# BSM MODELS FACING THE RECENT LHCb DATA: A FIRST LOOK\*

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During the last decade a number of detailed analyses of flavour observables and of their correlations within more than a dozen specific BSM models have been performed at the TUM. One of the goals of these analyses was to investigate which model is capable of obtaining large mixing induced CP asymmetry in the  $B_s$  system,  $S_{\psi\phi}$ , and to find out what this would imply for other flavour observables. In this context, also the rare decays  $B_{s,d} \to \mu^+ \mu^-$  have been considered. In some models, their branching ratios can be enhanced by orders of magnitude above the SM expectations. The recent data on  $S_{\psi\phi}$  and  $B_{s,d} \to \mu^+\mu^-$  from the LHCb put an end to these very optimistic hopes modifying significantly the allowed patterns of deviations from SM predictions for flavour observables in concrete BSM models. We make a first semi-quantitative assessment of the most important modifications in the predictions of the BSM models in question including also recently analyzed models and taking into account the most recent lattice input. Our presentation is dominated by quark flavour observables in  $B_{s,d}$  and  $K^+(K^0)$  meson systems. For some BSM models the LHCb data turned out to be a relief. On the other hand, the SM models with CMFV and MFV models without flavour blind phases appear to have significant difficulties in describing all  $\Delta F = 2$  observables in  $B_{s,d}$  and  $K^0$ meson systems simultaneously. However, definite conclusions will only be possible when  $|V_{ub}|$  and  $\gamma$  will be known from tree level decays with a much better precision and the lattice input will further improve. Finally, we propose to regard the stringent CMFV relations between various observables as standard candles of flavour physics. The pattern of deviations from these relations may help in identifying the correct NP scenario.

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## 1. Introduction

This decade should provide a much better understanding of the physics at the shortest distance scales explored by humans, that is scales of the order of  $5 \times 10^{-20}$ m explored by ATLAS and CMS and possibly even shorter distance scales explored by dedicated flavour physics experiments like LHCb, SuperKEKB, SuperB in Rome and kaon physics dedicated experiments like NA62, K<sup>0</sup>TO and ORKA. The main goal of these experiments is the search for New Physics (NP).

The most efficient way to uncover NP in weak decay processes is to identify correlations between flavour observables characteristic for a given extension of the SM. Such correlations being less sensitive to the model parameters can often allow a transparent distinction between various models proposed in the literature [1]. Also the so-called "DNA-Test" of a given model can give a global insight into the particular pattern of possible deviations from SM predictions for flavour observables [2]. Such studies are at the frontiers in our search for a fundamental theory of elementary particles in which flavour violating interactions will play undoubtedly a prominent role [3,4,5,6,7,8,9,10].

Now until recently, the number of flavour observables used efficiently to test the short distance structure of the SM and of various BSM scenarios was limited to tree level decays, particle—antiparticle mixing including CP-violating observables like  $\varepsilon_K$  and  $S_{\psi K_S}$  and a handful of  $\Delta F = 1$  loop induced processes like  $B \to X_s \gamma$  and  $B \to X_s \ell^+ \ell^-$ . This allowed in many models still significant deviations from the SM expectations summarized in [1]. In particular, a number of models having a multitude of free parameters were capable of obtaining large mixing induced CP asymmetry in the  $B_s$  system,  $S_{\psi\phi}$ , signalled initially by CDF and D0. This, in turn, implied often significant NP effects in some flavour observables and/or precluded in certain models large NP effects in other observables, in particular, in rare kaon decays like  $K^+ \to \pi^+ \nu \bar{\nu}$  and  $K_L \to \pi^0 \nu \bar{\nu}$ . In these analyses, the rare decays  $B_{s,d} \to \mu^+ \mu^-$  have also played a prominent role. In fact, in models with new heavy neutral scalars, like MSSM and 2HDMs of various type their branching ratios, could in the spring of 2011 still be enhanced by one order of magnitude above the SM expectations while satisfying all existing data.

While already the messages from Tevatron last summer indicated that NP effects are probably smaller in  $B_s$  flavour physics than initially expected and hoped for, the very recent data on  $S_{\psi\phi}$  and  $B_{s,d} \to \mu^+\mu^-$  from the LHCb put an end to these very optimistic hopes modifying significantly the allowed patterns of deviations from SM predictions for flavour observables in concrete BSM models.

Indeed, the most recent data on  $S_{\psi\phi}$  and  $B_{s,d} \to \mu^+\mu^-$  decays from LHCb read [11]

$$S_{\psi\phi} = 0.002 \pm 0.087, \qquad S_{\psi\phi}^{\text{SM}} = 0.035 \pm 0.002,$$
 (1)

$$\mathcal{B}\left(B_s \to \mu^+ \mu^-\right) \le 4.5 \times 10^{-9} , \quad \mathcal{B}\left(B_s \to \mu^+ \mu^-\right)^{\text{SM}} = (3.1 \pm 0.2) \times 10^{-9} , (2)$$

$$\mathcal{B}\left(B_d \to \mu^+ \mu^-\right) \le 8.1 \times 10^{-10}, \quad \mathcal{B}\left(B_d \to \mu^+ \mu^-\right)^{\text{SM}} = (1.0 \pm 0.1) \times 10^{-10}, (3)$$

where we have also shown updated SM predictions for these observables as discussed in Sec. 3. They differ only marginally from those quoted in [1] and given in (22) and (23). Our phase sign convention is such that in the SM  $S_{\psi\phi}$  is positive. See also (14). The experimental error on  $S_{\psi\phi}$  has been obtained by adding statistical and systematic errors in quadrature and the upper bounds on  $\mathcal{B}(B_{s,d} \to \mu^+\mu^-)$  are at 95% C.L.

Indeed, it looks like the SM still survived another test: mixing induced CP violation in  $B_s$  decays is significantly smaller than in  $B_d$  decays as expected in the SM already for 25 years. However, from the present perspective  $S_{\psi\phi}$  could still be found in the range

$$-0.20 \le S_{\psi\phi} \le 0.20$$
 (4)

and finding it to be negative would be not only a clear signal of NP but would rule out a number of models as we will see below. Moreover, finding it above 0.1 would also be a signal of NP but not as pronounced as a negative value.

Concerning  $B_s \to \mu^+ \mu^-$ , as pointed out recently in [12, 13], when comparing the theoretical branching ratio in (2) with experiment, a correction factor has to be included which takes care of  $\Delta \Gamma_s$  effects that influence the extraction of this branching ratio from the data

$$\mathcal{B}\left(B_s \to \mu^+ \mu^-\right)_{\text{th}} = r(\Delta \Gamma_s) \mathcal{B}\left(B_s \to \mu^+ \mu^-\right)_{\text{exp}}, \qquad r(0) = 1.$$
 (5)

The authors of [12, 13] find  $r(\Delta \Gamma_s) = 0.91 \pm 0.01$ . It is a matter of choice whether this factor should be included in the theoretical calculation or in the experimental branching ratio. We prefer to include it in the latter so that the experimental upper bound in (2) is reduced by 9% implying a more stringent upper bound of  $4.1 \times 10^{-9}$ . Thus finally, the SM central value is only by a factor of 1.3 below the experimental 95% C.L. upper bound.

Unfortunately, this bound on  $\mathcal{B}(B_s \to \mu^+ \mu^-)$  precludes a simple distinction between NP contributions coming from neutral scalars and neutral gauge bosons which would be possible if its value was larger than  $6 \times 10^{-9}$ .

Indeed, such an enhancement could most easily be attributed to heavy neutral scalar exchanges [14]. On the other hand, we should note that the upper bound on  $\mathcal{B}(B_d \to \mu^+\mu^-)$  is still one order of magnitude above its SM value and due to the smallness of  $\Delta \Gamma_d$  is not affected by the correction discussed above. It could turn out, after all, that it is  $B_d \to \mu^+\mu^-$  and not  $B_s \to \mu^+\mu^-$  that will most clearly signal NP in these decays. We will return to this point below.

However, the recent LHCb data listed above are not the only flavour highlights of the last half a year that are relevant for our presentation. In particular, improved values on the five non-perturbative parameters [15]<sup>1</sup>

$$\hat{B}_K = 0.767(10)$$
,  $F_{B_d} = (190.6 \pm 4.6) \text{ MeV}$ ,  $F_{B_s} = (227.7 \pm 6.2) \text{ MeV}$ , (6)

and

$$\xi = \frac{F_{B_s} \sqrt{\hat{B}_{B_s}}}{F_{B_d} \sqrt{\hat{B}_{B_d}}} = 1.237 \pm 0.032, \qquad F_{B_s} \sqrt{\hat{B}_{B_s}} = 279(13) \,\text{MeV}$$
 (7)

allowed for improved SM predictions for  $\varepsilon_K$ ,  $\Delta M_{s,d}$  and  $\mathcal{B}(B_{s,d} \to \mu^+ \mu^-)$ . The values quoted above are taken from a recent update of lattice averages in [15] that are based on a number of lattice calculations for which the references can be found in this paper. In particular, for  $B_{s,d} - \bar{B}_{s,d}$  physics one should refer to an impressive precision reached in [16,17,18,19] and in the case of  $K^0 - \bar{K}^0$  to [20]<sup>2</sup>.

In (7), following the recommendations of lattice experts, we use  $\xi$  and  $F_{B_s}\sqrt{\hat{B}_{B_s}}$  as basic lattice input which gives

$$F_{B_d} \sqrt{\hat{B}_{B_d}} = 226(13) \,\text{MeV} \,,$$
 (8)

that agrees well with the direct average in [15]  $F_{B_d}\sqrt{\hat{B}_{B_d}}=227(17)\,\mathrm{MeV}.$ 

The goal of this writing is to summarize first the possible deviations from SM predictions in flavour observables and investigate which models can remove these anomalies while being consistent with all available data, in particular with the LHCb data quoted above. However, as already advertised in the title, our main goal is to confront the models reviewed in [1] and few other models studied since then, with the LHCb data above and to investigate how the most important predictions of these models are modified

<sup>&</sup>lt;sup>1</sup> We thank Christine Davies for help in getting a better insight into these results.

<sup>&</sup>lt;sup>2</sup> The first author of this review expects on the basis of [21, 22] that the value of  $\hat{B}_K$  will eventually be below 0.75 or equal to it.

by these new messages from the nature. A recent analysis of the implications of the LHCb data on NP in  $B_{d,s}^0 - \bar{B}_{d,s}^0$  mixings has been presented from a different perspective in [23].

In view of space limitations, we have to develop a strategy for presenting our observations and results. First, as far as CP-violating observables are concerned, the main stars of our presentation will be

$$\varepsilon_K, \qquad S_{\psi K_{\rm S}}, \qquad S_{\psi \phi}, \qquad \mathcal{B}\left(K_L \to \pi^0 \nu \bar{\nu}\right).$$
 (9)

Among CP-conserving ones we will pay particular attention to

$$\Delta M_{s,d}$$
,  $\mathcal{B}\left(B_{s,d} \to \mu^+ \mu^-\right)$ ,  $\mathcal{B}\left(B^+ \to \tau^+ \nu_{\tau}\right)$ ,  $\mathcal{B}\left(K^+ \to \pi^+ \nu \bar{\nu}\right)$ . (10)

This means that several important decays like  $B \to K^* \ell^+ \ell^-$  and all  $b \to s \nu \bar{\nu}$  transitions will barely appear on the scene. We hope to improve on this in the future. For the time being, we refer to [14] and references therein. Similarly, we will not discuss CP violation in charm decays as in view of recent LHCb data charm experts have already presented several views in the literature and we have nothing to add here at present. Concerning lepton flavour violation, it will only show up in GUT models.

Our discussion will be at best semi-quantitative as a full-fledged analysis would require redoing all numerical analyses reviewed in [1] putting new constraints on the masses of new particles in these models that are being obtained from the LHC. This is clearly beyond the scope of this rather short review. Moreover, while the lower bounds on the masses of new particles increased during last year, in evaluating branching ratios this increase can be approximately compensated by the increase of the relevant couplings and mixing parameters so that the picture obtained through our rough analysis should be roughly correct. The reason why we can at all make any statements about the changes in predictions of various models without basically performing any new numerical analysis is that the strategy of the analyses discussed in [1] was to present the predictions for various observables as functions of  $S_{\psi\phi}$ . Therefore, simply inspecting the plots from different analyses in the corresponding papers one can get a rough idea of what is going on after new LHCb results have been taken into account. Other correlations presented there, involving this time the branching ratios  $\mathcal{B}(B_{s,d} \to \mu^+ \mu^-)$ , turned out to be very helpful in this respect as well.

Finally, our strategy will not be to repeat any details of the models considered here, because in [1] a rather compact presentation of most of them can be found. Interested readers are asked to read in parallel [1] and related original papers.

Our review has a very simple structure. In Sec. 2 we summarize briefly the present anomalies in the flavour data as seen from the point of view of the SM. In Sec. 3, the main section of this writing, we will have a first look at the modifications of the results of all models considered in [1], adding few models that have been analysed by us since then. A brief outlook consisting of observations, messages and a shopping list for coming years in Sec. 4 ends our review.

#### 2. Anomalies in the flavour data

Let us summarize the pattern of deviations from the SM expectations presently observed in the data. In this context, it should be emphasized that because of the  $\varepsilon_K - S_{\psi K_{\rm S}}$  tension [24,25,26,27,28,29,30] within the SM this pattern depends on whether  $\varepsilon_K$  or  $S_{\psi K_{\rm S}}$  is used as a basic observable to fit the CKM parameters. As both observables can receive important contributions from NP, none of them is optimal for this goal. The solution to this problem will be solved one day by precise measurements of the CKM parameters with the help of tree-level decays. Unfortunately, the tension between the inclusive and exclusive determinations of  $|V_{ub}|$  and the poor knowledge of the phase  $\gamma$  from tree-level decays preclude this solution at present. In view of this, it is useful to set  $\gamma \approx 70^{\circ}$  and consider two scenarios for  $|V_{ub}|$ :

- Exclusive (small)  $|V_{ub}|$  Scenario 1:  $|\varepsilon_K|$  is smaller than its experimental determination, while  $S_{\psi K_{\rm S}}$  is very close to the central experimental value.
- Inclusive (large)  $|V_{ub}|$  Scenario 2:  $|\varepsilon_K|$  is consistent with its experimental determination, while  $S_{\psi K_{\rm S}}$  is significantly higher than its experimental value.

Thus, dependently which scenario is considered, we need either constructive NP contributions to  $|\varepsilon_K|$  (Scenario 1) or destructive NP contributions to  $S_{\psi K_S}$  (Scenario 2). However, this NP should not spoil the agreement with the data for  $S_{\psi K_S}$  (Scenario 1) and for  $|\varepsilon_K|$  (Scenario 2).

In view of the fact that the theoretical precision on  $S_{\psi K_{\rm S}}$  is significantly larger than in the case of  $\varepsilon_K$ , one may wonder whether removing  $1-2\sigma$  anomaly in  $\varepsilon_K$  by generating a  $2-3\sigma$  anomaly in  $S_{\psi K_{\rm S}}$  is a reasonable strategy. However, one should take into account that in addition to the fact that large values of  $|V_{ub}|$  are found in inclusive B-decays there is still another tension within the SM that similarly to  $\varepsilon_K$  favours large  $|V_{ub}|$  scenario:

• The SM branching ratio for  $B^+ \to \tau^+ \nu_{\tau}$  in Scenario 1 is by a factor of two below the data, although the latter are not very precise and one

can talk only about a  $2.5\sigma$  discrepancy. In Scenario 2, the discrepancy is much smaller, about  $1\sigma$ . Models providing an *enhancement* of this branching ratio should be definitely favoured in the case of Scenario 1 but in Scenario 2 this is not so evident in view of large experimental error.

In any case, we think it is useful to concentrate on these two NP scenarios, even if precise definition of these scenarios depends on particular value of  $|V_{ub}|$ . We will be more specific about it in our numerical examples below.

Now models with many new parameters can face successfully both scenarios removing the deviations from the data for certain ranges of their parameters but as we will see below in simpler models often only one scenario can be admitted as only in that scenario for  $|V_{ub}|$  a given model has a chance to fit  $\varepsilon_K$  and  $S_{\psi K_{\rm S}}$  simultaneously. For instance, as we will see in the next section, models with constrained MFV select Scenario 1, while the 2HDM with MFV and flavour blind phases, 2HDM<sub>MFV</sub>, favours Scenario 2 for  $|V_{ub}|$ . What is interesting is that the future precise determination of  $|V_{ub}|$  through tree level decays will be able to distinguish between these two NP scenarios. We will see that there are other models which can be distinguished in this simple manner.

Now, the tensions within the SM discussed above constitute only a subset of visible deviations of its predictions from the data. A closer look at the measured quark flavour observables reveals the following deviations at the  $1-2\sigma$  level:

- The SM branching ratio for the inclusive decay  $B \to X_s \gamma$  is by  $1.2\sigma$  below the data so that models providing an *enhancement* appear to be favoured.
- The SM inclusive branching ratio for  $B\to X_s\ell^+\ell^-$  at high  $q^2$  is visibly below the data.
- The  $K^*$  longitudinal polarisation fraction  $F_L$  in  $B \to K^* \ell^+ \ell^-$  predicted by the SM is, on the other hand, larger than the data.
- The asymmetry  $A_{\rm SL}^b$  measured by D0 is by 3.9 $\sigma$  different from the SM value.

In Table I, we illustrate the SM predictions for some of these observables in both scenarios setting  $\gamma = 68^{\circ}$ . What is striking in this table is that with the new lattice input in (7) the predicted central values of  $\Delta M_s$  and  $\Delta M_d$ , although slightly above the data, are both in a good agreement with the latter when hadronic uncertainties are taken into account. In particular, the central value of the ratio  $\Delta M_s/\Delta M_d$  is very close to the data. These results

depend strongly on the lattice input and in the case of  $\Delta M_d$  on the value of  $\gamma$ . Therefore, to get a better insight, both lattice input and the tree level determination of  $\gamma$  have to improve. From the present perspective, models providing 10% suppression of both  $\Delta M_s$  and  $\Delta M_d$  appear to be slightly favoured.

TABLE I SM prediction for various observables for  $|V_{ub}|=3.4\times10^{-3}$  and  $|V_{ub}|=4.3\times10^{-3}$  and  $\gamma=68^\circ$  compared to experiment.

	Scenario 1:	Scenario 2:	Experiment
$ arepsilon_K $	$1.87(26) \times 10^{-3}$	$2.28(32) \times 10^{-3}$	$2.228(11) \times 10^{-3}$
$\mathcal{B}\left(B^+ \to  au^+  u_ au ight)$	$0.74(14) \times 10^{-4}$	$1.19(20) \times 10^{-4}$	$1.73(35) \times 10^{-4}$
$(\sin 2\beta)_{\rm true}$	0.676(25)	0.812(23)	0.679(20)
$\Delta M_s  \left[ \mathrm{ps}^{-1} \right]$	19.0(21)	19.1(21)	17.77(12)
$\Delta M_d  \left[ \mathrm{ps}^{-1} \right]$	0.55(6)	0.56(6)	0.507(4)
$\mathcal{B}(B \to X_s \gamma)$	$3.15(23) \times 10^{-4}$	$3.15(23) \times 10^{-4}$	$3.55(26) \times 10^{-4}$

We are aware of the fact that all these deviations are not yet very significant and could disappear. However, for the purpose of our presentation, it is useful to take them first seriously keeping in mind that the pattern of deviations from SM expectations could be modified in the future. This is, in particular, the case of observables, like  $\Delta M_{s,d}$ , that still suffer from nonperturbative uncertainties. It could turn out that suppressions (enhancements) of some observables required presently from NP will be modified to enhancements (suppressions) in the future.

#### 3. A new look at BSM scenarios

#### 3.1. Preliminaries

In this section, we will make a new look at a number of models analyzed in detail in the last decade in Munich in order to see how their patterns of flavour violation and CP violation are affected by the recent LHCb data. We will first of all discuss models that have been reviewed in some detail in [1]. These are: CMFV, MFV,  $2\text{HDM}_{\overline{\text{MFV}}}$ , the Littlest Higgs model with T-parity (LHT), the SM with sequential fourth generation (SM4), four classes of supersymmetric flavour models (SF), supersymmetric SU(5) GUT enriched through RH neutrinos SSU(5)<sub>RN</sub>, flavour blind

MSSM (FBMSSM), the minimal effective model with right-handed currents (RHMFV) and Randall–Sundrum model with custodial protection (RSc). In 2011 new models could be added to this list: left–right symmetric model based on the electroweak gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ , a maximal gauged flavour model (MGF) and a SO(10)-GUT.

Some of these analyses included also lepton flavour violations, EDMs and  $(g-2)_{\mu}$  but in the presentation below we will concentrate dominantly on flavour violating and CP-violating processes in the quark sector. The models to be discussed below are summarized in order of presentation in Fig. 1.

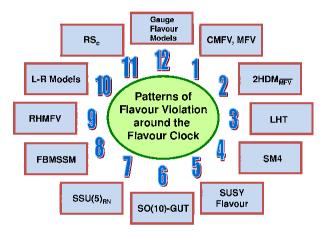


Fig. 1. Various patterns of flavour violation around the Flavour Clock.

# 3.2. CKM parameters

In our numerical examples, we will use

$$|V_{us}| = \lambda = 0.2252, \qquad |V_{cb}| = 0.0406$$
 (11)

which have been determined by means of tree level decays. The values of  $|V_{ub}|$  and  $\gamma$  will be specified in the context of our presentation.

We recall that once these four parameters of the CKM matrix have been fixed, the "true" values of the angle  $\beta$  and of the element  $|V_{td}|$  are obtained from the unitarity of the CKM matrix

$$|V_{td}| = |V_{us}||V_{cb}|R_t$$
,  $R_t = \sqrt{1 + R_b^2 - 2R_b \cos \gamma}$ ,  $\cot \beta = \frac{1 - R_b \cos \gamma}{R_b \sin \gamma}$ , (12)

where

$$R_b = \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \frac{|V_{ub}|}{|V_{cb}|}.$$
 (13)

# 3.3. Constrained Minimal Flavour Violation (CMFV)

The simplest class of extensions of the SM are models with CMFV [31, 32, 33]. They are formulated as follows:

- All flavour changing transitions are governed by the CKM matrix with the CKM phase being the only source of CP violation.
- The only relevant operators in the effective Hamiltonian below the weak scale are those that are also relevant in the SM.

There are basically three main implications of these assumptions:

•  $S_{\psi K_{\rm S}}$  and  $S_{\psi \phi}$  are as in the SM and, therefore, given by

$$S_{\psi K_S} = \sin(2\beta), \qquad S_{\psi \phi} = \sin(2|\beta_s|), \tag{14}$$

where  $\beta$  and  $\beta_s$  are defined by

$$V_{td} = |V_{td}|e^{-i\beta}, \qquad V_{ts} = -|V_{ts}|e^{-i\beta_s}.$$
 (15)

- For fixed CKM parameters determined in tree-level decays,  $|\varepsilon_K|$ ,  $\Delta M_s$  and  $\Delta M_d$ , if modified, can only be *enhanced* relative to SM predictions [34]. Moreover, this happens in a correlated manner [35].
- There are relations between various observables that are valid for the full class of the CMFV models including the SM. A review of these relations is given in [32]. We will list some of them now.

The most interesting relations in question are the following ones

$$\frac{\Delta M_d}{\Delta M_s} = \frac{m_{B_d}}{m_{B_s}} \frac{\hat{B}_d}{\hat{B}_s} \frac{F_{B_d}^2}{F_{B_s}^2} \left| \frac{V_{td}}{V_{ts}} \right|^2 r(\Delta M) = \frac{m_{B_d}}{m_{B_s}} \frac{1}{\xi^2} \left| \frac{V_{td}}{V_{ts}} \right|^2 r(\Delta M), \quad (16)$$

$$\frac{\mathcal{B}(B \to X_d \nu \bar{\nu})}{\mathcal{B}(B \to X_s \nu \bar{\nu})} = \left| \frac{V_{td}}{V_{ts}} \right|^2 r(\nu \bar{\nu}), \qquad (17)$$

$$\frac{\mathcal{B}(B_d \to \mu^+ \mu^-)}{\mathcal{B}(B_s \to \mu^+ \mu^-)} = \frac{\tau(B_d)}{\tau(B_s)} \frac{m_{B_d}}{m_{B_s}} \frac{F_{B_d}^2}{F_{B_s}^2} \left| \frac{V_{td}}{V_{ts}} \right|^2 r \left( \mu^+ \mu^- \right) , \tag{18}$$

where we have introduced the quantities  $r(\Delta M)$ ,  $r(\nu \bar{\nu})$  and  $r(\mu^+\mu^-)$  that are all equal unity in models with CMFV. They parametrize the deviations from these relations found in several models discussed by us below.

Eliminating  $|V_{td}/V_{ts}|$  from the three relations above allows to obtain three relations between observables that are universal within the CMFV models. In particular, from (16) and (18) one finds [36]

$$\frac{\mathcal{B}(B_s \to \mu^+ \mu^-)}{\mathcal{B}(B_d \to \mu^+ \mu^-)} = \frac{\hat{B}_d}{\hat{B}_s} \frac{\tau(B_s)}{\tau(B_d)} \frac{\Delta M_s}{\Delta M_d} r, \qquad r = \frac{r(\Delta M)}{r(\mu^+ \mu^-)}$$
(19)

that does not involve  $F_{B_q}$  and consequently contains smaller hadronic uncertainties than the formulae considered above. It involves only measurable quantities except for the ratio  $\hat{B}_s/\hat{B}_d$  that is now known already from lattice calculations with respectable precision [37,15]

$$\frac{\hat{B}_s}{\hat{B}_d} = 1.05 \pm 0.07, \qquad \hat{B}_d = 1.26 \pm 0.11, \qquad \hat{B}_s = 1.33 \pm 0.06.$$
 (20)

Finally, one can derive the relations [36]

$$\mathcal{B}\left(B_q \to \mu^+ \mu^-\right) = 4.36 \times 10^{-10} \frac{\tau_{B_q}}{\hat{B}_a} \frac{Y^2(v)}{S(v)} \Delta M_q,$$
 (21)

where Y(v) and S(v) are two master functions of CMFV models [32] that are universal with respect to the flavour (q = d, s, K). The argument vindicates that they depend on specific parameters of a given model. In the SM  $v = x_t$ . Once  $\mathcal{B}(B_q \to \mu^+\mu^-)$  will be measured one day, it will be possible to measure the ratio  $Y^2/S$  and compare the result with model predictions of various CMFV models. The important test for CMFV will be the same value for  $Y^2/S$  obtained from  $\mathcal{B}(B_s \to \mu^+\mu^-)$  and  $\mathcal{B}(B_d \to \mu^+\mu^-)$ .

The relations (16)–(19), (21) and other relations discussed in [32] can be regarded as *standard candles of flavour physics* and the deviations from them may help in identifying the correct NP scenario. In particular, the parameter r in (19) can deviate significantly from unity if non-MFV sources are present as demonstrated by us in LHT, RSc and SM4 models.

The relations in (21) allowed already some time ago to predict  $\mathcal{B}(B_{s,d} \to \mu^+\mu^-)$  in a given CMFV model with substantially smaller hadronic uncertainties than found by using directly the formulae for the branching ratios in question. Using the lattice input, in particular (20), known in 2010, and inserting the experimental values of  $\Delta M_{d,s}$  allowed to find in the SM [1]

$$\mathcal{B}\left(B_s \to \mu^+ \mu^-\right)_{SM} = (3.2 \pm 0.2) \times 10^{-9},$$
 (22)

$$\mathcal{B}\left(B_d \to \mu^+ \mu^-\right)_{\text{SM}} = (1.0 \pm 0.1) \times 10^{-10}.$$
 (23)

However, as the uncertainties on the  $F_{B_q}$  given in (6) have been reduced in 2011 significantly, it is more appropriate to calculate the branching ratios in question directly without using experimental data  $\Delta M_{s,d}$ . The result of this calculation is given in (2) and (3) and the new values are very close to the ones in (22) and (23) obtained two years ago using the experimental values of  $\Delta M_{s,d}$ . We believe that the results given in (2) and (3), that have been obtained in two ways, are closer to the true values than  $3.6 \times 10^{-9}$  in the case of  $\mathcal{B}(B_s \to \mu^+\mu^-)$  quoted recently by some authors.

Let us then confront the CMFV relations listed above and predictions with the present data. We observe:

- 1. As there are no new CP-violating phases in this framework and formulae in (14) apply, CMFV selects solution 1 for  $|V_{ub}|$ , i.e. small  $|V_{ub}|$ . It should be noted that the small value of  $S_{\psi\phi} = 0.035$  in this framework is fully consistent with the LHCb data in (1).
- 2. As seen in Table I, with small value of  $|V_{ub}|$  the central value of  $\varepsilon_K$  in the SM is by 16% lower than its experimental value. But in CMFV such an enhancement can be naturally obtained simply by increasing the value of the one-loop box function S. In fact, only the increase of S is possible in CMFV [34].
- 3. Yet, the enhancement of  $|\varepsilon_K|$  in models with CMFV implies automatically in a correlated manner enhancements of  $\Delta M_d$  and  $\Delta M_s$  with their ratio unchanged with respect to the SM as seen in (16)<sup>3</sup>. As seen in Table I, the SM values of  $\Delta M_{s,d}$  are slightly above the data and their necessary increase required by  $|\varepsilon_K|$  worsens the agreement of the theoretical values of  $\Delta M_{s,d}$  with data significantly even if their ratio agrees well with the data.

We conclude, therefore, that there is a serious difficulty in bringing  $\Delta M_{s,d}$  and  $|\varepsilon_K|$  to agree with the data simultaneously within the full class of CMFV models. In Fig. 2, we plot  $\Delta M_s$  and  $\Delta M_d$  as functions of  $|\varepsilon_K|$ . In obtaining this plot we have simply varied the master one-loop  $\Delta F = 2$  function S keeping CKM parameters and other input parameters fixed. The value of S at which central experimental value of  $|\varepsilon_K|$  is reproduced turns out to be S = 2.9 to be compared with  $S_{\rm SM} = 2.31$ . At this value the central values of  $\Delta M_{s,d}$  read

$$\Delta M_d = 0.69(6) \text{ ps}^{-1}, \qquad \Delta M_s = 23.9(2.1) \text{ ps}^{-1}.$$
 (24)

They both differ from experimental values by  $3\sigma$ . The error on  $|\varepsilon_K|$  coming dominantly from the error of  $|V_{cb}|$  and the error of the QCD factor  $\eta_1$  in the charm contribution [38] is however disturbing. Clearly this plot gives

<sup>&</sup>lt;sup>3</sup> We assume that CKM parameters  $|V_{ub}|$  and  $\gamma$  have been determined in tree-level decays and as small  $|V_{ub}|$  value is also favoured by SM fits, also values of  $|V_{td}|$  and  $|V_{ts}|$  remain unchanged with respect to the SM.

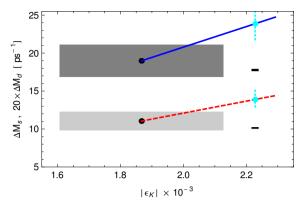


Fig. 2.  $\Delta M_s$  (upper solid blue line) and  $20 \Delta M_d$  (lower dashed red line) as functions of  $|\varepsilon_K|$  in models with CMFV for Scenario 1 chosen by these models. The short black lines represent the data, while the large dark grey and light grey regions represent the SM predictions. The vertical dotted lines corresponds to the error of  $\Delta M_s$  and  $20 \Delta M_d$ , where  $|\varepsilon_K|$  equals the experimental value. More information can be found in the text.

only some indication for possible difficulties of the CMFV and we need a significant decrease of theoretical errors in order to see how solid this result is.

**4.** Concerning the  $B_{s,d} \to \mu^+\mu^-$  decays, the LHCb upper bound on  $\mathcal{B}(B_s \to \mu^+\mu^-)$  implies within CMFV models an upper bound on  $\mathcal{B}(B_d \to \mu^+\mu^-)$  that is much stronger than the bound in (3)

$$\mathcal{B}(B_d \to \mu^+ \mu^-) \le 1.3 \times 10^{-10}, \quad \text{(CMFV)},$$

where we took into account the correction in (5). In Fig. 3, we show  $\mathcal{B}(B_d \to \mu^+\mu^-)$  versus  $\mathcal{B}(B_s \to \mu^+\mu^-)$  as predicted by CMFV. This result and the plot in Fig. 2 constitute important tests of CMFV. In particular, improved data on  $\mathcal{B}(B_d \to \mu^+\mu^-)$  are very important in this respect. The LHCb bound on  $\mathcal{B}(B_d \to \mu^+\mu^-)$  is still outside this plot.

In summary, we find that while in the SM  $\Delta M_{s,d}$  are consistent with the data and  $|\varepsilon_K|$  is visibly below the data, a model with CMFV characterized by an enhanced box function  $S \approx 2.9$ , while obtaining the correct value of  $|\varepsilon_K|$  predicts  $\Delta M_{s,d}$  significantly above the data. As this result depends sensitively on lattice input and the chosen  $|V_{ub}|$  and  $\gamma$ , it will be interesting to see whether improved lattice calculations and the tree level determination of  $|V_{ub}|$  and  $\gamma$  will confirm our findings with higher accuracy. In this context, the measurements of all observables listed in (9) and (10) will be of crucial importance. Only then we will know whether this simplest class of extensions of the SM is ruled out and NP contributions beyond the CMFV framework

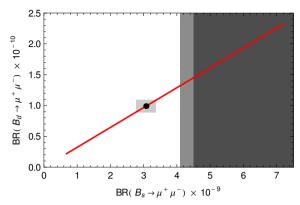


Fig. 3.  $\mathcal{B}(B_d \to \mu^+\mu^-)$  versus  $\mathcal{B}(B_s \to \mu^+\mu^-)$  in models with CMFV. SM is represented by the light grey area with black dot. The excluded range by LHCb bound in (2) is in dark and the additional excluded grey area corresponds to (5).

are at work. However, if CMFV should remain a viable NP scenario also the experimental branching ratio for  $\mathcal{B}(B^+ \to \tau^+ \nu_{\tau})$  has to go down by a factor of 2.

# 3.4. Minimal Flavour Violation at large

We have already formulated what we mean by CMFV. Let us first add here that the models with CMFV generally contain only one Higgs doublet and the top Yukawa coupling dominates. On the other hand, general models with MFV contain more scalar representations, in particular, two Higgs doublets. Moreover, the operator structure in these models can differ from the SM one. This is the case when bottom and top Yukawa couplings are of comparable size. A well known example is the MSSM with MFV and large  $\tan \beta$ .

In the more general case of MFV, the formulation with the help of global symmetries present in the limit of vanishing Yukawa couplings [39, 40] as formulated in [41] is elegant and useful. Other discussions of various aspects of MFV can be found in [42, 43, 44, 45, 46, 47, 48].

The hypothesis of MFV amounts to assuming that the Yukawas are the only sources of the breakdown of flavour and CP violation. The phenomenological implications of the MFV hypothesis formulated in this more grander manner than the CMFV formulation given above can be found model independently by using an effective field theory approach (EFT) [41]. In this framework, the SM Lagrangian is supplemented by all higher dimensional operators consistent with the MFV hypothesis, built using the Yukawa couplings as spurion fields. NP effects in this framework are then parametrized in terms of a few flavour-blind free parameters and SM Yukawa couplings that are solely responsible for flavour violation and also CP violation if these

flavour-blind parameters are chosen as *real* quantities as done in [41]. This approach naturally suppresses FCNC processes to the level observed experimentally, even in the presence of new particles with masses of a few hundreds GeV. It also implies specific correlations between various observables, which are not as stringent as in the CMFV but are still very powerful.

Yet, it should be stressed that the MFV symmetry principle in itself does not forbid the presence of flavour blind CP violating sources [49, 50, 51, 47, 52, 42, 53, 44, 45, 46]. Effectively, this makes the flavour blind free parameters complex quantities having flavour-blind phases (FBPs). These phases can in turn enhance the electric dipole moments EDMs of various particles and atoms and in the interplay with the CKM matrix can have also profound impact on flavour violating observables, in particular the CP-violating ones. In the context of the so-called aligned 2HDM model such effects have also been emphasized in [54].

Before turning to a specific model with FBPs let us just mention that in this more general framework, when FBPs are absent, several relations of CMFV remain. This is, in particular, the case of (18), where  $r(\mu^+\mu^-) \approx 1$  is found. On the other hand, (19) can be violated in the presence of new operators that can affect  $\Delta M_d$  and  $\Delta M_s$  differently.

3.5. 
$$2HDM_{\overline{MFV}}$$

#### 3.5.1. Preliminaries

We will next discuss a specific 2HDM model, namely 2HDM with MFV accompanied by flavour blind CP phases that we will call for short  $2\text{HDM}_{\overline{\text{MFV}}}$  [55] with the "bar" on MFV indicating the presence of FBPs.

Let us first list few important points of the  $2HDM_{\overline{MFV}}$  framework.

• The presence of FBPs in this MFV framework modifies through their interplay with the standard CKM flavour violation the usual characteristic relations of the MFV framework. In particular, the mixing induced CP asymmetries in  $B_d^0 \to \psi K_{\rm S}$  and  $B_s^0 \to \psi \phi$  take the form known from non-MFV frameworks like LHT, RSc and SM4

$$S_{\psi K_{\mathcal{S}}} = \sin(2\beta + 2\varphi_{B_d}), \qquad S_{\psi \phi} = \sin(2|\beta_s| - 2\varphi_{B_s}), \qquad (26)$$

where  $\varphi_{B_q}$  are NP phases in  $B_q^0 - \bar{B}_q^0$  mixings. Thus in the presence of non-vanishing  $\varphi_{B_d}$  and  $\varphi_{B_s}$ , originating here in non-vanishing FBPs, these two asymmetries do not measure  $\beta$  and  $\beta_s$  but  $(\beta + \varphi_{B_d})$  and  $(|\beta_s| - \varphi_{B_s})$ , respectively.

• The FBPs in the 2HDM<sub>MFV</sub> can appear both in Yukawa interactions and in the Higgs potential. While in [55] only the case of FBPs in Yukawa interactions has been considered, in [56] these considerations

have been extended to include also the FBPs in the Higgs potential. The two flavour-blind CPV mechanisms can be distinguished through the correlation between  $S_{\psi K_{\rm S}}$  and  $S_{\psi \phi}$  that is strikingly different if only one of them is relevant. In fact, the relation between generated new phases are very different in each case

$$\varphi_{B_d} = \frac{m_d}{m_s} \varphi_{B_s} \quad \text{and} \quad \varphi_{B_d} = \varphi_{B_s}$$
(27)

for FBPs in Yukawa couplings and Higgs potential, respectively.

- The heavy Higgs contributions to  $\varepsilon_K$  are negligible and consequently this model in contrast to CMFV favours the high value of  $|V_{ub}|$  for which the SM is consistent with the data on  $\varepsilon_K$ . But this time the presence of the phase  $\varphi_{B_d}$  allows, in principle, to remove the  $|\varepsilon_K| S_{\psi K_S}$  anomaly. Simultaneously,  $S_{\psi \phi}$  is enhanced over the SM values with the size of enhancement depending on whether FBPs in Yukawas or Higgs potential are at work.
- The selection of the large value of  $|V_{ub}|$  softens significantly the problem with the experimental value of  $\mathcal{B}(B \to \tau^+ \nu_{\tau})$  that is by a factor of two larger than in the SM.
- The branching ratios for  $B_{s,d} \to \mu^+ \mu^-$  can be sizably enhanced over the SM values but in a correlated manner given by (18) with  $r(\mu^+ \mu^-) \approx 1$ . Moreover, for  $S_{\psi\phi} \geq 0.25$  lower bounds on both branching ratios are found that are above the SM values and become stronger with increasing  $S_{\psi\phi}$ .
- Sizeable FBPs, necessary to explain possible sizable non-standard CPV effects in  $B_s$  mixing could, in principle, be forbidden by the upper bounds on EDMs of the neutron and the atoms. However, even for  $S_{\psi\phi} = \mathcal{O}(1)$  consistency with present bounds is obtained [56].

What is nice about this model is that while having new sources of CP violation it has a small number of free parameters and a number of definite predictions and correlations between various flavour observables that provide very important tests of this model. The question then arises how this simple model faces most recent LHCb data. Here we only provide the first observations. A more detailed study is in progress [57].

1. The removal of the  $\varepsilon_K - S_{\psi K_S}$  anomaly, which proceeds through the negative phase  $\varphi_{B_d}$ , is only possible with the help of FBPs in the Higgs potential so that, optimally,  $\varphi_{B_s} = \varphi_{B_d}$  implying the full dominance of the  $Q_{1,2}^{\rm SLL}$  operators as far as CP-violating contributions are concerned.

**2.** As shown in Fig. 4, the size of  $\varphi_{B_d}$  that is necessary for this removal implies in turn  $S_{\psi\phi} \geq 0.15$  which is  $2\sigma$  away from the LHCb central value in (1). Finding in the future that nature chooses a *negative* value of  $S_{\psi\phi}$  and/or small (exclusive) value of  $|V_{ub}|$  would practically rule out  $2\text{HDM}_{\overline{\text{MFV}}}$ .

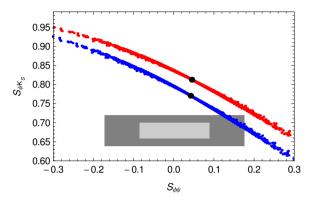


Fig. 4.  $S_{\psi K_{\rm S}}$  versus  $S_{\psi\phi}$  in 2HDM<sub>MFV</sub> for  $|V_{ub}| = 4.0 \times 10^{-3}$  (light grey/blue) and  $|V_{ub}| = 4.3 \times 10^{-3}$  (dark grey/red). SM is represented by black points while  $1\sigma$  ( $2\sigma$ ) experimental range by the grey (dark grey) area [57].

- **3.** As the CMFV relation (18) with  $r(\mu^+\mu^-) \approx 1$  also applies, the upper bound on  $\mathcal{B}(B_d \to \mu^+\mu^-)$  in (25) is also valid in this model.
- 4. In the case of the full dominance of NP effects from the Higgs potential, represented by the operators  $Q_{1,2}^{\rm SLL}$ , also in the case of  $\Delta M_{s,d}$  the CMFV relation (16) with  $r(\Delta M) \approx 1$  applies and consequently (19) with r=1 is valid. Yet, the CMFV correlation between  $\varepsilon_K$  and  $\Delta M_{s,d}$  is absent and  $\Delta M_{s,d}$  can be both suppressed and enhanced if necessary. The predicted value of  $\Delta M_s/\Delta M_d$  is close to the one found in the data unless large contributions of operators  $Q_{1,2}^{\rm LR}$  are present. They suppress  $\Delta M_s$  with basically no effect on  $\Delta M_d$ . Then  $r(\Delta M) \geq 1$  and consequently for this range of parameters  $r \geq 1$ . Thus, at first sight, this model provides a better description of  $\Delta F = 2$  data than the SM and models with CMFV. A more definite statement will be provided soon [57].
- 5. As already seen in the plots in [56] in the range of the presently allowed values for  $S_{\psi\phi}$ , there is basically no correlation between this CP-asymmetry and  $\mathcal{B}(B_q \to \mu^+\mu^-)$ . Consequently, the latter, even if already rather constrained by the data, can be still smaller or larger than the SM values. More work is needed to provide more definite statements [57].

We are looking forward to improved experimental data and improved lattice calculations to find out whether this simple model can satisfactorily describe the observables considered by us.

# 3.6. Littlest Higgs model with T-parity

We will next discuss two models having the operator structure of the SM but containing new sources of flavour and CP violation. This is the Littlest Higgs model with T-parity (LHT) and the SM4, the SM extended by a fourth sequential generation of quarks and leptons.

The Littlest Higgs model without [58] T-parity has been invented to solve the problem of the quadratic divergences in the Higgs mass without using supersymmetry. In this approach, the cancellation of divergences in  $m_H$  is achieved with the help of new particles of the same spin-statistics. Basically, the SM Higgs is kept light because it is a pseudo-Goldstone boson of a spontaneously broken global symmetry

$$SU(5) \to SO(5)$$
. (28)

Thus the Higgs is protected from acquiring a large mass by a global symmetry, although in order to achieve this the gauge group has to be extended to

$$G_{\text{LHT}} = \text{SU}(3)_c \times [\text{SU}(2) \times \text{U}(1)]_1 \times [\text{SU}(2) \times \text{U}(1)]_2$$
 (29)

and the Higgs mass generation properly arranged (collective symmetry breaking).

In order to make this model consistent with electroweak precision tests and simultaneously having the new particles of this model in the reach of the LHC, a discrete symmetry, T-parity, has been introduced [59,60]. Under T-parity all SM particles are even. Among the new particles only a heavy +2/3 charged T quark belongs to the even sector. Its role is to cancel the quadratic divergence in the Higgs mass generated by the ordinary top quark. The even sector and also the model without T-parity (LH model) belong to the CMFV class if only flavour violation in the down-quark sector is considered [61,62]. But it should be stressed that the T-even sector of the LHT model differs from the LH model considered in the latter papers, where also loop diagrams with heavy gauge bosons contribute. The high masses of new particles implied by electroweak precision tests in the LH model allow only for small deviations from the SM. In the LHT model, all NP effects in the T-even sector come from the T-quark interactions with standard quarks mediated by the SM gauge bosons and the effects can be, in principle, larger.

Yet, from the point of view of FCNC processes more interesting is the T-odd sector. Because of T-parity, it contains, first of all, three doublets of heavy mirror quarks and three doublets of mirror leptons that correspond to the SM fermions and communicate with the latter by means of heavy  $W_H^{\pm}$ ,  $Z_H^0$  and  $A_H^0$  gauge bosons that are also odd under T-parity. These interactions are governed by new mixing matrices that bring in new flavour parameters, in particular, new CP phases [63, 64]. The T-parity partner of

the T quark does not play any role in FCNC processes. It should be observed that in the limit of degenerate mirror quark masses T-odd sector does not contribute to FCNC processes and the LHT model represented then in these processes only by T-even sector belongs to the class of CMFV models. The problems of CMFV models identified before indicate that the T-odd sector is crucial for this model to achieve the agreement with data.

Let us summarize the main implications of this model for flavour phenomenology which are based on [65,66,67,68,69]. The plots given in these papers allow to answer, at least qualitatively, how this model faces the recent LHCb data. Fortunately, our new analysis triggered by new LHCb data allows to see the present status of the flavour physics in the LHT model even at a quantitative level [70]:

- 1. The difference between the CMFV models and the LHT model, originating in the presence of mirror quarks and new mixing matrices, is the violation of the usual CMFV relations between K,  $B_d$  and  $B_s$  systems of which we have shown some above. This allows to remove the  $\varepsilon_K S_{\psi K_S}$  anomaly for both scenarios of  $|V_{ub}|$  and also improve agreement with  $\Delta M_s$  and  $\Delta M_d$ . As this can be done simultaneously, the LHT model provides a better description of the data than the CMFV models.
- **2.** Interestingly, in this model it was not possible to obtain  $S_{\psi\phi}$  of  $\mathcal{O}(1)$  and values above 0.3 were rather unlikely. The LHCb result in (1) can, therefore, be considered as a relief for this model in which also negative values for  $S_{\psi\phi}$  as opposed to  $2\text{HDM}_{\overline{\text{MFV}}}$  are possible.
- 3.  $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})$  and  $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})$  can be enhanced up to factors of 3 and 2.5, respectively but not simultaneously with  $S_{\psi\phi}$ . Therefore, the small values for  $S_{\psi\phi}$  found by LHCb are good news for  $K_L \to \pi^0 \nu \bar{\nu}$  and  $K^+ \to \pi^+ \nu \bar{\nu}$  in the LHT model, although large enhancements of their branching ratios although possible are not guaranteed.
- 4. Rare B-decays turn out to be SM-like but still some enhancements are possible. In particular  $\mathcal{B}(B_s \to \mu^+ \mu^-)$  is predicted to be larger than its SM value but it can only be enhanced by 30%, where a significant part of this enhancement comes from the T-even sector. The effects in  $\mathcal{B}(B_d \to \mu^+ \mu^-)$  can be larger and also suppression is possible.

The plots presented in [70] should facilitate monitoring the future confrontations of the LHT model with the data and to find out whether this simple model can satisfactorily describe the observables considered by us. Further phenomenological discussions of LHT model can be found in original papers quoted above and in [71,72].

# 3.7. The SM with sequential fourth generation

One of the simplest extensions of the SM3 is the addition of a sequential fourth generation (4G) of quarks and leptons [73] (hereafter referred to as SM4). Therefore, it is of interest to study its phenomenological implications. Beyond flavour physics possibly the most interesting implications of the presence of 4G would be the viability of electroweak baryogenesis [74, 75, 76] and dynamical breakdown of electroweak symmetry triggered by the presence of 4G quarks [77, 78, 79, 80, 81, 82, 83, 84].

Yet, the LHC data indicate that our nature seems to have only three sequential generations of quarks and leptons, although the story of SM4 is not over yet. Selected recent papers demonstrating that SM4 is in trouble outside the flavour physics can be found in [85,86].

During the last ten years a number of flavour analyses of SM4 have been performed [87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100]. The SM4 introduces three new mixing angles  $s_{14}$ ,  $s_{24}$ ,  $s_{34}$  and two new phases in the quark sector and can still have a significant impact on flavour phenomenology. Similarly to the LHT model, it does not introduce any new operators but brings in new sources of flavour and CP violation that originate in the interactions of the four generation fermions with the ordinary quarks and leptons that are mediated by the SM electroweak gauge bosons. Thus in this model, as opposed to the LHT model, the gauge group is the SM one. This implies smaller number of free parameters.

An interesting virtue of the SM4 model is the non-decoupling of new particles. Therefore, unless the model has non-perturbative Yukawa interactions, the 4G fermions are bound to be observed at the LHC with masses below 600 GeV. This did not happen yet. In spite of this, it is of interest to see what is the impact of the recent LHCb data on the correlations found in our analyses of K and B flavour physics [96, 101].

In what follows, we list the most interesting patterns of quark flavour violation in the SM4 we have found in these papers and indicate how they are modified through LHCb at a qualitative level.

- 1. As before, the presence of new sources of flavour violations allows to solve all existing tensions related to  $\Delta F = 2$  observables.
- 2. We have also found that the desire to explain large values of  $S_{\psi\phi}$  signalled in 2010 implies uniquely the suppressions of the CP asymmetries  $S_{\phi K_{\rm S}}$  and  $S_{\eta'K_{\rm S}}$  below their SM values that are equal to  $S_{\psi K_{\rm S}}$ . Such suppressions were still visible in the 2010 data. This correlation has been pointed out in [88, 90], however we observed that for  $S_{\psi\phi}$  significantly larger than 0.6 the values of  $S_{\phi K_{\rm S}}$  and  $S_{\eta'K_{\rm S}}$  are below their central values indicated by the data, although some non-perturbative uncertainties were involved here.

With the new value for  $S_{\psi\phi}$  in (1) this suppression is absent and the values of  $S_{\phi K_{\rm S}}$  and  $S_{\eta' K_{\rm S}}$  in SM4 are compatible with the value of  $S_{\psi K_{\rm S}}$  as meanwhile also seen in the data.

3. The enhanced value of  $S_{\psi\phi}$  would imply a sizable enhancement of  $\mathcal{B}(B_s \to \mu^+\mu^-)$  over the SM3 prediction although this effect is much more modest than in SUSY models, where the Higgs penguin with large  $\tan \beta$  is at work. Yet, values as high as  $8 \times 10^{-9}$  were certainly possible in the SM4 in 2010. On the other hand, large values of  $S_{\psi\phi}$  would preclude non-SM values of  $\mathcal{B}(B_d \to \mu^+\mu^-)$ . Consequently, the CMFV relations in (18) and (19) can be strongly violated in this model. The small value of  $S_{\psi\phi}$  and the stringent upper bound on  $\mathcal{B}(B_s \to \mu^+\mu^-)$  from LHCb implies now that  $\mathcal{B}(B_d \to \mu^+\mu^-)$  can significantly depart from the SM value. On the other hand,  $\mathcal{B}(B_s \to \mu^+\mu^-)$  is SM-like with values below SM prediction being more likely than above it. In any case, the deviations from CMFV relations in (18) and (19) could still be sizable. All these features are clearly seen in the plots of our 2010 paper [96] and can be considered as predictions of SM4 found prior to the LHCb results. See also Fig. 5.

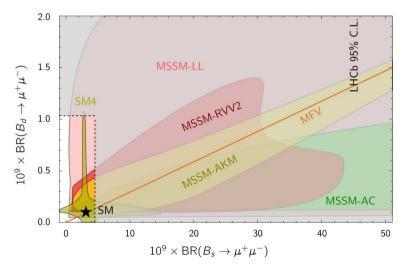


Fig. 5. Results in different SF models [2] as collected in [107]. The impact of the new LHCb bounds in (2) and (3) is shown.

4. Possible enhancements of  $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})$  and  $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})$  over the SM3 values are still possible as these branching ratios were not strongly correlated with  $S_{\psi\phi}$  and  $B_{s,d} \to \mu^+ \mu^-$ . Even in the presence of SM-like values for  $S_{\psi\phi}$  and  $\mathcal{B}(B_s \to \mu^+ \mu^-)$ , significant effects in the other decays like  $K_L \to \pi^0 \ell^+ \ell^-$  and  $K_L \to \mu^+ \mu^-$  are possible.

5. We have found that for large positive values of  $S_{\psi\phi}$  the predicted value of  $\varepsilon'/\varepsilon$  is significantly below the data, unless the hadronic matrix elements of the electroweak penguins are sufficiently suppressed with respect to the large N result and the ones of QCD penguins enhanced. With the small value of  $S_{\psi\phi}$  from LHCb this is not an issue anymore.

In summary, provided the four generation quarks will still be found, our qualitative discussion shows that if present, the new quarks can still have a potential impact on quark flavour physics. The same comment applies to new heavy leptons [98].

# 3.8. Supersymmetric Flavour models (SF)

None of the supersymmetric particles has been seen so far. However, one of the important predictions of the simplest realization of this scenario, the MSSM with R-parity, is a light Higgs with  $m_H \leq 130 \,\text{GeV}$ . The events at the LHC around 125 GeV could indeed be the first hints for a Higgs of the MSSM but it will take some time to verify it. In any case, MSSM remains still a viable NP scenario at scales  $\mathcal{O}(1 \,\text{TeV})$ .

Concerning the FCNC processes squarks, sleptons, gluinos, charginos, neutralinos, charged Higgs particles  $H^{\pm}$  and additional heavy neutral scalars can contribute to FCNC transitions through box and penguin diagrams. New sources of flavour and CP violation come from the misalignment of quark and squark mass matrices and similar new flavour and CP-violating effects are present in the lepton sector. Some of these effects can be strongly enhanced at large  $\tan \beta$  and the corresponding observables provide stringent constraints on the parameters of the MSSM. In particular  $B_{s,d} \to \mu^+ \mu^-$  can be strongly enhanced and the CP asymmetry  $S_{\psi\phi}$  can be  $\mathcal{O}(1)$ .

The SUSY dreams of large  $\mathcal{B}(B_s \to \mu^+\mu^-)$  and  $S_{\psi\phi}$  have not been realized at the LHCb and the data from LHCb listed in (1), (2) and (3) have certainly an impact on SUSY predictions.

In what follows, we will only make a first look at the impact of the LHCb data on the supersymmetric flavour (SF) models having flavour symmetries that allow for some understanding of the flavour structures in the Yukawa couplings and in SUSY soft-breaking terms, adequately suppressing FCNC and CP violating phenomena and solving SUSY flavour and CP problems.

The SF models can be divided into two broad classes depending on whether they are based on Abelian or non-Abelian flavour symmetries. Moreover, their phenomenological output crucially depends on whether the flavour and CP violations are governed by left-handed (LH) currents or if there is an important new right-handed (RH) current component [2].

In [2] we have performed an extensive study of processes governed by  $b \to s$  transitions in the SF models and of their correlations with processes governed by  $b \to d$  transitions,  $s \to d$  transitions,  $D^0 - \bar{D}^0$  mixing, LFV decays, electric dipole moments and  $(g-2)_{\mu}$ . Both Abelian and non-Abelian flavour models have been considered as well as the flavour blind MSSM (FBMSSM) and the MSSM with MFV. It has been shown how the characteristic patterns of correlations among the considered flavour observables allow to distinguish between these different SUSY scenarios and also to distinguish them from RSc and LHT scenarios of NP.

Of particular importance in our study were the correlations between the CP asymmetry  $S_{\psi\phi}$  and  $B_s \to \mu^+\mu^-$ , between the observed anomalies in  $S_{\phi K_s}$  and  $S_{\psi\phi}$ , between  $S_{\phi K_s}$  and  $d_e$ , between  $S_{\psi\phi}$  and  $(g-2)_{\mu}$  and also those involving LFV decays.

In the context of our study of the SF models, we have analysed the following representative scenarios:

- 1. Dominance of RH currents (Abelian model by Agashe and Carone (AC) [102]).
- 2. Comparable LH and RH currents with CKM-like mixing angles represented by the special version (RVV2) of the non-Abelian SU(3) model by Ross, Velasco and Vives [103] as discussed in [104].
- 3. In the second non-Abelian SU(3) model by Antusch, King and Malinsky (AKM) [105] analyzed by us the RH contributions are CKM-like but new LH contributions in contrast to the RVV2 model can be suppressed arbitrarily at the high scale. Still, they can be generated by RG effects at lower scales. To first approximation, the version of this model considered by us can be characterized by NP being dominated by CKM-like RH currents.
- 4. Dominance of CKM-like LH currents in non-Abelian models [106].

The distinct patterns of flavour violation found in each scenario have been illustrated with several plots that can be found in figures 11–14 of [2]. The power of these plots lies in the fact that even without a detailed numerical analysis one can, on a qualitative level, state what is the impact of the LHCb data on these results. A more quantitative analysis would require full numerical analysis taking also collider constraints on the masses of new particles involved. Before the situation with SUSY searches at the LHC is settled down, such an analysis appears premature to us. Keeping this in mind we will just recall some of the main messages from [2], stating how at first sight they are modified by the new data.

- 1. Supersymmetric models with RH currents (AC, RVV2, AKM) and those with exclusively LH currents can be, in principle, globally distinguished by the values of the CP-asymmetries  $S_{\psi\phi}$  and  $S_{\phi K_{\rm S}}$  with the following important result: none of the models considered in [2] could simultaneously explain the  $S_{\psi\phi}$  and  $S_{\phi K_{\rm S}}$  anomalies observed in the data in 2009. In the models with RH currents,  $S_{\psi\phi}$  can naturally be much larger than its SM value, while  $S_{\phi K_{\rm S}}$  remains either SM-like or its correlation with  $S_{\psi\phi}$  is inconsistent with the data. On the contrary, in the models with LH currents only,  $S_{\psi\phi}$  remains SM-like, while the  $S_{\phi K_{\rm S}}$  anomaly could easily be explained in 2009. The data on  $S_{\psi\phi}$  and  $S_{\phi K_{\rm S}}$  in 2012 do not indicate any potential anomalies in both observables and the distinction between these two classes of models on the basis of these two asymmetries will be difficult unless  $S_{\psi\phi}$  is finally found visibly different from the SM value. This could only be explained in models with RH currents.
- 2. The desire to explain large values of  $S_{\psi\phi}$  in 2009 within the models with RH currents unambiguously implied, in the case of the AC and the AKM models, values of  $\mathcal{B}(B_s \to \mu^+ \mu^-)$  as high as several  $10^{-8}$ . In the RVV2 model such values were also possible but not necessarily implied by the large value of  $S_{\psi\phi}$ . With the new range for  $S_{\psi\phi}$ , a lower bound on  $\mathcal{B}(B_s \to \mu^+ \mu^-)$  slightly above the SM value in AC and the AKM models is still present for  $|S_{\psi\phi}| \approx 0.2$ . It should also be emphasized that all these models can provide negative values of  $S_{\psi\phi}$  which is not possible in  $2\text{HDM}_{\overline{\text{MFV}}}$ .
- 3. As seen in Fig. 5, a better distinction between different SF models is offered by the ratio  $\mathcal{B}(B_d \to \mu^+\mu^-)/\mathcal{B}(B_s \to \mu^+\mu^-)$  that in the AC and RVV2 models turns out to be dominantly below its MFV prediction (straight line) and could be much smaller than the latter. In the AKM model, this ratio stays much closer to the MFV value of roughly 1/32 [36,108] and can be smaller or larger than this value with equal probability. Interestingly, in the LH-current-models, the ratio  $\mathcal{B}(B_d \to \mu^+\mu^-)/\mathcal{B}(B_s \to \mu^+\mu^-)$  cannot only deviate significantly from its MFV value of approximately 1/32, but in contrast to the models with RH currents considered by us can also be much larger than the latter value. Consequently,  $\mathcal{B}(B_d \to \mu^+\mu^-)$  as high as  $1 \times 10^{-9}$  is still possible, saturating the present upper bound in (3) while being consistent with the bounds on  $\mathcal{B}(B_s \to \mu^+\mu^-)$  in (2). Evidently, the new LHCb data had a significant impact on the ratio in question and from the present perspective there is more room for NP contributions dominated by LH currents.
- 4. Next, while the Abelian AC model resolves the present UT tensions [24, 25, 27, 26, 109, 28, 29, 110] through the modification of the ratio  $\Delta M_d/\Delta M_s$ , the non-Abelian flavour models RVV2 and AKM provide the solution through NP contributions to  $\varepsilon_K$ . As the ratio  $\Delta M_d/\Delta M_s$  within

the SM is roughly correct and cannot be changed by much, it appears, at first sight, that the AC model cannot remove the  $|\varepsilon_K| - S_{\psi K_{\rm S}}$  anomaly. However, in order to be sure a new analysis of this model has to be performed.

5. The branching ratios for  $K \to \pi \nu \bar{\nu}$  decays in the supersymmetric models considered by us remain SM-like and can be distinguished from RSc and LHT models where they can still be significantly enhanced.

In summary, although the large range of departures from SM expectations found in [2] has been significantly narrowed, still significant room for novel SUSY effects is present in quark flavour data. Assuming that SUSY particles will be found, the future improved data for  $B_{s,d} \to \mu^+ \mu^-$  and  $S_{\psi\phi}$  as well as  $\gamma$  combined with  $|V_{ub}|$  should help in distinguishing between various supersymmetric flavour models.

# 3.9. Supersymmetric SO(10) GUT model

GUTs open the possibility to transfer the neutrino mixing matrix  $U_{\rm PMNS}$  to the quark sector. This is accomplished in a controlled way in a SUSY GUT model proposed by Chang, Masiero and Murayama (CMM model) where the atmospheric neutrino mixing angle induces new  $b \to s$  and  $\tau \to \mu$  transitions [111, 112]. We have performed a global analysis in the CMM model including an extensive renormalisation group (RG) analysis to connect Planck-scale and low-energy parameters [113,114]. Since this work has been done before the new LHCb data on  $S_{\psi\phi}$  and  $\mathcal{B}(B_s \to \mu^+\mu^-)$ , we want to comment on the implications of these data for this model.

The basic properties of this model can be summarized as follows: the flavour symmetry which is exact at the Planck scale is broken at the SO(10) scale which manifests itself in the appearance of a non-renormalizable operator in the SO(10) superpotential. As a consequence, we get a natural hierarchy between top and bottom Yukawa couplings which finally leads to small  $\tan \beta$  between 3 and 10 (as opposed to other SO(10) GUT models with  $y_t \approx y_b$  and  $\tan \beta \approx 50$ ). While at  $M_{\rm Pl}$  the soft masses are universal, we get a large splitting between the masses of the 1st/2nd and 3rd down-squark and charged-slepton generation at the electroweak scale due to RG effects of  $y_t$ . The SO(10) symmetry is broken down to the SM gauge group via SU(5), where the right-handed down quarks and the lepton doublet are unified in the  $\bar{\bf 5}$ . Then, not only the neutrinos are rotated with the PMNS matrix, but the whole 5-plet and the corresponding supersymmetric partners. In addition, a model parameter — a CP violating phase  $\xi$  — enters the rotation of right-handed down (s)quarks. Consequently, the neutrino mixing angles are transferred to the right-handed down-squark/charged-slepton sector which then induces  $b \to s$  and  $\tau \to \mu$  transitions and CP violation in  $B_s - \overline{B}_s$ 

mixing via SUSY loops. The flavour effects in the CMM model are mainly determined by the generated mass splitting and the structure of the PMNS matrix.

Whereas effects in  $K-\overline{K}$  mixing,  $B_d-\overline{B}_d$  mixing, and  $\mu \to e\gamma$  are very small in the original version of the model, large contributions are predicted in observables connecting the 2nd and 3rd generation. However, since the CMM model at low energies appears as a special version of the MSSM with small  $\tan \beta$ , effects in  $B_s \to \mu^+\mu^-$  are negligible such that this branching ratio stays SM-like consistent with the recent upper bound from LHCb. In [113] we, therefore, focused on  $b \to s\gamma$ ,  $\tau \to \mu\gamma$ ,  $\Delta M_s$  and  $S_{\psi\phi}$ , where e.g.  $\mathcal{B}(\tau \to \mu\gamma)$  alone puts a lower bound on  $M_{\tilde{q}}$  (mass of the 1st/2nd squark generation). Here, we will concentrate on  $\Delta M_s$  and  $S_{\psi\phi}$ .

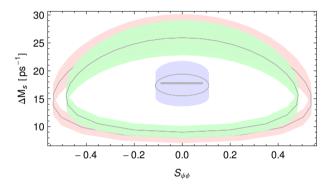


Fig. 6. Correlation between  $\Delta M_s$  and  $S_{\psi\phi}$  for Scenario 1 (dark grey/blue), 2 (light grey/green), 3 (grey/rose) (from inside to outside). The width of the coloured bands comes from the error of  $F_{B_s}^2 \hat{B}_{B_s}$  (the solid grey line corresponds to the central value).

In the CMM model, two operators contribute to  $B_s - \overline{B}_s$  mixing:  $Q_1^{\text{VLL}}$  and  $Q_1^{\text{VRR}}$ . The corresponding Wilson coefficient can then be written as  $C = C_L + e^{-2i\xi} |C_R^{\text{CMM}}|$  which makes clear that there can be new CP violating effects in  $S_{\psi\phi}$ . In view of the data from CDF and D0 on  $S_{\psi\phi}$ , this property was very welcomed in 2010. As an example, we show in Fig. 6 the correlation between  $\Delta M_s$  and  $S_{\psi\phi}$  for three selected points in the GUT parameter space that are consistent with  $b \to s\gamma$  and  $\tau \to \mu\gamma$ , namely for  $\tan\beta = 7$ ,  $\arg(\mu) = 0$  and

- 1.  $M_{\tilde{q}} = 1500 \text{ GeV}, m_{\tilde{q}} = 900 \text{ GeV}, a_1^d/M_{\tilde{q}} = 1.5 \text{ (dark grey/blue)},$
- 2.  $M_{\tilde{q}} = 1500 \text{ GeV}, m_{\tilde{q}} = 600 \text{ GeV}, a_1^d/M_{\tilde{q}} = 1.5 \text{ (light grey/green)},$
- 3.  $M_{\tilde{q}} = 2000 \text{ GeV}, m_{\tilde{g}} = 700 \text{ GeV}, a_1^d/M_{\tilde{q}} = 1.8 \text{ (grey/rose)},$

while simultaneously scanning over  $\xi \in [0, \pi]$ . As one can see, it is easily possible to get large CP violation in  $B_s - \overline{B}_s$  mixing, such that Scenario 2 and 3 were consistent with the experimental values of  $\Delta M_s$  and the large  $S_{\psi\phi}$  found in CDF and D0. However, now only Scenario 1 is consistent with the new LHCb measurement of  $S_{\psi\phi}$  and this implies new constraints on the model parameters, e.g. on the ratio  $m_{\tilde{g}}/M_{\tilde{q}}$ . Consequently, one previous advantage of the CMM model over mSUGRA/CMSSM scenarios—the ability to generate a large  $S_{\psi\phi}$ —is now gone.

The authors of [115] studied the implications of corrections to the unification of down-quark and charged-lepton Yukawa couplings  $Y_d = Y_\ell$  in the CMM model. This relation works remarkably well for the third generation but not for the two lighter ones. Therefore, one has to include corrections that are generated by higher-dimensional Yukawa operators suppressed by powers of  $M_{\rm Pl}$  which do not spoil the successful bottom-tau unification but can a priori have arbitrary flavour structure. Consequently, these operators have implications for  $K - \overline{K}$  and  $B_d - \overline{B}_d$  mixing but do not change the CMM predictions for  $b \to s$  and  $\tau \to \mu$  transitions. One main result of [115] is that the flavour structure of the higher-dimensional Yukawa operators is very much constrained due to  $|\varepsilon_K|$  which means that the dimension-5-Yukawa couplings and tree level Yukawa couplings must be nearly aligned. A similar result using  $\mu \to e\gamma$  was found in [116]. In [115] it was also shown that the tension in the SM between  $\sin 2\beta$  predicted from  $|\varepsilon_K|$  and  $\Delta M_s/\Delta M_d$ , and its direct measurement from  $S_{\psi K_S}$  can be removed with the help of higher-dimensional Yukawa couplings. CMM effects can then appear either in the UT side  $R_t$  through contributions from  $\Delta M_s$  or both in  $R_t$  and  $S_{\psi K_S}$ through an additional phase.

The CMM model can still serve as an alternative benchmark scenario to the popular constraint MSSM. It has only seven input parameters, is universal at  $M_{\rm Pl}$  and not at  $M_{\rm GUT}$  as in the CMSSM, it has a very clear flavour structure and in contrast to the CMSSM hadronic and leptonic observables are related.

# 3.10. Supersymmetric SU(5) GUT with RH neutrinos (RN): $SSU(5)_{RN}$

We will next consider a supersymmetric SU(5) GUT enriched by right-handed neutrinos ( $SSU(5)_{RN}$ ) accounting for the neutrino masses and mixing angles by means of a type-I see-saw mechanism. Since SUSY-GUTS generally predict FCNC and CP violating processes to occur both in the leptonic and hadronic sectors, we have performed in [117] an extensive study of FCNC and CP violation in both sectors, analyzing possible hadron/lepton correlations among observables. In particular, we have monitored how in this framework the tensions observed in the UT analysis can be resolved.

Here, the correlations between leptonic and hadronic processes taking place between the same generations like  $\mu \to e \gamma$  and  $s \to d$  or  $\tau \to \mu \gamma$  and  $b \to s$  transitions exist.

The main results of our study of the  $s \to d$  transitions and of their correlations with the  $\mu \to e$  transitions remains basically unchanged except that the effects are likely to be smaller as the lower bounds on supersymmetric particles increased. We refer to [117] for details. Here, we concentrate on the impact of LHCb data on selected results of our study of the  $b \to s$  transitions and of their correlations with the  $\tau \to \mu$  transitions. They are:

- 1. Non-standard values for  $S_{\psi\phi}$  implied in 2010 a lower bound for  $\mathcal{B}(\tau \to \mu\gamma)$  within the reach of SuperKEKB and SuperB. However, we also found that the  $(g-2)_{\mu}$  anomaly can be solved only for large  $\tan\beta$  values, where we found  $|S_{\psi\phi}| \leq 0.2$  for  $\Delta a_{\mu}^{\text{SUSY}} \geq 1 \times 10^{-9}$ , while being still compatible with the constraints from  $\mathcal{B}(\tau \to \mu\gamma)$ . Now, even if our analysis implied already at that time only moderate values of  $|S_{\psi\phi}| \leq 0.2$ , the fact that large values of  $\tan\beta$  are not welcome anymore in view of the LHCb upper bound on  $\mathcal{B}(B_s \to \mu^+\mu^-)$  changes the analysis of  $(g-2)_{\mu}$ . It appears that SUSY models do not provide the explanation for this anomaly anymore [118].
- 2. We recall that in this model  $S_{\psi K_{\rm S}}$  remains SM-like to a very good extent and consequently the solution of the UT anomalies by means of CPV effects in  $b \to d$  mixing is not possible. However, the UT anomalies can be solved by means of a negative NP contribution to  $\Delta M_d/\Delta M_s$ , implying a lower bound for  $\mathcal{B}(\tau \to \mu \gamma)$  within the reach of SuperKEKB and SuperB and large values for the angle  $\gamma$ . This scenario will be probed or falsified in due time at the LHCb through a precise tree level measurement of the latter UT angle. However, already now, it appears to us that the modification of  $\Delta M_d/\Delta M_s$  through the increase of  $\gamma$  is not favoured by the present data and the model may have a problem similarly to the AC supersymmetric flavour model in removing the anomalies.
- **3.** Both  $\mathcal{B}(B_s \to \mu^+ \mu^-)$  and  $\mathcal{B}(B_d \to \mu^+ \mu^-)$  can reach large non-standard values. While for  $\mathcal{B}(B_s \to \mu^+ \mu^-)$  this is no longer possible, this could still be the case for  $\mathcal{B}(B_d \to \mu^+ \mu^-)$ . In such a case, sizable departures from the MFV prediction  $\mathcal{B}(B_s \to \mu^+ \mu^-)/\mathcal{B}(B_d \to \mu^+ \mu^-) \approx |V_{ts}/V_{td}|^2$  will be present and as our analysis shows this could allow enhanced values of  $\mathcal{B}(\tau \to \mu \gamma)$ , possibly in the reach of SuperKEKB and SuperB.

# 3.11. The flavour blind MSSM (FBMSSM)

The flavour blind MSSM (FBMSSM) scenario [49, 50, 51, 52, 53] having new FBPs in the soft sector belongs actually to the class of MFV models or, even better, is of  $\overline{\text{MFV}}$  type. However, it is a supersymmetric framework and we mention this model here.

The FBMSSM has fewer parameters than the general MSSM implying striking correlations between various observables analyzed in [53]. Here we only make a few remarks on the  $\Delta F = 2$  observables as the recent LHCb results are actually good news for this model, even if the model suffers from some tension caused by the correlation between  $\Delta M_{s,d}$  and  $|\varepsilon_K|$ .

- 1. Indeed, only small effects in  $S_{\psi\phi}$  have been predicted, which in 2008 was a problem that disappeared now.
- 2. The NP effects in  $S_{\psi K_{\rm S}}$  and  $\Delta M_d/\Delta M_s$  turn out to be very small so that within this model these observables determine the coupling  $V_{td}$ , its phase  $-\beta$  and its magnitude  $|V_{td}|$ , without significant NP pollution. In particular, we found  $\gamma = 63.5^{\circ} \pm 4.7^{\circ}$  and  $|V_{ub}| = (3.5 \pm 0.2) \times 10^{-3}$ . Thus in this model, Scenario 1 for  $|V_{ub}|$  is favoured implying a value of  $|\varepsilon_K|$  that is visibly below the data if NP contributions are not included.
- 3. Fortunately, in this model  $|\varepsilon_K|$  turns out to be uniquely enhanced over its SM value and similarly to CMFV models also  $\Delta M_{s,d}$  are enhanced in a correlated manner. Simply the plots in Fig. 2 apply here. In 2008 it was possible to enhance  $\varepsilon_K$  up to a level of 15% basically removing the corresponding anomaly but no definite statements could be made about  $\Delta M_{s,d}$  due to large hadronic uncertainties present at that time. In 2012, as  $\Delta M_d/\Delta M_s$  is SM like in this model, it looks like the model is in a good shape at least from this point of view. However, the first look at  $\varepsilon_K$  and the values of  $\Delta M_{d,s}$ indicates that not everything is optimal in this model. Indeed, taking the squarks of first two generations to be degenerate in mass and above 1 TeV and imposing the lower bound on the stop mass of  $300 \text{ GeV}^4$  shows that in 2012  $\varepsilon_K$  can only be enhanced by at most 7% which only softens the problem with  $\varepsilon_K$  in the SM. Moreover, then, also  $\Delta M_{d,s}$  increases automatically by 10% worsening the agreement with the data for these observables relative to the SM. Simply the correlation between  $\Delta M_{d,s}$  and  $\varepsilon_K$  being in this model CMFV-like is not supported by the data. Still, when hadronic uncertainties are taken into account one cannot claim yet that this model fails to achieve consistency with the measured  $\Delta F = 2$  observables.

Finally, we notice that this model can be easily distinguished from  $SSU(5)_{RN}$  on the basis of the value of  $\gamma$  alone.

#### 3.12. The minimal effective model with right-handed currents: RHMFV

The recent phenomenological interest in making another look at the right-handed currents in general, and not necessarily in the context of a given left-right symmetric model, originated in tensions between inclusive and exclusive determinations of the elements of the CKM matrix  $|V_{ub}|$  and  $|V_{cb}|$ .

<sup>&</sup>lt;sup>4</sup> We thank Wolfgang Altmannshofer for making this first look.

It could be that these tensions are due to the underestimate of theoretical and/or experimental uncertainties. Yet, it is a fact, as pointed out and analyzed in particular in [119,120,121,122], that the presence of right-handed currents could either remove or significantly weaken some of these tensions, especially in the case of  $|V_{ub}|$ .

Assuming that RH currents provide the solution to the problem at hand, there is an important question whether the strength of RH currents required for this purpose is consistent with other observables and whether it implies new effects somewhere else that could be used to test this idea more globally.

This question has been addressed in [123]. The starting point of this analysis is the assumption that the SM is the low-energy limit of a more fundamental theory and consequently an effective theory is a useful approach to analyze the implications of RH currents. In [123] the central role is played by a left–right symmetric flavour group  $SU(3)_L \times SU(3)_R$ , commuting with an underlying  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  global symmetry and broken only by two Yukawa couplings. The model contains a new unitary matrix  $\tilde{V}$  controlling flavour-mixing in the RH sector and can be considered as the minimally flavour violating generalization to the RH sector. Thus bearing in mind that this model contains non-MFV interactions from the point of view of the standard MFV hypothesis that includes only LH charged currents it can be called RHMFV.

It should be stressed from the start that similarly to  $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$  it is the high inclusive value of  $|V_{ub}|$  that is selected by the model as the true value of this element providing simultaneously the explanation of the smaller  $|V_{ub}|$  found in SM analysis of exclusive decays. The latter explanation is not offered in  $2\mathrm{HDM}_{\overline{\mathrm{MFV}}}$  but in both models the true large value of  $|V_{ub}|$  implies automatically a value of  $\sin 2\beta$  above 0.80 and, therefore, significantly larger than the measured value of  $S_{\psi K_{\mathrm{S}}}$ . The question then arises how RHMFV solves this problem and what are its implications for  $S_{\psi\phi}$  and  $B_{s,d} \to \mu^+\mu^-$ .

Now, whereas the phenomenology of  $2\text{HDM}_{\overline{\text{MFV}}}$  is governed by flavour blind phases in Yukawas and Higgs potential, this role in RHMFV is taken by the new mixing matrix  $\widetilde{V}$  that can be parametrized in terms of 3 real mixing angles and 6 complex phases. A detailed phenomenology of this matrix taking all tree level constraints into account and solving the  $|V_{ub}|$  problem in this manner allowed us to identify few favourite shapes for this matrix. Subsequently a detailed FCNC analysis has been performed [123].

For our discussion of 2012 only the information about mixing structures relevant to the three down-type  $\Delta F = 2$  and FCNC amplitudes in the RH sector is important. Denoting by  $\tilde{c}_{ij}$  and  $\tilde{s}_{ij}$  the parameters of the RH matrix, one finds that the  $\tilde{c}_{12}$  and  $\tilde{s}_{12}$  dependencies in the three systems considered are non-universal with the observables in the K mixing,  $B_d$  mixing and  $B_s$  mixing dominated by  $\tilde{c}_{12}\tilde{s}_{12}$ ,  $\tilde{c}_{12}$  and  $\tilde{s}_{12}$ , respectively.

We should emphasize, probably for the first time, that this pattern allows to cope with the present data in a different manner than  $2\text{HDM}_{\overline{\text{MFV}}}$  does.

However, as both  $\Delta S=2$  and  $B_d$  mixing are strongly constrained, and the data from CDF and D0 gave some hints for sizable NP contributions to the CP violation in the  $B_s$  mixing, it was natural to assume in 2010 that  $\tilde{c}_{12}\ll 1$ . The phenomenological analysis was then rather constrained but evidently with  $\tilde{c}_{12}\ll 1$  the problem with the high value of  $S_{\psi K_S}$  could not be removed. This forced the authors of [123] to the following statement that we repeat here verbally:

Thus our analysis casts a shadow on the explanation of the  $|V_{ub}|$ -problem with the help of RH currents alone unless the  $S_{\psi\phi}$  anomaly goes away and  $\tilde{c}_{12}$  can be large solving the problem with  $S_{\psi K_{\rm S}}$  naturally.

As of 2012 the RHMFV model of [123] is no longer under the shadow of a large value of  $S_{\psi\phi}$ . With its value given in (1) the structure of the RH matrix changes and a large  $\tilde{c}_{12}$  can be chosen bringing  $S_{\psi K_{\rm S}}$  down to its experimental value and introducing only a small modification in  $S_{\psi\phi}$ . It will be interesting to see whether RHMFV model works in detail when the LHCb data on  $S_{\psi\phi}$  and other observables improve. However, this will require a new numerical analysis.

As far as the decays  $B_{s,d} \to \mu^+\mu^-$  are concerned, already in 2010 the constraint from  $B \to X_s \ell^+ \ell^-$  precluded  $\mathcal{B}(B_s \to \mu^+\mu^-)$  to be above  $1 \times 10^{-8}$ . Moreover, NP effects in  $B_d \to \ell^+\ell^-$  have been generally found to be smaller than in  $B_s \to \ell^+\ell^-$ . With the new structure of the RH matrix the opposite is true and the NP effects in  $B_d \to \ell^+\ell^-$  can now be much larger than in  $B_s \to \ell^+\ell^-$  in accordance with the room left for NP in the LHCb data.

Finally, let us mention that in this model, similar to  $2\text{HDM}_{\overline{\text{MFV}}}$ , the large value of  $|V_{ub}|$  softens significantly the problem with  $B^+ \to \tau^+ \nu_{\tau}$ .

There are other interesting consequences of this NP scenario that can be found in [123] even if some of them will be modified due to changes in the structure of the RH matrix. But let us stop here. It looks like the sun is again shining for RHMFV but it is not guaranteed that this will remain after new experimental informations will be available.

# 3.13. Left-right symmetric models (LRM)

The question then arises whether similar results can be obtained in a concrete BSM model with RH currents like the left-right symmetric model (LRM) based on the weak gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  [124, 125, 126, 127, 128]. This question has been addressed in [129], where a complete study of  $\Delta S = 2$  and  $\Delta B = 2$  processes in a LRM including, in particular,  $\varepsilon_K$ ,  $\Delta M_{s,d}$  and the mixing induced CP asymmetries  $S_{\psi K_S}$  and  $S_{\psi \phi}$  has been performed. Compared to the SM these observables are affected

in this model by tree level contributions from heavy neutral Higgs particles  $(H^0)$  as well as new box diagrams with  $W_R$  gauge boson and charged Higgs  $(H^{\pm})$  exchanges. We also analysed the  $B \to X_{s,d} \gamma$  decays that receive important new contributions from the  $W_{\rm L} - W_{\rm R}$  mixing and  $H^{\pm}$  exchanges. Compared to the previous literature, the novel feature of our analysis was the search for correlations between various observables that could help us to distinguish this model from other extensions of the SM and to obtain an insight into the structure of the mixing matrix  $V^{R}$  that governs righthanded currents. Moreover, we performed the full phenomenology including both gauge boson and Higgs boson contributions. We found that even for  $M_{H^0} \approx M_{H^\pm} \sim \mathcal{O}(20) \, \text{TeV}$ , the tree level  $H^0$  contributions to  $\Delta F = 2$  observables are by far dominant and the  $H^\pm$  contributions to  $B \to X_q \gamma$ can be very important, even dominant for certain parameters of the model. While in a large fraction of the parameter space this model has to struggle with the experimental constraint from  $\varepsilon_K$ , we demonstrated that there exist regions in parameter space which satisfy all existing  $\Delta F = 2$ ,  $B \to X_{s,d} \gamma$ , tree level decays and electroweak precision constraints for scales  $M_{W_{\rm R}} \simeq$ 2–3 TeV in the reach of the LHC. We also showed that the  $S_{\psi K_S} - \varepsilon_K$  tension present in the SM can be removed in the LRM. Simultaneously  $\mathcal{B}(B \to X_s \gamma)$ can be brought closer to the data. However, we pointed out that with the increased lower bound on  $M_{W_{\rm R}}$ , the LRM cannot help in explaining the difference between the inclusive and exclusive determinations of  $|V_{ub}|$ , when all constraints are taken into account, unless allowing for large fine-tuning.

The present impact of a decreased value of  $S_{\psi\phi}$  on this model can be significant in certain cases. In particular, the allowed shape of the matrix  $V^{\rm R}$  is modified<sup>5</sup>.

Also simple scenarios for this matrix considered in Sec. 7 of [129] will be affected. However, to assess these changes an analysis of rare K and B decays in this model constraining better the free parameters of this model should be performed. The other general findings of this paper are still valid.

#### 3.14. A Randall-Sundrum model with custodial protection

Models with a warped extra dimension first proposed by Randall and Sundrum (RS) [130] provide a geometrical explanation of the hierarchy between the Planck scale and the EW scale. Moreover, when the SM fields, except for the Higgs field, are allowed to propagate in the bulk [131,132,133], these models naturally generate the hierarchies in the fermion masses and mixing angles [133,131] through different localisations of the fermions in the bulk. Yet, this way of explaining the hierarchies in masses and mixings necessarily implies FCNC transitions at the tree level [134,135,136,137]. Most

<sup>&</sup>lt;sup>5</sup> Tillmann Heidsieck private communication.

problematic is the parameter  $\varepsilon_K$  which receives tree level KK gluon contributions and some fine-tuning of parameters in the flavour sector is necessary in order to achieve consistency with the data for KK scales in the reach of the LHC [137, 138].

Once this fine-tuning is made, the RS-GIM mechanism [135, 136], combined with an additional custodial protection of flavour violating Z couplings [138, 139, 140], allows yet to achieve the agreement with existing data for other observables without an additional fine tuning of parameters<sup>6</sup>. New theoretical ideas addressing the issue of large FCNC transitions in the RS framework and proposing new protection mechanisms occasionally leading to MFV can be found in [141, 142, 143, 144, 145, 146].

In order to avoid problems with electroweak precision tests (EWPT) and FCNC processes, the gauge group is generally larger than the SM gauge group [147, 148, 149]

$$G_{\rm RSc} = SU(3)_c \times SU(2)_{\rm L} \times SU(2)_{\rm R} \times U(1)_X, \qquad (30)$$

and similarly to the LHT model new heavy gauge bosons are present. The increased symmetry provides a custodial protection.

The lightest new gauge bosons in this so-called RSc framework are the KK-gluons, the KK-photon and the electroweak KK gauge bosons  $W_H^{\pm}$ ,  $W'^{\pm}$ ,  $Z_H$  and Z', all with masses  $M_{\rm KK}$  around 2–3 TeV as required by the consistency with the EWPT [147,148,149]. The fermion sector is enriched through heavy KK-fermions (some of them with exotic electric charges) that could, in principle, be discovered at the LHC. The fermion content of this model is explicitly given in [150], where also a complete set of Feynman rules has been worked out. Detailed analyses of electroweak precision tests and of the parameter  $\varepsilon_K$  in a RS model without and with custodial protection can also be found in [151, 152]. These authors analyzed also rare and non-leptonic decays in [153]. Possible flavour protections in warped Higgsless models have been presented in [145].

We will now summarize the impact of LHCb data on the results presented in [138, 139]:

- 1. The CP asymmetry  $S_{\psi\phi}$  was found in RSc in 2008 to reach values as high as 0.8. Such values are clearly excluded at present but RSc can have also SM-like values for this asymmetry and if necessary the asymmetry can be negative.
- 2. The smallness of  $S_{\psi\phi}$  are good news for rare K decays as in RSc simultaneous large NP effects in  $S_{\psi\phi}$  and  $K \to \pi\nu\bar{\nu}$  channels are very unlikely and this feature is even more pronounced than in the LHT model. Thus as stated by the authors of [138, 139] on many occasions: SM-like value of

<sup>&</sup>lt;sup>6</sup> See, however, comments in the final part of this subsection.

 $S_{\psi\phi}$  would open the road to large enhancements of these branching ratios that could be tested by K<sup>0</sup>TO at J-Parc, NA62 at CERN and ORKA at Fermilab. It looks like the LHCb opened this road this year.

- 3. Indeed, in the absence of a large  $S_{\psi\phi}$  the branching ratios for  $K^+ \to \pi^+ \nu \bar{\nu}$ ,  $K_L \to \pi^0 \nu \bar{\nu}$ ,  $K_L \to \pi^0 \ell^+ \ell^-$  can be enhanced relative to the SM expectations up to factors of 1.6, 2.5 and 1.4, respectively, when only moderate fine tuning in  $\varepsilon_K$  is required. Otherwise the enhancements can be larger.  $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})$  and  $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})$  can be simultaneously enhanced but this is not necessary as the correlation between these two branching ratios is not evident in this model. On the other hand,  $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})$  and  $\mathcal{B}(K_L \to \pi^0 \ell^+ \ell^-)$  ( $\ell = e, \mu$ ) are strongly correlated and the enhancement of one of these three branching ratios implies the enhancement of the remaining two.
- **4.** The branching ratios for  $B_{s,d} \to \mu^+ \mu^-$  and  $B \to X_{s,d} \nu \bar{\nu}$  remain SM-like: the maximal enhancements of these branching ratios amount to 15%. This is clearly consistent with the present LHCb data but the situation may change in the future.
- 5. The CMFV relations (16)–(18) and (19) between various observables can be strongly violated.

Next, let us just mention that large NP contributions in the RS framework that require some tunings of parameters in order to be in agreement with the experimental data have been found in  $\mathcal{B}(B \to X_s \gamma)$  [154],  $\mathcal{B}(\mu \to e \gamma)$  [155, 156, 157] and EDMs [136, 158], that are all dominated by dipole operators. Also the new contributions to  $\varepsilon'/\varepsilon$  can be large [159]. Moreover, it appears that the fine tunings in this ratio are not necessarily consistent with the ones required in the case of  $\varepsilon_K$ .

Finally, we would like to mention a very recent study of  $B \to X_s \gamma$ ,  $B \to K^* \mu^+ \mu^-$ ,  $B \to K^* \gamma$  and of the related observables in the RSc model [160]. As these processes were not the main stars of the present review we refer to a very systematic summary section of this paper for details.

Now many of the ideas and concepts that characterize most of the physics discussed in the context of RS scenario do not rely on the assumption of additional dimensions and as indicated by AdS/CFT correspondence we can regard RS models as a mere computational tool for certain strongly coupled theories. Therefore, in spite of some tensions in this NP scenario, the techniques developed in the last decade will certainly play an important role in the phenomenology, in particular, if Higgs will not be found to be an elementary particle and a new strong dynamics will show up at the LHC.

## 3.15. Gauged flavour models

In [161, 162, 163], a MFV-like ansatz has been implemented in the context of maximal gauge flavour (MGF) symmetries: in the limit of vanishing Yukawa interactions these gauge symmetries are the largest non-Abelian ones allowed by the Lagrangian of the model. The particle spectrum is enriched by new heavy gauge bosons, carrying neither colour nor electric charges, and exotic fermions, to cancel anomalies. Furthermore, the new exotic fermions give rise to the SM fermion masses through a see-saw mechanism, in a way similar to how the light left-handed (LH) neutrinos obtain masses by the heavy RH ones. Moreover, the MFV spurions are promoted to scalar fields — called flavons — invariant under the gauge group of the SM, but transforming as bi-fundamental representations of the non-Abelian part of the flavour symmetry. Once the flavons develop suitable VEVs, the SM fermion masses and mixings are correctly described.

Even if this approach has some similarities to the usual MFV description, the presence of flavour-violating neutral gauge bosons and exotic fermions introduces modifications of the SM couplings and tends to lead to dangerous contributions to FCNC processes mediated by the new heavy particles. Consequently, the MGF framework goes beyond the standard MFV and a full phenomenological analysis of this NP scenario is mandatory to judge whether it is consistent with all available data.

In [164], we have presented a detailed analysis of  $\Delta F = 2$  observables and of  $B \to X_s \gamma$  in the framework of a specific MGF model of Grinstein *et al.* [161] including all relevant contributions, in particular tree-level heavy gauge boson exchanges. The number of parameters in this model is much smaller than in some of the extensions of the SM discussed above and, therefore, it is not obvious that the present tensions on the flavour data can be removed or at least softened. Our findings are as follows:

- 1. We find that large corrections to the CP observables in the meson oscillations,  $\varepsilon_K$ ,  $S_{\psi K_{\rm s}}$  and  $S_{\psi \phi}$ , are allowed. However, requiring  $\varepsilon_K$  to be in agreement with experiment only small deviations from the SM values of  $S_{\psi K_{\rm s}}$  and  $S_{\psi \phi}$  are allowed. While at the time of our analysis this appeared as a possible problem as far as  $S_{\psi \phi}$  was concerned, this result is now fully consistent with present LHCb data.
- **2.** Consequently, we find that this model selects the scenario with exclusive (small) value of  $|V_{ub}|$ .
  - **3.**  $|\varepsilon_K|$  is enhanced without modifying  $S_{\psi K_{\rm S}}$ .
- 4. The values of  $\Delta M_d$  and  $\Delta M_s$  being strongly correlated in this model with  $\varepsilon_K$  turned out to be enhanced. In our original paper, were 2011 lattice input has been used, they were much larger than the data for the central values of input parameters:  $\Delta M_d \approx 0.75 \text{ ps}^{-1}$  and  $\Delta M_s \approx 27 \text{ ps}^{-1}$ . Mean-

while the lattice values for the relevant non-perturbative parameters have been modified so that this problem has been softened:  $\Delta M_d \approx 0.69 \text{ ps}^{-1}$  and  $\Delta M_s \approx 23.9 \text{ ps}^{-1}$  as in CMFV (see (24)). Therefore, after the inclusion of theoretical and parametric uncertainties these central values are within  $3\sigma$  from the data. Further decrease of non-perturbative uncertainties is necessary to fully assess whether this model fails to describe the data on  $\Delta F = 2$  processes properly.

**5.** Also problematic for this model at present appears to be the branching ratio  $\mathcal{B}(B^+ \to \tau^+ \nu_{\tau})$  for which the model does not provide any improvement with respect to the SM.

In summary, the new LHCb data and new lattice input provided a relief for MGF as far as  $S_{\psi\phi}$  is concerned but a satisfactory simultaneous description of  $\Delta M_{s,d}$  and  $|\varepsilon_K|$  has not been yet achieved in this model. Also the experimental value of  $\mathcal{B}(B^+ \to \tau^+ \nu_\tau)$  remains still a big problem due to the small  $|V_{ub}|$  selected by this model. Yet,  $\mathcal{B}(B^+ \to \tau^+ \nu_\tau)$  could turn out to be smaller one day.

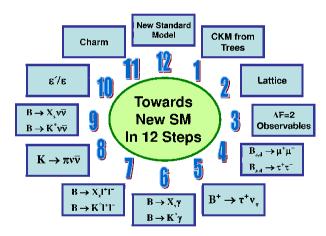


Fig. 7. Towards the New Standard Model in 12 Steps.

# 4. Observations, messages and a shopping list

Our BSM story is approaching the end. We have seen that in certain cases the recent LHCb data on  $S_{\psi\phi}$  and  $B_{s,d} \to \mu^+\mu^-$  had a profound impact on some extensions of the SM. Also, most recent update on lattice input modified predictions for  $\Delta M_{s,d}$  and  $B^+ \to \tau^+\nu_\tau$  not only within several BSM scenarios but also within the SM. It is to be expected that in the coming years the new data from LHCb combined with direct searches for NP at the LHC and further improvements coming from lattice calculations

will have an important impact on the landscape of BSM scenarios reviewed here, reducing it to a few oases. We should then hope that the measurements performed during the second LHC phase, by upgraded LHCb and in particular SuperKEKB, SuperB in Rome and kaon physics dedicated experiments NA62, K<sup>0</sup>TO and ORKA will select one oasis represented by a new SM (NSM). For recent reviews see [9, 165].

The route to the NSM will not be easy and will involve within the quark flavour physics at least *twelve* steps depicted in Fig. 7. This route will be complemented by another one involving the lepton flavour violation and if we are lucky the two routes will meet in the NSM oasis. But the description of these routes is another story.

For the coming years we are less ambitious. Nevertheless our shopping list includes some steps that in 2020 will surely be classified as mile stones in quark flavour physics.

Our shopping list has been constructed on the basis of what we have seen on previous pages and we summarize here our observations and related messages:

- First of all, we have identified a number of models that select either small or large value of  $|V_{ub}|$ : they simply have only a chance to describe the  $\Delta F = 2$  observables properly for such values. These are models in which NP can contribute significantly to either  $\varepsilon_K$  or  $S_{\psi K_S}$  but not to both of them. The small value of  $|V_{ub}|$  is chosen by CMFV, MFV without FBPhs, FBMSSM and MGF models. Large value of  $|V_{ub}|$  is selected by 2HDM<sub>MFV</sub> and RHMFV. Thus already clarification what is the true value of  $|V_{ub}|$  will tell us which of these two classes of models should be favoured.
- There are models like  $SSU(5)_{RN}$  in which there are no significant contributions to  $\varepsilon_K$  or  $S_{\psi K_S}$ . Such models can only provide satisfactory description of the  $\Delta F = 2$  data through an increased value of  $\gamma$ , typically above 80°. Therefore, future tree level measurements of  $\gamma$  will tell us whether such models offer good description of  $\Delta F = 2$  data or not. As we remarked in the corresponding section, from the present perspective we do not expect this to be the case.
- We have seen that in models with CMFV and MFV without FBPhs,  $S_{\psi\phi}$  had the SM value. On the other hand, in  $2\text{HDM}_{\overline{\text{MFV}}}$  it could have a different value but only a positive one and larger than the SM one. Models with new sources of flavour violation like LHT, SM4, SF models with RH-currents and the CMM model could generate  $S_{\psi\phi}$  with both signs. Finding  $S_{\psi\phi}$  to be negative would clearly indicate new sources of flavour and CP violation at work.

- The ratio  $\Delta M_s/\Delta M_d$  within SM, CMFV, 2HDM<sub>MFV</sub> and MGF models is roughly the same and consistent with experiment. On the other hand, while in SM the values of  $\Delta M_s$  and  $\Delta M_d$  are only slightly above the data, the desire to lift up  $|\varepsilon_K|$  over its SM value shifts automatically  $\Delta M_s$  and  $\Delta M_d$  in CMFV and MGF models so that we have to conclude that in these models it is not possible to obtain a good fit simultaneously to  $\Delta M_{s,d}$  and  $\varepsilon_K$ . This problem does not exist in 2HDM<sub>MFV</sub> because there is no correlation between  $\Delta M_{s,d}$  and  $\varepsilon_K$  in this model.
- As seen in Fig. 5,  $B_{s,d} \to \mu^+\mu^-$  can distinguish between various scenarios, in particular, when considered simultaneously. In SM4 and SF models with LH-currents  $\mathcal{B}(B_d \to \mu^+\mu^-)$  can still be enhanced by one order of magnitude with respect to the SM value. We have also found that in SM4 the suppression of  $\mathcal{B}(B_s \to \mu^+\mu^-)$  relative to the SM is rather likely, while in the LHT model only enhancement is possible and in the CMM model both decays  $B_{s,d} \to \mu^+\mu^-$  stay SM-like. Evidently, already the enhancements or suppressions of  $\mathcal{B}(B_{s,d} \to \mu^+\mu^-)$  with respect to SM values will select favourite scenarios of NP. In this context, the CMFV relations (16)–(19), (21) and other relations discussed in [32] can be regarded as standard candles of flavour physics and the deviations from them may help in identifying the correct NP scenario.
- Concerning  $B^+ \to \tau^+ \nu_{\tau}$  we do not yet take the discrepancy with the SM prediction as seriously as some authors do. It should be kept in mind that these are the first measurements of this decay and it could well be that its branching ratio is much closer to the SM value. In this context, precise determinations of  $|V_{ub}|$  and  $F_{B^+}$  are very important.
- Our discussion of  $K^+ \to \pi^+ \nu \bar{\nu}$ ,  $K_L \to \pi^0 \nu \bar{\nu}$  and  $B \to K^* \ell^+ \ell^-$  was marginal in view of space imitations but these decays together with  $b \to s \nu \bar{\nu}$  transitions are among superstars of this decade.

Having all this in mind our shopping list for the coming years looks as follows:

- Clarification of the discrepancy between inclusive and exclusive values of  $|V_{ub}|$ .
- $\bullet$  Precise measurement of the angle  $\gamma$  in tree-level decays.
- Improved lattice input for  $\Delta F = 2$  observables.

- Precise measurement of  $S_{\psi\phi}$ , in particular the determination of its sign.
- Improved calculations of  $\Delta M_s$  and  $\Delta M_d$  in order to test CMFV relations (16) and (19).
- Improved measurements of  $\mathcal{B}(B_{s,d} \to \mu^+ \mu^-)$  with the goal to test the MFV relation (18).
- Improved measurement of  $\mathcal{B}(B^+ \to \tau^+ \nu_{\tau})$  combined with improved  $|V_{ub}|$  and weak decays constant  $F_{B^+}$ .
- Measurements of the branching ratio  $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})$  by NA62 at CERN and ORKA at Fermilab and of  $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})$  by K<sup>0</sup>TO.
- Precise study of angular observables in  $B \to K^* \ell^+ \ell^-$ .

We could continue like this around the clock in Fig. 7 but let us stop here. Certainly the coming years will be exciting for flavour physics.

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