | 0.86 ± 0.01 | 0.13 ± 0.003 | −0.88 ± 0.02 | 1.10 ± 0.008 |
| 1988–1992 | −4.79 ± 0.17 | 0.61 ± 0.05 | 1.01 ± 0.02 | 0.93 ± 0.03 | 0.26 ± 0.001 | −0.84 ± 0.01 | 1.03 ± 0.01 |
| 1993–1997 | −15.06 ± 0.25 | 0.13 ± 0.03 | 0.97 ± 0.01 | 0.508 ± 0.007 | 0.46 ± 0.01 | −1.01 ± 0.01 | 1.08 ± 0.006 |
| 1998–2002 | −4.84 ± 0.14 | −0.30 ± 0.03 | 1.02 ± 0.02 | 0.92 ± 0.02 | 0.26 ± 0.003 | −0.66 ± 0.02 | 1.0 ± 0.006 |
| 2003–2007 | −17.83 ± 0.62 | 0.95 ± 0.09 | 0.96 ± 0.02 | 0.47 ± 0.01 | 0.33 ± 0.004 | −0.83 ± 0.01 | 1.08 ± 0.006 |
| 2008–2011 | −9.1 ± 0.25 | −0.43 ± 0.04 | 1.01 ± 0.01 | 0.67 ± 0.02 | 0.17 ± 0.01 | −0.79 ± 0.01 | 0.88 ± 0.006 |

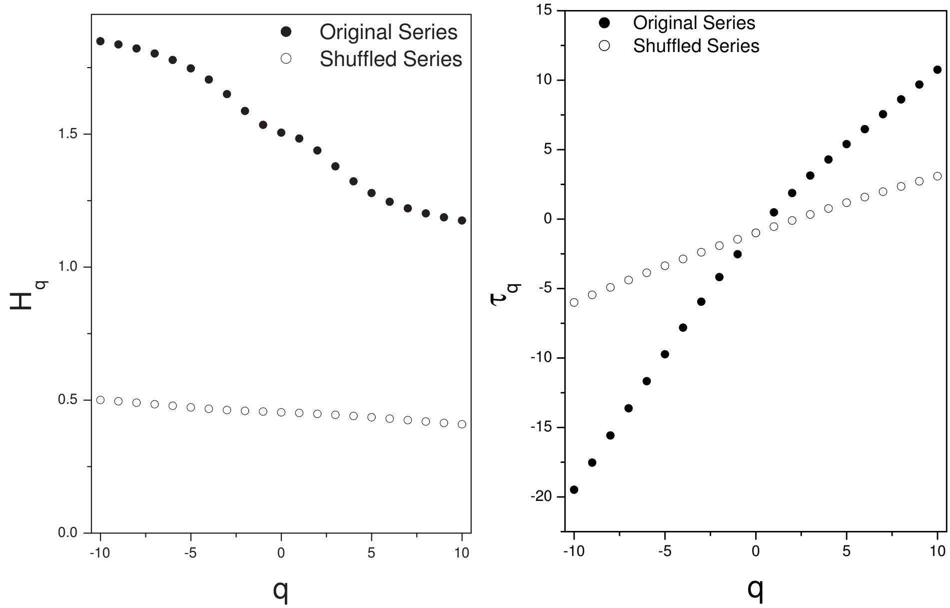


Fig. 4. Left: Plot of $h(q)$ *versus* $q$ for original and shuffled series for the period 1983–87. Right: Plot of $\tau(q)$ *versus* $q$ for original and shuffled series for the period 1983–87.

are destroyed in the shuffling procedure. For the set 1993–97 the values of $W_{\rm shuff}$ are quite high indicating that the origin of multifractality is due to both — broad probability distribution and long range correlation. However, for all the other sets, the shuffling procedure has reduced the degree of multifractality substantially. Thus, for these sets, both types of multifractality are present but the more dominant factor is the long range correlations. In an ideal case, for a sufficiently long series the shuffled series would have monofractal properties with a value of $\alpha$ close to 0.5. Fig. 5 shows that for the shuffled series $f(\alpha)$ *versus* $\alpha$ has a peak at $\alpha_0$ close to 0.5. Ideally, $f(\alpha)$ should be independent of $\alpha$. In this case, the series is comparatively short. Drozdz *et al.* [39] have shown that a relatively short series may reveal traces of multifractality. However, with increase in number of data points the results systematically and steadily approach monofractal behaviour.

We have mentioned before that the lower value of $\gamma$ indicates the higher degree of correlation. Fig. 6 shows the variation of correlation over the years. We have obtained a negative value for the autocorrelation. Drozdz *et al.* [39] have also observed negative values of autocorrelation. It is observed that degree of correlation seems to be less in recent years. It is found to be relatively constant during the period 1983–1992 and 2003–2011. During the period 1983–2011 the autocorrelation coefficient shows a similar pattern of variation as the degree of multifractality depicted in Fig. 7. Thus, for this period of time, with decrease in degree of multifractality the degree of correlation increases and *vice versa*.

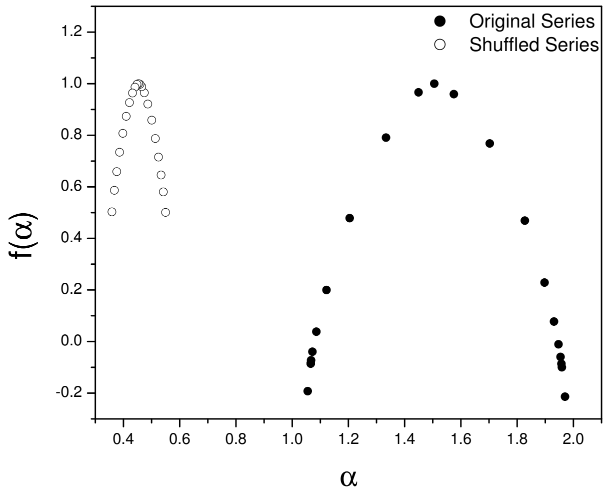


Fig. 5. Multifractal spectrum $f(\alpha)$ *versus* $\alpha$ for original and shuffled series for the period 1983–87.

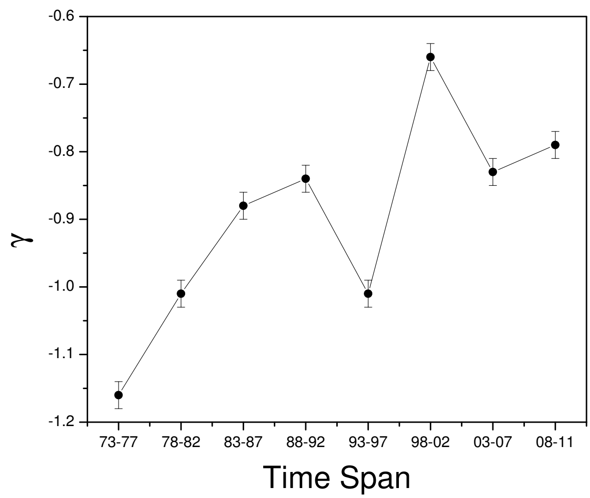


Fig. 6. Variation of autocorrelation $\gamma$ with time.

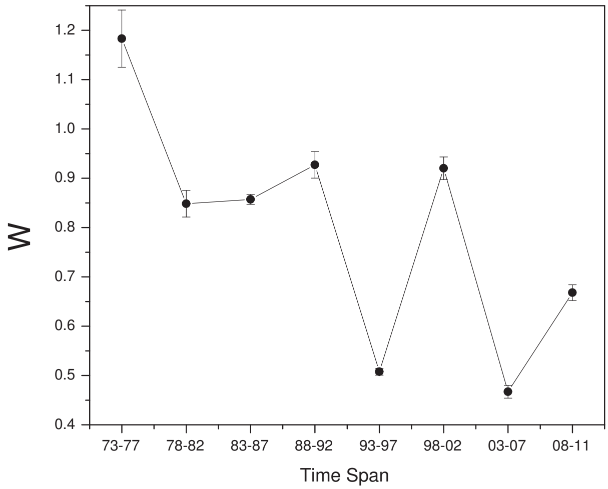


Fig. 7. Variation of multifractal width $W$ with time.

Telesca *et al.* [17] performed a multifractal detrended fluctuation analysis of geoelectrical time series. They have observed an increase in multifractal behaviour of the signal prior to occurrence of largest earthquake. However, other enhancements were also observed without occurrence of earthquake. Dutta [40] has also observed an increase in degree of multifractality prior to volatile behaviour in the values of SENSEX. The variation of degree of multifractality with time is depicted in Fig. 7. From the figure we can draw conclusions which are consistent with the references stated above. The degree of multifractality had its highest value in years 1973–77. Years 1978–82 experienced a huge fluctuation in gold price. Similarly after a huge increase in multifractal behaviour in 1998–02, gold price hugely increased in 2003–07 and continued to increase in the period 2008–11 in spite of a fall in multifractal width in years 2003–07. Thus the increase in multifractal width may indicate a possibility of huge fluctuation of gold price in future but the reverse is not true.

As for future predictions, the degree of multifractality apparently increased in years 2008–11. At present, since the price has an increasing trend, we may conclude that the price of gold are expected to increase in near future due to correlated behaviour and apparent increase in degree of multifractality.

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