PARTICLE CLUSTERS AND MULTIPLICITY FLUCTUATIONS IN NARROW RAPIDITY BINS

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(Received May 2, 2012)

Effects of resonance, or cluster production on the random (Poissonian) fluctuations of multiplicity in rapidity bins has been analyzed. It is found that for narrow bins (up to some 0.25 in rapidity), and for realistic assumptions on the resonances or clusters, this effect is reasonably small.

DOI:10.5506/APhysPolB.43.1357
PACS numbers: 25.75.Gz, 13.65.+i

1. In our recent paper with Bzdak [1], we considered fluctuations of the particle numbers in well-separated rapidity bins. One reason for such fluctuations are the fluctuations of the initial conditions and/or dynamics of the evolution. *E.g.* in a nucleus–nucleus collision, the number and rapidity distribution of the final state particles depends on the impact parameter of the collision, on the number of wounded nucleons in each nucleus *etc.* These are the dynamic fluctuations which were studied since the early times of high-energy physics [2, 3, 4] and are still of interest [5, 6, 7, 8, 9, 10, 11, 12, 13]. On top of them, however, there are purely statistical, random fluctuations. This is well known to users of Monte Carlo programs, where the initial conditions can be fully specified, but this is not enough to predict the numbers of particles in the rapidity bins of a single event.

In [1], following [14, 15], we have chosen the simplest assumption that these random fluctuations are Poissonian. Under this assumption, it is possible to correct for the random fluctuations and extract the dynamic fluctuations which are much more interesting. The Poissonian Ansatz, however, corresponds to particles produced independently. Thus, it neglects the production of resonances and/or particle clusters (in the following, we use the term clusters for both resonances and clusters). In the present paper, we discuss a simple model, where all the particles are produced from cluster decays. This certainly exaggerates the effect of clusters, but even so we find that the Poissonian Ansatz is an acceptable approximation, provided the bin is sufficiently narrow. Our analysis shows that, at the present precision of the data, bins of width up to about 0.25 rapidity units can be safely used. We also discuss the bin-size dependence of such measurements and show that they can be used to estimate the amount of clustering in particle production.

2. Let us consider a rapidity bin of width Δ and one cluster with at least one particle among its decay products detected in the bin. Denoting by p_m the probability that exactly m particles from the cluster are detected in the bin, we have the generating function

$$\phi(z) = \sum_{m=1}^{\max} p_m z^m \,, \tag{1}$$

where 'max' is the maximum number of the decay products of the cluster which can be registered. Putting $p_1 = 1$, and consequently $p_m = 0$ for all $m > 1$, we would reproduce the results of the model without clusters, considered in [1]. Here we will discuss the implications of the assumption $p_1 < 1$.

Assume now that the distribution of the number of clusters contributing to the population of the bin is a superposition of Poissonians with a distribution of average multiplicities ν given by the weight $W(\nu)$. Then the generating function for the number of particles in the bin is

$$\Phi(z) = \int d\nu \; W(\nu) e^{\nu(\phi(z)-1)} \,. \tag{2}$$

Differentiating over z and putting $z = 1$, we get the moments of the distribution of the total number of particles n in the bin

$$\langle n\rangle = \langle m\rangle\langle\nu\rangle \,; \qquad \langle n(n-1)\rangle = \langle m\rangle^2 \left\langle\nu^2\right\rangle + \langle m(m-1)\rangle\langle\nu\rangle \,, \tag{3}$$

and so on.

As an illustration, consider the special case when only the probabilities p_1 and p_2 are different from zero. Then

$$p_1 + p_2 = 1 \,; \qquad \langle m\rangle = p_1 + 2p_2 = 1 + p_2 \,; \qquad \langle m(m-1)\rangle = 2p_2 \,. \tag{4}$$

Consequently,

$$\langle n\rangle = (1 + p_2)\langle\nu\rangle \,; \qquad \langle n(n-1)\rangle = (1 + p_2)^2 \left\langle\nu^2\right\rangle + 2p_2\langle\nu\rangle \,,$$

and the variance

$$\sigma^2(n) \equiv \langle n^2 \rangle - \langle n \rangle^2 = (1+p_2)^2 \sigma^2(\nu) + \langle \nu \rangle (1+3p_2)\,. \tag{5}$$

The value for $p_2 = 0$ is the result without clusters. In order to see how important are the corrections, it is necessary to estimate p_2.

3. Consider a cluster of mass M decaying into exactly two (detectable) particles of mass μ each. The z axis is along the beam direction and, if the transverse momentum of the cluster $\boldsymbol{P}_{\mathrm{T}} \neq 0$, the x axis is parallel to $\boldsymbol{P}_{\mathrm{T}}$. The spherical angles θ, ϕ are defined in the rest frame of the cluster. We need the probability p_2 that both decay products of a cluster fall into the rapidity bin $[-\frac{1}{2}\Delta, +\frac{1}{2}\Delta]$.

Let us denote the rapidity of the cluster by y and the rapidities of its two decay products by y_f and y_b respectively, with y_f corresponding to the particle with $\cos\theta \geq 0$. For $\cos\theta = 0$ this definition is ambiguous, but since we will be interested only in integrals over $\cos\theta$ this ambiguity is harmless. It is assumed that the decay distribution is spherically symmetric in the rest frame of the cluster and that the rapidity distribution for the clusters is flat. Then the probability of a set of events is proportional to the corresponding volume V in the $\{y, \cos\theta, \phi\}$ space.

Denoting by y_f^0 and y_b^0 the absolute values of the rapidities of the particles in the frame, where the cluster rapidity vanishes, we have

$$y_f = y + y_f^0\,, \qquad y_b = y - y_b^0\,. \tag{6}$$

Therefore, the probabilities that the decay products fall into the bin $[-\frac{1}{2}\Delta, +\frac{1}{2}\Delta]$ are respectively non-zero if and only if

$$-\tfrac{1}{2}\Delta - y_f^0 < y < \tfrac{1}{2}\Delta - y_f^0\,, \qquad -\tfrac{1}{2}\Delta + y_b^0 < y < \tfrac{1}{2}\Delta + y_b^0\,. \tag{7}$$

The range of rapidity for each of the two final particles is of length Δ, independently of the angles θ, ϕ. Defining the unnormalized probability p_{u} as the reduced volume $\frac{V}{2\pi}$, we have for each particle $p_{\mathrm{u}} = \Delta$. For both particles the sum of the reduced volumes is 2Δ but the region, where both particles fall into the bin is counted twice. Thus finally

$$p_{\mathrm{u}1} + 2p_{\mathrm{u}2} = 2\Delta\,. \tag{8}$$

The normalizing factor for the probabilities is

$$p_{\mathrm{u}1} + p_{\mathrm{u}2} = 2\Delta - p_{\mathrm{u}2}\,. \tag{9}$$

Therefore,

$$p_2 = \frac{p_{\mathrm{u}2}}{2\Delta - p_{\mathrm{u}2}}\,. \tag{10}$$

It remains to calculate $p_{\mathrm{u}2}$. Both particles fall into the bin if and only if

$$-\tfrac{1}{2}\Delta + y_b^0 = y_{\min} \le y \le y_{\max} = +\tfrac{1}{2}\Delta - y_f^0 \,. \tag{11}$$

4. We begin with the simplest case when $\boldsymbol{P}_{\mathrm{T}} = 0$. Then $y_f^0 = y_b^0$ and we have

$$y_f^0 + y_b^0 = \ln \frac{1+\alpha_0 \cos\theta}{1-\alpha_0\cos\theta} \,, \tag{12}$$

where

$$\alpha_0 = \sqrt{1 - \frac{4\mu^2}{M^2}} \,. \tag{13}$$

From the condition (11) we deduce that, provided that $y_{\max} > y_{\min}$, the allowed range of y is

$$\Delta y = y_{\max} - y_{\min} = \Delta - \left(y_b^0 + y_f^0\right) = \Delta - \ln \frac{1+\alpha_0\cos\theta}{1-\alpha_0\cos\theta} \,. \tag{14}$$

This range is different from zero only if

$$0 \le \cos\theta \le \cos\theta_m = \frac{1}{\alpha_0} \tanh \frac{\Delta}{2} \,. \tag{15}$$

Consequently,

$$p_{\mathrm{u}2} = \int\limits_0^{\frac{1}{\alpha_0}\tanh\frac{\Delta}{2}} d\cos\theta \left(\Delta - \ln \frac{1+\alpha_0\cos\theta}{1-\alpha_0\cos\theta}\right) = \frac{2}{\alpha_0} \ln\cosh \frac{\Delta}{2} \tag{16}$$

and

$$p_2 = \frac{\ln\cosh\frac{\Delta}{2}}{\alpha_0\Delta - \ln\cosh\frac{\Delta}{2}} = z + z^2 + \left(1 - \frac{8}{3}\alpha_0^2\right) z^3 + \left(1 - \frac{16}{3}\alpha_0^2\right) z^4 + \dots \,, \tag{17}$$

where the convenient parameter

$$z = \frac{\Delta}{8\alpha_0} \tag{18}$$

has been introduced.

5. For $\boldsymbol{P}_{\mathrm{T}} \neq 0$, formula (12) gets replaced by

$$y_f^0 + y_b^0 = \frac{1}{2} \ln \frac{(1+\alpha\cos\theta)^2 - \beta^2\sin^2\theta\cos^2\phi}{(1-\alpha\cos\theta)^2 - \beta^2\sin^2\theta\cos^2\phi} \,, \tag{19}$$

where

$$\alpha = \frac{\alpha_0}{\sqrt{1+\frac{P_\mathrm{T}^2}{M^2}}}\,, \qquad \beta = \alpha\frac{P_\mathrm{T}}{M}\,. \tag{20}$$

One sees that both α and β depend only on the ratio P_T/M (*i.e.* only on the transverse velocity of the cluster).

Formula (11) yields the accessible rapidity interval for given θ and ϕ

$$\Delta y = \Delta - \frac{1}{2}\ln\frac{(1+\alpha\cos\theta)^2 - \beta^2\sin^2\theta\cos^2\phi}{(1-\alpha\cos\theta)^2 - \beta^2\sin^2\theta\cos^2\phi}\,; \qquad \Delta y \geq 0\,. \tag{21}$$

This can be integrated numerically, but for $P_\mathrm{T} \leq 1$ GeV a narrow bin approximation, analogous to (17), deviates from the exact results by less than one percent. One finds

$$p_2(P_\mathrm{T}) = z(P_\mathrm{T}) + z^2(P_\mathrm{T}) + O\left(\Delta^3\right)\,, \tag{22}$$

where

$$z(P_\mathrm{T}) = z\frac{\sqrt{P_\mathrm{T}^2+M^2}}{M}\left(1 - \frac{\alpha_0^2 P_\mathrm{T}^2}{2\left(P_\mathrm{T}^2+M^2\right)}\right)\,, \tag{23}$$

This function increases with increasing P_T, but rather slowly: $z(0) = 0.0335$ and $z(1) = 0.0401$.

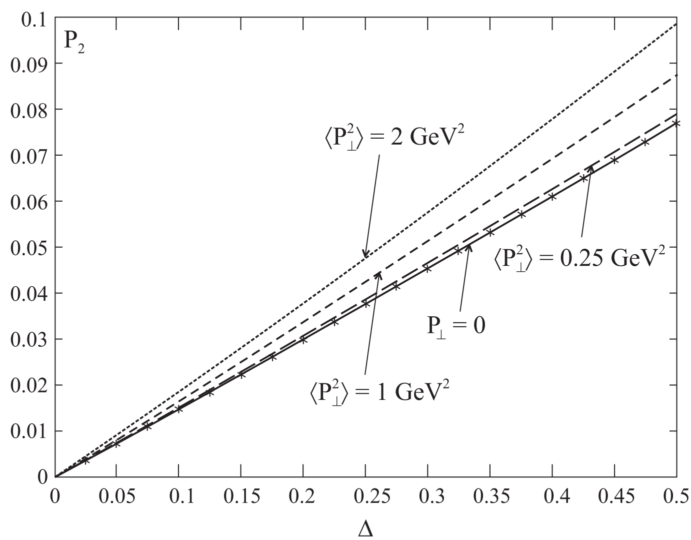

Fig. 1. p_2 *versus* Δ for Gaussian distributions of P_T with $\langle P_\mathrm{T}^2\rangle$ as shown in the figure.

To see what happens if the transverse momentum of the cluster is not fixed, we also calculated numerically the correction assuming the Gaussian distribution of P_{T}. In Fig. 1, the probability p_2 is plotted *versus* the bin width Δ up to $\Delta = 0.5$ for various average transverse momenta of the cluster. One sees that p_2 increases almost linearly with Δ and that it is pretty small in this Δ range.

6. Our analysis can be easily extended to more bins. For B bins the generating function generalizing (2) is

$$\Phi(z_1,\dots,z_B) = \int d\nu_1,\dots,d\nu_B\, W(\nu_1,\dots,\nu_B) \prod_{j=1}^{B} e^{\nu_j(\phi_j(z)-1)}. \tag{24}$$

As an illustrative example, we will estimate the correction from cluster formation to left–right asymmetric fluctuations as measured by the dispersion

$$D_-^2 \equiv \tfrac{1}{4}\left\langle [\nu_1-\nu_2]^2 \right\rangle = \tfrac{1}{2}\left(\langle\nu_1^2\rangle - \langle\nu_1\nu_2\rangle\right) \tag{25}$$

evaluated in [1]. We consider $B=2$ symmetric bins

$$\phi_1(z) = \phi_2(z)\,, \qquad W(\nu_1,\nu_2) = W(\nu_2,\nu_1)\,. \tag{26}$$

The only additional average we need is $\langle\nu_1\nu_2\rangle$. Differentiating the generating function with respect to z_1 and z_2 we find

$$\langle n_1 n_2\rangle = \frac{\partial^2\Phi(z_1,z_2)}{\partial z_1\partial z_2} = \langle\nu_1\nu_2\rangle\langle m\rangle^2\,. \tag{27}$$

Combining the formulae given in the present paper, we find

$$D_-^2 = \frac{1}{\langle m\rangle^2}\left[\tilde{D}_-^2 - \frac{1}{2}\langle m(m-1)\rangle\langle\nu\rangle\right] \approx \tilde{D}_-^2\left[1-2p_2\left(1+\frac{\langle n_1\rangle}{2\tilde{D}_-^2}\right)\right], \tag{28}$$

where $\tilde{D}_-^2$ is the dispersion evaluated without the correction due to cluster production, *i.e.* at $p_2=0$ [1]

$$\tilde{D}_-^2 \equiv \tfrac{1}{2}\left[\langle n_1(n_1-1)\rangle - \langle n_1n_2\rangle\right] = \tfrac{1}{2}\left[D_{ff}^2 - D_{bf}^2 - \langle n_1\rangle\right] \tag{29}$$

and

$$D_{ff}^2 = \langle n_1^2\rangle - \langle n_1\rangle^2 = \sigma^2(n_1)\,, \qquad D_{bf}^2 = \langle n_1n_2\rangle - \langle n_1\rangle^2\,. \tag{30}$$

These two parameters and $\langle n_1\rangle$ can be evaluated from the data of the STAR Collaboration for AuAu collisions at $\sqrt{s} = 200$ GeV [11, 12]. For the most central collisions and bins $0.8 < |y| < 1.0$ one finds [1]

$$D_{ff}^2 = 350\pm17\,, \qquad D_{bf}^2 = 202\pm17\,, \qquad \langle n_1\rangle = 96\pm5\,. \tag{31}$$

This yields

$$D_-^2 = \tfrac{1}{2}(\langle n_1(n_1-1)\rangle - \langle n_1 n_2\rangle)(1-3.7p_2)\,. \tag{32}$$

For $p_2 = 0.03$ this is an eleven percent correction, which is much smaller than the experimental uncertainty, which is about 46%. Note that the assumption that all the particles falling into the bins are decay products of clusters certainly overestimates the correction.

We conclude that at the present accuracy of the data and bin width in rapidity of some 0.25 or less, the corrections due to cluster formation are unimportant for D_-^2. With increasing accuracy of the data, it may be necessary to include this correction, however. We show that it is negative and can be roughly estimated.

7. Eq. (28) implies a specific dependence of the measured D_-^2 on the bin width $\varDelta$. We first observe that, since one expects $\langle\nu\rangle \sim \varDelta$, $\tilde{D}_-^2 \sim \langle\nu\rangle^2$ and, as seen from Fig. 1, $p_2 \sim \varDelta$, the ratio

$$-\frac{1}{2}\frac{\langle m(m-1)\rangle}{\tilde{D}_-^2}\langle\nu\rangle \approx -\frac{p_2}{\tilde{D}_-^2}\langle\nu\rangle \tag{33}$$

representing the relative correction, is independent of $\varDelta$. It follows that this correction persists even in the limit of very small $\varDelta$.

Furthermore, we have

$$\frac{\tilde{D}_-^2}{\langle n_1\rangle^2} \sim \langle m\rangle^{-2} \sim (1+p_2)^{-2} \sim (1+\omega\varDelta)^{-2}\,, \tag{34}$$

where ω is a constant. As seen in Fig. 1, ω is a small number (of the order of 0.15), therefore, the result may be difficult to observe for small $\varDelta$, where our approximations are expected to be valid. With increasing precision of data, however, it will be interesting to look for this signal, as it provides a possibility to estimate the clustering effects in particle production.

8. In conclusion, we have estimated the effect of clustering in particle production for the random fluctuations of particle numbers in small, well separated rapidity bins. The effect is small but non-negligible and with increasing accuracy of data it may turn out important. It implies a characteristic dependence of the fluctuations on the bin size which may be used as a measure of the amount of clustering and thus seems to be interesting to look for.

We thank Adam Bzdak for the fruitful collaboration on the subject of multiplicity fluctuations and for interesting discussions on the issues related to the present investigation. This work was supported in part by the grant N N202 123437 of the Polish Ministry of Science and Higher Education (2009–2012).

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