ENERGY LOSS AND JET QUENCHING PARAMETER IN A THERMAL NON-RELATIVISTIC NON-COMMUTATIVE YANG–MILLS PLASMA

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In this paper, we consider the problem of a moving heavy quark through a hot non-relativistic, non-commutative Yang–Mills plasma. We discuss the configuration of the static and dynamic quarks, and also obtain the quasi-normal modes. The main goal of this study is calculating the jet-quenching parameter for the non-relativistic, non-commutative theory and comparing it with drag forces which were recently obtained in an independent work, K.L. Panigrahi, S. Roy, *J. High Energy Phys.* **04**, 003 (2010).

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1. Introduction

The drag force of a heavy moving quark through a hot non-relativistic, non-commutative Yang–Mills plasma has been recently studied by using the AdS/CFT correspondence [1]. As we know, the AdS/CFT correspondence [2, 3, 4, 5, 6, 7, 8] is a powerful mathematical tool for simplification of some complicated calculations in QCD. However, in reality, QCD itself is not directly amenable to this correspondence which permits access to various other interesting strongly-coupled gauge theories. The problem of the drag force has been studied in the ordinary $\mathcal{N}=4$ super Yang–Mills thermal plasma [9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25] and in the $\mathcal{N}=2$ supergravity theory [26, 27, 28], and in the various other backgrounds [29]. We have already found that moving a heavy quark through $\mathcal{N}=2$ supergravity [30] thermal plasma with non-extremal black hole and finite chemical potential corresponds to the case of $\mathcal{N}=4$ super Yang–Mills thermal plasma

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with near-extremal black hole without chemical potential [26, 27, 28]. The holographic picture of the moving heavy quark through the thermal QGP (quark gluon plasma) with the momentum P, the mass m, the constant velocity v and an external force F is the stretched string in the AdS space. In that case, the drag force is given by $\dot{P} = F - \mu P$, where μ is called the friction coefficient. Another interesting problem in the strongly coupled plasma is the jet-quenching parameter [22,23,24,25,31,32,33,34,35,36,37,38,39,40,41]. In the ultra-relativistic heavy-ion collisions at LHC or RHIC, interactions between the high-momentum parton and the QGP are expected to lead to jet energy loss, which is called the jet quenching. The jet-quenching parameter provides a measurement of the dispersion of the plasma. The jet-quenching parameter is usually calculated by using the perturbation theory, but by using the AdS/CFT correspondence it is possible to compute the jet-quenching parameter in the non-perturbative quantum field theory. The perturbative QCD is not very reliable for current experimental temperature, therefore, the non-perturbative calculations have been used in recent literature. It is known that the non-perturbative definition of the jet-quenching parameter may be obtained in terms of light-like Wilson loop [31]. When fitting the current data, it seems that the value of the jet-quenching parameter is likely to be within the range 5–25 $\mathrm{GeV}^2/\mathrm{fm}$ [39].

The shear viscosity in the strongly coupled plasma can be calculated by using the AdS/CFT correspondence. In that case, the universality of the ratio of the shear viscosity to the entropy density has been studied in various backgrounds [42, 43, 44, 45, 46, 47, 48]. In Ref. [46], the shear viscosity of the $\mathcal{N} = 2$ supergravity thermal plasma and strong coupling limit of the shear viscosity for the $\mathcal{N} = 4$ super-Yang–Mill theory with a chemical potential computed.

In Ref. [1], the non-relativistic non-extremal (D1, D3) bound state solution of type IIB string theory is constructed by using the standard procedure of Null Melvin Twist [49, 50, 51, 52]. A particular low energy limit of this configuration is reduced to a non-commutative Yang–Mills theory [53,54,55], as well as coincident non-relativistic (D1, D3) bound state system. In some unified theories, such as great unification theories (GUT), it has been proposed that space-time coordinates could be non-commutative. Thus the non-commutativity became an interesting subject in modern physics. We have, therefore, strong motivation to study the plasma that simultaneously incorporates non-relativistic and non-commutative features. The origin of the non-commutativity in the D3-brane is the large magnetic field existing in the background. Besides the CFT usually has relativistic nature. However, in the context of condensed matter systems, it is useful to find holographic descriptions of CFT with the non-relativistic nature [56, 57, 58, 59]. These systems sometimes can be produced in the laboratory and, indeed, there exists such a strongly coupled non-relativistic system such as cold fermions. Therefore, it is interesting to study non-relativistic, non-commutative QGP. In Ref. [1], it is found that in the non-relativistic, non-commutative Yang–Mills theory explicit expression of the drag force does not have a closed form but in various limits has different forms. We deal with this difficulty in calculating the jet-quenching parameter, too. Therefore, we discuss the effect of the non-relativistic and non-commutative nature of the theory for large and small corresponding parameter. However, up to the constant, we succeed to find a closed form for the jet-quenching parameter in the relativistic case.

In this paper, we complement the discussion of the drag force and quasi-normal modes. An important difference between our work about drag force with Ref. [1] is the discussion of static quark and zero temperature limit. Moreover, we calculate the jet-quenching parameter in the non-relativistic, non-commutative Yang–Mills theory.

This paper is organized as follows. In Sec. 2, we obtain the equation of motion and momentum densities. We discuss the straightforwardly stretched string at zero and finite temperatures. In Sec. 3, we study the quasi-normal modes of the string and obtain the lowest modes. In Sec. 4, we compute the jet-quenching parameter and discuss the effect of non-commutativity parameter and non-relativistic nature of the theory. Finally in Sec. 5, we summarize our results.

2. Drag force

We begin with the following background metric in the original coordinates [1]

$$\begin{aligned} ds^2 \; = \; & \frac{r^2}{KR^2}\left[\left(1-\beta^2 r^2 f\right)\left(dx^-\right)^2 - \left(1+\beta^2 r^2\right) f (dx^+)^2 + 2\beta^2 r^2 f dx^- dx^+\right] \\ & + \frac{hr^2}{R^2}\left(\left(dx^2\right)^2 + \left(dx^3\right)^2\right) + \frac{R^2}{fr^2} dr^2 \,, \end{aligned} \tag{1}$$

where we have neglected the 5-sphere (S^5) part of the metric (it has no contribution in our calculations because we limited the calculation to the AdS_5 space only). Indeed, the metric (1) represents the AdS_5 space, and r denotes vertical direction to D-branes. In the above solution, $K \equiv 1 + \beta^2 \frac{r_{\mathrm{h}}^4}{r^2}$ and $R^2 = r_{\mathrm{h}}^2 \sinh\varphi$, r_{h} denotes the horizon radius and φ is called the boost parameter, β is a physical parameter related to the chemical potential of the Yang–Mills theory on the boundary. Moreover,

$$f = 1 - \frac{r_{\mathrm{h}}^4}{r^4}\,, \qquad h = \frac{1}{1+a^4 r^4}\,, \tag{2}$$

where

$$a^4=\frac{1}{r_{\mathrm{h}}^4\sinh^2\varphi\cos^2\theta}\,. \tag{3}$$

D3-branes are lying along x^1, x^2, x^3, and D1-branes along x^1. The angle θ in the relation (3) measures the relative numbers of D-branes. So, for N D3-branes and M D1-branes one can write, $\cos\theta=\frac{N}{\sqrt{N^2+M^2}}$.

In this configuration, there is a large magnetic field in the x^2-x^3 directions so these directions satisfy the non-commutativity relation $[x^2,x^3]=i\Theta$, where Θ is called the non-commutativity parameter [53]. It has been shown that $a^4r_{\mathrm{h}}^4\sim\Theta^2$ [1], so the parameter a measures the non-commutativity. Also $\beta\to 0$ limit recovers the relativistic cases, so the parameter β specifies the non-relativistic feature.

Therefore, an external quark moves non-relativistically at non-commutative plasma. Dual picture of this configuration is the stretched string from the brane to the horizon. The end point of string on the brane represents the external quark and string may move along the non-commutative direction $x^2\equiv x$. The open string is described by the following Nambu–Goto action

$$S=-T_0\int d\tau d\sigma\sqrt{-g}\,, \tag{4}$$

where T_0 is the string tension and (τ,σ) are the string world-sheet coordinates and g is the determinant of the world-sheet metric g_{ab}. In the static gauge, $x^+\equiv t=\tau$ and $r=\sigma$, the string world-sheet is described by the function $x(t,r)$, so the Lagrangian density is given by

$$\mathcal{L}=\sqrt{-g}=\left[\frac{1+\beta^2r^2}{K}-\frac{h}{f}\dot{x}^2+\frac{r^4}{R^4}\frac{1+\beta^2r^2}{K}hfx'^2\right]^{\frac{1}{2}}\,, \tag{5}$$

where the prime and dot denote derivative with respect to r and t, respectively. Then, by using the Euler–Lagrange equation one can find the string equation of motion

$$\frac{\partial}{\partial r}\left[\frac{r^4}{R^4}\frac{1+\beta^2r^2}{K}hf\frac{x'}{\sqrt{-g}}\right]=\frac{h}{f}\frac{\partial}{\partial t}\left[\frac{\dot{x}}{\sqrt{-g}}\right]\,. \tag{6}$$

In order to obtain the canonical momentum densities associated with the string, we use the following expressions

$$\begin{aligned}\pi_\mu^0 &= -T_0G_{\mu\nu}\frac{\left(\dot{X}\cdot X'\right)(X^\nu)'-(X')^2\dot{X}^\nu}{\sqrt{-g}}\\ \pi_\mu^1 &= -T_0G_{\mu\nu}\frac{\left(\dot{X}\cdot X'\right)\dot{X}^\nu-\left(\dot{X}\right)^2(X^\nu)'}{\sqrt{-g}}\,,\end{aligned} \tag{7}$$

where the metric $G_{\mu\nu}$ is given by relation (1). Therefore, for $\mu, \nu = x, r, t$ one can obtain

$$\begin{pmatrix} \pi_x^0 & \pi_x^1 \\ \pi_r^0 & \pi_r^1 \\ \pi_t^0 & \pi_t^1 \end{pmatrix} = -\frac{T_0}{\sqrt{-g}} \begin{pmatrix} -\frac{h}{f}\dot{x} & \frac{r^4}{R^4}\frac{1+\beta^2 r^2}{K} hfx' \\ \frac{h}{f}\dot{x}x' & \frac{1+\beta^2 r^2}{K} - \frac{h}{f}\dot{x}^2 \\ \frac{1+\beta^2 r^2}{K}\left(1 + \frac{r^4}{R^4} hfx'^2\right) & \frac{r^4}{R^4}\frac{1+\beta^2 r^2}{K} hf\dot{x}x' \end{pmatrix} . \tag{8}$$

Relation (8) is the general expression for the canonical momentum of the string stretched on the brane from $r = r_\mathrm{m}$ to $r = r_\mathrm{h}$. In that case, the total energy and momentum of the string are calculated by the following relations, respectively

$$E = -\int_{r_\mathrm{h}}^{r_\mathrm{m}} dr \pi_t^0 \,, \qquad P = \int_{r_\mathrm{h}}^{r_\mathrm{m}} dr \pi_x^0 \,. \tag{9}$$

The component π_x^1 is interpreted as the drag force on the quark due to thermal plasma. The simplest solution of the equation of motion (6) is $x = x_0$, where x_0 is a constant. In this case the string is stretched straightforwardly from $r = r_\mathrm{m}$ to the $r = r_\mathrm{h}$. This configuration is a dual picture of the static quark in the thermal non-commutative plasma. In this case $-g = \frac{1+\beta^2 r^2}{K}$, and the drag force vanishes which is expected for the static quark. Only non-zero components of the momentum density can be obtained as

$$\pi_r^1 = \pi_t^0 = -T_0 \sqrt{\frac{1+\beta^2 r^2}{K}} \,. \tag{10}$$

By using the first relation of (9) one can obtain the total energy of the string. For the case of $\beta \ll 1$ we get

$$E = T_0 \left[r_\mathrm{m} - r_\mathrm{h} + \frac{\beta^2}{2} \left(\frac{r_\mathrm{h}^4}{r_\mathrm{m}} + \frac{r_\mathrm{m}^3}{3} - \frac{4 r_\mathrm{h}^2}{3} \right) \right] + \mathcal{O}\left(\beta^4\right) \,. \tag{11}$$

It is clear that at $\beta \to 0$ limit we recover the result of Ref. [9], where the total energy of the string was obtained as $E \sim r_\mathrm{m} - r_\mathrm{h}$. Therefore, $\beta \to 0$ limit of these calculations is corresponding to the moving heavy quark through $\mathcal{N} = 4$ super Yang–Mills plasma.

The temperature of the non-relativistic, non-commutative Yang–Mills theory is given by [1]

$$T = \frac{r_\mathrm{h}}{\pi R^2} = \frac{r_\mathrm{h}}{\sqrt{2\hat{\lambda}\pi\alpha'}} \,, \tag{12}$$

where α' is the slope parameter ($\alpha' = \frac{1}{2\pi T_0}$), and $\hat{\lambda}$ is the 't Hooft coupling of the non-relativistic, non-commutative theory, which is related to the 't Hooft coupling of the ordinary Yang–Mills theory by the relation $\lambda = \frac{\alpha'}{\Theta}\hat{\lambda}$. Therefore, the zero-temperature limit is obtained by taking $r_{\rm h} \to 0$. At the zero-temperature limit one can interpret E as the physical mass of the quark, hence in this limit one can obtain

$$(E)_{T=0} = m = T_0 r_{\rm m}\left[1 + \frac{\beta^2 r_{\rm m}^2}{6}\right] . \tag{13}$$

As we expected, at the $r_{\rm m} \to \infty$ limit (brane position moves to the boundary) the quark mass will be infinite. Also, increasing the temperature decreases the quark mass, so one can write

$$(E)_{T\neq 0} = M_{\rm rest}(T) = m - \Delta m(T) , \tag{14}$$

where thermal rest mass shift is defined as

$$\Delta m(T) = E = T_0 r_{\rm h}\left[1 + \frac{\beta^2}{2}\left(4\frac{r_{\rm h}}{3} - \frac{r_{\rm h}^3}{r_{\rm m}}\right)\right] . \tag{15}$$

We summarized these results in Table I.

TABLE I

AdS/CFT translation table. Expressions of $\Delta m(T)$, $M_{\rm rest}(T)$ and m obtained for the infinitesimal β and $\mathcal{O}(\beta^4)$ have been neglected. These results agree with [9] at $\beta \to 0$ limit.

Quantity	NR-NC YM	Type IIB string
Slope parameter	$\frac{R^2}{\sqrt{2\hat{\lambda}}}$	α'
't Hooft coupling	$\hat{\lambda}$	$\frac{R^4}{2\alpha'^2}$
Temperature	T	$\frac{r_{\rm h}}{\pi R^2}$
Horizon radius	$\pi R^2 T$	$r_{\rm h}$
Thermal rest mass shift	$\Delta m(T)$	$T_0 r_{\rm h}\left[1 + \frac{\beta^2}{2} r_{\rm h}\left(\frac{4}{3} - \frac{r_{\rm h}^2}{r_{\rm m}}\right)\right]$
Physical mass	m	$T_0 r_{\rm m}\left[1 + \frac{\beta^2 r_{\rm m}^2}{6}\right]$
Static thermal mass	$M_{\rm rest}(T)$	$T_0\left[r_{\rm m} - r_{\rm h} + \frac{\beta^2}{2}\left(\frac{r_{\rm h}^4}{r_{\rm m}} + \frac{r_{\rm m}^3}{3} - \frac{4r_{\rm h}^2}{3}\right)\right]$

On the other hand, for the case of $\beta \gg 1$ one can obtain

$$E \approx T_0 \frac{\beta^2}{2} r_{\rm h}\left[r_{\rm m} - r_{\rm h} - \beta r_{\rm h} \ln\frac{r_{\rm m} + \beta r_{\rm h}}{r_{\rm h} + \beta r_{\rm h}}\right] . \tag{16}$$

In this case, the non-commutative parameter a is not important because the static quark has no motion along the non-commutative directions.

Another solution of the equation of motion (6) may be written as $x = vt + x_0$, and is corresponding to the moving straightforward string which is not a physical solution [9, 26].

The other solution, which satisfies the equation of motion, may be written as $x(t,r) = vt + \xi(r)$. Such a solution has been recently considered [1] and the drag force was obtained in the standard form as the following

$$\pi_x^1 = -T_0 C v r_{\mathrm{c}}^2\,, \tag{17}$$

where constant C is given by

$$C = \frac{1}{r_{\mathrm{h}}^2\left(1+a^4 r_{\mathrm{c}}^4\right)\sinh\varphi}\,, \tag{18}$$

and the critical radius r_{c} is the root of the following equation

$$\left(r^4 - r_{\mathrm{h}}^4\right)\left(1+a^4r^4\right)\left(1+\beta^2r^2\right) - v^2\left(r^4+\beta^2 r_{\mathrm{h}}^4 r^2\right) = 0\,. \tag{19}$$

Expression (17) for the drag force is in agreement with the previous works such as [9, 26, 27, 28], the only differences are the definition of the constant C and the critical radius r_{c}. In the special case of $\beta = a = 0$ one can find

$$\pi_x^1 = -T_0 \frac{v}{\sqrt{1-v^2}\,\sinh\varphi}\,. \tag{20}$$

Particularly, if we set $\sinh\varphi = -\sqrt{1-v^2}$, this result coincides with [26] at $\eta \to 0$ limit and $\Lambda^2 = 1$, where η is called the non-extremality parameter and Λ denotes the cosmological constant. As we know, the non-extremality parameter is related to the black hole charge q [28], and the black hole charge q is related to the chemical potential ($\eta \sim q \sim \frac{1}{\beta}$). Thus, it is reasonable that results of heavy quark in non-relativistic, non-commutative Yang–Mills plasma at $\beta \to 0$ and $a \to 0$ limits agree with the results of the moving heavy quark through $\mathcal{N} = 2$ supergravity thermal plasma at extremal limit without B-field [26, 27, 28]. Both theories at mentioned limits are corresponding to moving heavy quark through $\mathcal{N} = 4$ super Yang–Mills thermal plasma without the chemical potential.

3. Quasi-normal modes

In this section, we would like to consider behavior of the curved string at the late time and in the low velocity limit. In that case, the string has small fluctuations around the straight string. It means that $\dot{x}^2$ and x'^2 are infinitesimal, so one can neglect them in the expression (5). Therefore, the equation of motion reduces to the following equation

$$\frac{\partial}{\partial r}\left[\frac{r^4}{R^4}\sqrt{\frac{1+\beta^2 r^2}{K}} f x'\right] = \frac{1}{f}\sqrt{\frac{K}{1+\beta^2 r^2}}\ddot{x}\,, \tag{21}$$

where $a = 0$ (there are no movements along the non-commutative direction). Then, one may choose the time-dependent solution of the form

$$x(r,t) = \xi(r) e^{-\mu t}\,. \tag{22}$$

In that case, the equation of motion (21) reduces to the following differential equation

$$O\xi(r) = \mu^2 \xi(r)\,, \tag{23}$$

where we define

$$O \equiv f\sqrt{\frac{1+\beta^2 r^2}{K}}\frac{\partial}{\partial r}\frac{r^4}{R^4} f\sqrt{\frac{1+\beta^2 r^2}{K}}\frac{\partial}{\partial r}\,. \tag{24}$$

Ansatz (22) satisfies the Neumann boundary condition at $r = r_{\rm m} (\xi'(r_{\rm m}) = 0)$. In order to obtain the friction coefficient μ, it is convenient to expand $\xi(r)$ as power series of μ

$$\xi(r) = \xi_0(r) + \mu\xi_1(r) + \mu^2\xi_2(r) + \ldots\,. \tag{25}$$

Substituting expansion (25) in the equation (23) tells that

$$O\xi_0 = 0\,, \qquad O\xi_1 = 0\,, \qquad O\xi_2 = \xi_0\,. \tag{26}$$

The Neumann boundary condition causes to choose $\xi_0 = A$, where A is a constant, therefore,

$$\xi'(r_{\rm m}) = \mu\xi_1'(r_{\rm m}) + \mu^2\xi_2'(r_{\rm m}) = 0\,. \tag{27}$$

At the $\beta \to 0$ limit, by using equation (27), one can obtain

$$\mu = \left[r_{\rm m} + \frac{r_{\rm h}}{4}\ln\frac{r_{\rm m}-r_{\rm h}}{r_{\rm m}+r_{\rm h}} - \frac{r_{\rm h}}{2}\tan^{-1}\frac{r_{\rm m}}{r_{\rm h}}\right]^{-1}\,. \tag{28}$$

On the other hand, for $\beta \neq 0$, by using the near horizon behavior ($r \to r_{\rm h}$ approximation which yields $f \approx 1$ and $\frac{K}{1+\beta^2 r^2} \approx 1$), one can obtain

$$\xi_1'(r) = -A\frac{R^4}{r^4}\,, \qquad \xi_2'(r) = A\frac{R^4}{r^3}\,. \tag{29}$$

Therefore, applying the Neumann boundary condition yields to the following expression for the friction coefficient

$$\mu = \frac{1}{r_{\rm m}} \tag{30}$$

which agrees with the first term of expression (28). This is the smallest eigenvalue of operator O obtained for the given boundary condition.

As mentioned above, for the infinitesimal $\dot{x}$ and x' one can write $\sqrt{-g} \approx \sqrt{\frac{1+\beta^2 r^2}{K}}$, and then the momentum density is given by

$$\pi_x^0 = -\frac{T_0}{\mu}\left[\frac{r^4}{R^4} f \sqrt{\frac{1+\beta^2 r^2}{K}} x'\right]' , \tag{31}$$

where we used the equation of motion (21) and solution (22). Then, we use the second relation (9) to obtain the total momentum of the string

$$P = \frac{T_0}{\mu}\left[\frac{r_{\rm min}^4}{R^4}\left(1-\frac{r_{\rm h}^4}{r_{\rm min}^4}\right)\sqrt{\frac{1+\beta^2 r_{\rm min}^2}{1+\beta^2 \frac{r_{\rm h}^4}{r_{\rm min}^2}}} x'(r_{\rm min})\right] , \tag{32}$$

where we used the Neumann boundary condition, and $r_{\rm min} > r_{\rm h}$ as an IR cutoff at a lower limit of the integral.

In order to obtain the total energy, we expand $\sqrt{-g}$ to the second order of $\dot{x}$ and x', and obtain

$$\pi_t^0 = -T_0\left[\sqrt{\frac{1+\beta^2 r^2}{K}} + \frac{1}{2}\left(\frac{r^4}{R^4} f \sqrt{\frac{1+\beta^2 r^2}{K}} x x'\right)'\right] , \tag{33}$$

where we used the equation of motion (21). Then, we use the first relation (9) to obtain the total energy of the string

$$\begin{aligned} E \;=\; & T_0\left[r_{\rm m} - r_{\rm min} + \frac{\beta^2}{2}\left(\frac{r_{\rm h}^4}{r_{\rm m}} + \frac{r_{\rm m}^3}{3} - \frac{4 r_{\rm h}^2}{3}\right)\right] \\ & + \frac{T_0}{2}\frac{r_{\rm min}^4}{R^4}\left(1-\frac{r_{\rm h}^4}{r_{\rm min}^4}\right)\sqrt{\frac{1+\beta^2 r_{\rm min}^2}{1+\beta^2 \frac{r_{\rm h}^4}{r_{\rm min}^2}}} x(r_{\rm min}) x'(r_{\rm min}) , \end{aligned} \tag{34}$$

where we assume that the parameter β is infinitesimal and use the Neumann boundary condition. Combining relations (32) and (34) and using $\dot{x} = -\mu m x$ yields to the simple relationship $E = M_{\rm rest} + \frac{P^2}{2m}$, where m is the kinetic mass of the quark.

4. Jet-quenching parameter

In order to calculate the jet-quenching parameter, we should consider an open string whose endpoints lie on the brane. This is corresponding to quark–antiquark configuration. In the light cone coordinates, the string profile is given by the function $r(t, y)$, where we used static gauge, $\tilde{x}^- \equiv \tau = t\,(L^- \leq \tilde{x}^- \leq 0)$ and $x^2 \equiv \sigma = y\,(-\frac{L}{2} \leq y \leq \frac{L}{2})$, and all other coordinates are constant. Because of the condition $L^- \gg L$, the world-sheet is invariant along the $\tilde{x}^-$ direction, and one can consider the function $r(y)$ as the string profile, so $r(\pm\frac{L}{2}) = \infty$. Moreover, we use the light cone coordinates $\tilde{x}^- = \frac{1}{\sqrt{2}}(x^+ - x^-)$ and $\tilde{x}^+ = \frac{1}{\sqrt{2}}(x^+ + x^-)$ in the metric (1) [1, 56, 57, 58, 59]. In that case, the Nambu–Goto action (4) takes the following form

$$S = 2T_0 L^- \int\limits_0^{\frac{L}{2}} dy \sqrt{\frac{\left(\frac{r_{\rm h}^4}{2r^4} - 2r^2\beta^2 f\right)}{K} \left(\frac{r^4}{R^4}h + \frac{r'^2}{f}\right)}\,, \tag{35}$$

where prime denotes derivative with respect to y. Equation of motion ($\mathcal{H} = \mathcal{L} - \frac{\partial \mathcal{L}}{\partial r'} r' = \varepsilon$) yields to the following expression

$$r'^2 = \frac{fh}{\varepsilon^2}\frac{r^4}{R^4}\left[\frac{\left(\frac{r_{\rm h}^4}{2r^4} - 2r^2\beta^2 f\right)}{K}\frac{r^4}{R^4}h - \varepsilon^2\right]\,, \tag{36}$$

where we interpreted the constant ε as the string energy. By using relation (36) in (35) and $dy = dr/r'$ one can rewrite the action (35) as the following

$$S = 2T_0 L^- \int\limits_{r_{\rm h}}^{\infty} dr \sqrt{\frac{\left(\frac{r_{\rm h}^4}{2r^4} - 2r^2\beta^2 f\right)}{fK}} \left(1 - \frac{\varepsilon^2}{\left(\frac{r_{\rm h}^4}{2r^4} - 2r^2\beta^2 f\right)}\frac{R^4}{r^4}\frac{K}{h}\right)^{-\frac{1}{2}}\,. \tag{37}$$

For the low energy limit ($\varepsilon \ll 1$), which is corresponding to $L \ll L^-$, one can obtain

$$S = S_0 + T_0 L^- \varepsilon^2 \int\limits_{r_{\rm h}}^{\infty} dr \frac{R^4}{r^4}\sqrt{\frac{K}{fh^2\left(\frac{r_{\rm h}^4}{2r^4} - 2r^2\beta^2 f\right)}}\,, \tag{38}$$

where

$$S_0 = 2T_0 L^- \int\limits_{r_{\rm h}}^{\infty} dr \sqrt{\frac{fh^2\left(\frac{r_{\rm h}^4}{2r^4} - 2r^2\beta^2 f\right)}{fK}} \tag{39}$$

can be interpreted as self energy of the isolated quark and antiquark. Furthermore, one can integrate equation (36) and obtain the following relation

$$L=\varepsilon\int\limits_{r_{\rm h}}^{\infty}dr\frac{R^4}{r^4}\sqrt{\frac{K}{fh^2\left(\frac{r_{\rm h}^4}{2r^4}-2r^2\beta^2f\right)}}\,. \tag{40}$$

Therefore, one can find

$$S-S_0=T_0L^-L^2\left[\int\limits_{r_{\rm h}}^{\infty}dr\frac{R^4}{r^4}\sqrt{\frac{K}{fh^2\left(\frac{r_{\rm h}^4}{2r^4}-2r^2\beta^2f\right)}}\right]^{-1}. \tag{41}$$

Finally, by using the following relation [25]

$$\hat{q}\equiv 2\sqrt{2}\frac{S-S_0}{L^-L^2}\,, \tag{42}$$

we find the expression of the jet-quenching parameter

$$\hat{q}=2\sqrt{2}T_0\left[\int\limits_{r_{\rm h}}^{\infty}dr\frac{R^4}{r^4}\sqrt{\frac{K}{fh^2\left(\frac{r_{\rm h}^4}{2r^4}-2r^2\beta^2f\right)}}\right]^{-1}. \tag{43}$$

Before anything else, we check the validity of the above expression at $\beta\to 0$ and $a\to 0$ limits. In these limits, one find $K=1$ and $h=1$, so one can obtain

$$\hat{q}_{\rm SYM}=\frac{\pi^2}{b}\sqrt{\lambda}T^3\,, \tag{44}$$

where $T_0=\frac{1}{2\pi\alpha'}$, $T=\frac{r_{\rm h}}{\pi R^2}$, $R^2=\alpha'\sqrt{\lambda}$ and $b=\sqrt{\pi}\frac{\Gamma(\frac{5}{4})}{\Gamma(\frac{3}{4})}\approx 1.311$. Equation (44) is the well known relation of the jet-quenching parameter in the hot $\mathcal{N}=4$ supersymmetric QCD [31].

From Ref. [1] it is found that, for the case of $\beta=0$ and $a=0$ the drag force $\dot{P}$ is proportional to $\sqrt{\lambda}\pi T^2v$. Therefore, in this case, one can obtain

$$\left(\frac{\hat{q}}{\dot{P}}\right)_{\rm SYM}\sim T\,. \tag{45}$$

It means that the ratio of the jet-quenching parameter to the drag force is linear for the temperature in the $\mathcal{N}=4$ super Yang–Mills plasma without the chemical potential.

One can consider another case with $\beta \to 0$ and $ar_{\rm h} \gg 1$ which yields to the following expression

$$\hat{q}_{\rm NC\ SYM} = \frac{12\pi^2}{5}\sqrt{2\hat{\lambda}}T^3\,, \tag{46}$$

where we assume $a^2 r_{\rm h}^2 = \Theta$. This assumption is consistent with the relation (3) with $\Theta = (\sinh\varphi\cos\theta)^{-1}$. The large non-commutativity parameter a means that $r_{\rm h}$ will be a small parameter, so in the above relation the $\mathcal{O}(r_{\rm h}^4)$ terms can be neglected. Therefore, the value of temperature in relation (46) is lower than the value of temperature in relation (44), hence it seems that the jet-quenching parameter for the case of $\beta \to 0$, $a \gg 1$ is smaller than the jet-quenching parameter for the case of $\beta \to 0$ and $ar_{\rm h} \to 0$. However, we should note that $\hat{\lambda} > \lambda$ for a large non-commutativity parameter. So, in order to compare $\hat{q}_{\rm NC\ SYM}$ with $\hat{q}_{\rm SYM}$, we exam their value at $T = 300$ MeV. In that case, one can obtain $\hat{q}_{\rm SYM} \approx 4.5$ GeV^2/fm, and $\hat{q}_{\rm NC\ SYM} \approx 19$ GeV^2/fm, which is nicely in the experimental range [39]. Therefore, the presence of the non-commutativity increases the value of the jet-quenching parameter. In order to obtain numerical value of $\hat{q}_{\rm NC\ SYM}$, we assumed a to be of the order of 10^5, which is consistent with the value of Θ in Ref. [60], to find a significant correction due to non-commutativity in collider experiments. Such value is also in agreement with spin statistics violations in non-commutative QED from Gran Sasso and Super-Kamiokande [61]. Moreover, we can impose other limits on non-commutativity from experiments, for example quantum mechanics, Lamb shift in non-commutative QED and non-commutative extensions of standard model give $a \sim 10$ [62], and non-commutative symplectic structure in classical mechanics and perihelion of mercury give $a \sim 10^{13}$ [63].

In both cases, we find the standard form of the jet-quenching parameter proportional to T^3 times square root of the 't Hooft coupling. Although the drag force for the large non-commutativity parameter is proportional to $(\sqrt{\hat{\lambda}}T^2)^{-1}$, the jet-quenching parameter saves its shape. However, authors in Ref. [1], for the large non-commutativity, concluded that the drag force on quark is very small and this result is in agreement with our result.

The next case which we consider in this paper is the case of $\beta \gg 1$ at $a \to 0$ limit. In this case, one can obtain

$$\hat{q}_{\rm NR\ SYM} = \frac{\bar{\mu}^2}{T\sqrt{2\hat{g}_{\rm YM}^2 N}} I^{-1}\,, \tag{47}$$

where $\bar{\mu} = (\beta\alpha')^{-1}$ is defined as the chemical potential of the non-relativistic, non-commutative Yang–Mills theory, and

$$I=\int\limits_{r_{\rm h}}^{\infty}\frac{dr}{r\sqrt{\left(r^4-r_{\rm h}^4\right)\beta^2\left(r_{\rm h}^4-4r^2\beta^2\left(r^4-r_{\rm h}^4\right)\right)}}\,. \tag{48}$$

In the case of high temperature ($r_{\rm h}\to\infty$), one can obtain $I\propto\beta^{-2}r_{\rm h}^{-4}$. Therefore, we found that $\hat{q}_{\rm NR\ SYM}\propto T$, comparing the jet-quenching parameter with the drag force yields to the following relation

$$\left(\frac{\hat{q}}{F}\right)_{\rm NR\ SYM}\sim T\,. \tag{49}$$

In Ref. [1], it is found that the drag force for the case of $\beta\gg 1$ and $ar_{\rm h}\ll 1$ does not depend on the temperature. It means that the ratio of the jet-quenching parameter to the drag force at high temperature limit is proportional to the temperature which was already obtained for the case of ordinary theory (see relation (45)).

Finally, we consider the case of $\beta\ll 1$ and $ar_{\rm h}\ll 1$ and obtain

$$\hat{q}_{\rm NC\ NR\ SYM}\propto\frac{\pi^2a^2\sqrt{2\hat{\lambda}}T^3}{a^2+\pi^2\hat{\lambda}\alpha'^2\beta^2T^2}\,. \tag{50}$$

It is clear that $a\to 0$ limit of equation (50) yields to $\hat{q}_{\rm NR\ SYM}\propto T$ and $\beta\to 0$ limit of equation (50) yields to $\hat{q}_{\rm NC\ SYM}\propto T^3$, which agree with the results of relations (45) and (47).

5. Conclusion

In this paper, we considered non-relativistic, non-commutative Yang–Mills plasma and studied the problem of the moving heavy quark through the thermal plasma. As we mentioned in the introduction, the non-relativistic nature of CFT is important for the condensed matter theory. Besides, the large magnetic field existing in the background yields to non-commutativity in the background which is important for some unified theories. Therefore, the study of non-relativistic, non-commutative QGP is interesting. We have obtained the full components of the momentum density and discussed the static quark configuration. Then, we discussed the quasi-normal modes. Finally, we have computed the jet-quenching parameter for the non-relativistic, non-commutative theory. For the large chemical potential (for both infinitesimal and large non-commutativity parameter) the jet-quenching parameter is obtained in its standard form, but for the case of infinitesimal non-commutativity parameter and large β the jet quenching parameter is proportional to temperature. However, in this case, the ratio of the jet-quenching parameter to the drag force is similar to the ordinary theory. We have found that the presence of non-commutativity is necessary to obtain the jet-quenching parameter in the experimental range. As we know, the experimental data show that the value of the jet-quenching should be in the range (15 ± 10) GeV^2/fm [39]. We have found that $\hat{q}_{\mathrm{NC\ SYM}} \approx 19$ GeV^2/fm at $T = 300$ MeV which is in the experimental range. We should note that our results differ from the usual results obtained by means of calculation in quantum field theory involving the usual mechanisms of multiple scattering and radiative energy loss [64]. For example, the value of the jet-quenching parameter obtained by using multiple scattering mechanism for $T = 400$ MeV was about 2.3 GeV^2/fm which is clearly lower than our result and experimental data. The reason is that the AdS/CFT correspondence gives more exact solutions than other methods.

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