NLO CORRECTIONS IN THE INITIAL-STATE PARTON SHOWER MONTE CARLO*

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The decade-old technique of combining the NLO-corrected hard process with LO-level parton shower Monte Carlo is now mature and used in practice of the QCD calculations in the LHC data analysis. The next step, its extension to an NNLO-corrected hard process combined with the NLO-level parton shower Monte Carlo, will require development of the latter component. It does not exist yet in a complete form. In this note, we describe recent progress in developing the NLO parton shower for the initial-state hadron beams. The technique of adding NLO corrections in the fully exclusive form (defined in recent years) is now simplified and tested numerically, albeit for a limited set of NLO diagrams in the evolution kernels.

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1. Introduction

Perturbative Quantum Chromodynamics (pQCD) [1–3] is the basic and indispensable tool for analyzing experimental data in the LHC experiments. The technique of combining an NLO-corrected hard process with an LO-level parton shower Monte Carlo (replacing collinear PDFs), such as MC@NLO [4] and POWHEG [5, 6], is now used in practice of the QCD calculations in the LHC data analysis. Its logical extension, providing higher-precision QCD predictions, would be an NNLO-corrected hard process combined with the

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NLO-level parton shower Monte Carlo (MC). However, the NLO-level parton shower Monte Carlo does not exist yet. In addition, the methods of NLO-correcting the hard process used in the above methodologies are quite complicated and it would be desirable to simplify them before going to the NNLO level. The authors of this note are developing solutions to the both above problems. On the one hand, in Refs. [7, 8], see also Refs. [9, 10], they have developed a simpler method of introducing the NLO corrections to the hard process. On the other hand, completely new techniques of NLO-correcting parton shower MC are developed, see Refs. [11, 12].

In the present note, we show that the technique used to simplify and speedup inclusion of the NLO corrections in the hard process [7, 8] can also be applied for the same purpose within the methods of Refs. [11, 12] to introduce the NLO corrections in the parton shower MC. Some similarities (and differences) to the POWHEG [5] method are discussed in Refs. [8, 10], in the case of the hard process.

2. Overview of method of NLO-correcting parton shower MC

For the detailed description of the methodology of NLO-correcting the parton shower MC, we refer the Reader to Refs. [9, 12]. Reference [11] presents an older variant of the method — on the other hand, it provides many details of the differential cross sections of the NLO corrections to the ladder. The above studies and this work, are limited to the non-singlet component of the QCD evolution of the quark distributions in the hadron beam, using the non-running α_S. The DGLAP evolution equation is solved *exactly* using a simple Markovian MC with the relevant inclusive LO or LO+NLO evolution kernels[1]. The newly developed methods use fully exclusive (unintegrated) evolution kernels and their results, at the inclusive level (evolved quark x-distributions), are compared with the exact inclusive MC calculation.

The algebraic structure of the NLO-corrected exclusive distributions of the simplified parton shower MC reads as follows[2]

[1] We shall refer to this calculation as an *"inclusive benchmark MC"*. See Ref. [8] for details.

[2] This is Eq. (1) in Ref. [12].

$$\rho_n(k_l) = e^{-S_{\mathrm{ISR}}} \left\{ \left| \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \right|^2 + \sum_{p_1=1}^{n} \sum_{j_1=1}^{p_1-1} \left| \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \right|^2 + \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1 \\ j_1 \neq p_2}}^{p_1-1} \sum_{\substack{j_2=1 \\ j_2 \neq p_1, j_2}}^{p_2-1} \left| \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array} \right|^2 \right\}$$

$$= e^{-S_{\mathrm{ISR}}} \left[\beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W\left(\tilde{k}_p, \tilde{k}_j\right) \right.$$

$$\left. + \sum_{p_1=1}^{n} \sum_{p_2=1}^{p_1-1} \sum_{\substack{j_1=1 \\ j_1 \neq p_2}}^{p_1-1} \sum_{\substack{j_2=1 \\ j_2 \neq p_1, j_2}}^{p_2-1} W\left(\tilde{k}_{p_1}, \tilde{k}_{j_1}\right) W\left(\tilde{k}_{p_2}, \tilde{k}_{j_2}\right) + \ldots \right]$$

$$\times \prod_{i=1}^{n} \theta_{a_i > a_{i-1}} \rho_1^{(1)}(k_i) \beta_0^{(1)}(z_i) \,. \tag{1}$$

Notation and definitions can be found in Ref. [12]. For the purpose of the following discussion, let us only recall the definition of the weight W. It introduces the 2-real NLO correction involving C_{F}^2 subtracted part of the exact matrix element for the emission of two gluons from the quark line (including interference)

$$W(k_2, k_1) = \frac{\left| \begin{array}{c} \\ \\ \end{array} \right|^2}{\left| \begin{array}{c} \\ \\ \end{array} \right|^2} = \frac{\left| \begin{array}{c} \\ \\ \end{array} \right|^2}{\left| \begin{array}{c} \\ \\ \end{array} \right|^2} - 1 \,. \tag{2}$$

The other, triple, vertex aggregates the LO kernel with all unresolved (virtual+soft) corrections (excluding the Sudakov part)[3]

$$\left| \begin{array}{c} \\ \\ \end{array} \right|^2 = \left(1 + 2\Re\left(\Delta_{\mathrm{ISR}}^{(1)}\right)\right) \left| \begin{array}{c} \\ \\ \end{array} \right|^2 \,. \tag{3}$$

It is very important that both the above building blocks of the NLO corrections are free of any infrared or collinear singularities.

[3] See also Ref. [13].

In Eq. (1) the summations over indices p_1 and p_2 are over the positions of the so-called "NLO insertions", which upgrade one and two kernels to the NLO level. The triple and higher order summations, which are upgrading three and more kernels could be included, but we have checked that they are numerically unimportant. On the other hand, summations over "spectator gluons" j_1 and j_2 are important and they are regarded as a landmark of our method. (They are similar to the sums over $\tilde{\beta}$ non-infrared terms in the QED exponentiation scheme of Ref. [14].) These sums may slow down the generation of the MC events and are rendering the evaluation of the MC weight quite complicated.

In Refs. [8, 10] it was shown how to reduce the sums j_i over spectator gluons just to one or two terms, limiting these sums to contributions from one or two gluons with maximum transverse momentum[4], without loosing the completeness of the NLO approximation. Hence, it is obvious to ask whether a similar "trick" is possible here, in Eq. (1). The key point is to invent within the ladder kinematics some new variable which could be used to define easily a spectator gluon as the hardest one — the only one which "saturates" the sum j_i over spectators[5]. We cannot use directly k_j^{T} of the spectator, because the phase space of the NLO correction is really the two-gluon phase space — a new variable u_{pj} has to involve momenta of both gluons, the "head" p and the "spectator" j. Moreover, similarly as k^{T} in the hard process, it has to provide the "Sudakov suppression" in the limit $u_{pj} \to 0$.

In Fig. 1 we illustrate the problem and the solution in a graphical way. The solution is the following

$$u_{pj} = \eta_p - \eta_j + \lambda \ln(1 - z_j)\,. \tag{4}$$

The parameter $\lambda \simeq 1$ will provide an extra optimization in the following numerical exercises. The direction of u_{pj} is marked in Fig. 1 — it points towards the tip of the shaded triangle which marks the endpoint of the allowed phase space of the spectator gluon j. In the essence variable $\exp(u_{pj})$ represents the rescaled k^{T} of the spectator gluon j. The above kinematics describes a parton shower MC with the angular ordering, however, the kinematics of the parton shower with the k^{T}-ordering is quite similar.

In the above double-gluon phase space with fixed rapidity of the head gluon p, the Sudakov phase space is 3-dimensional, $(\eta_p - \eta_j, \ln(1 - z_j)$, $\ln(1 - z_p))$, and the volume of the underlying 3-dimensional LO gluon phase space is equal to the triple Sudakov log. In the 2-dimensional visualization

[4] Similarly as in POWHEG, but without complicated "vetoed" and "truncated" MC showers.

[5] In the sense of protecting completeness of the NLO.

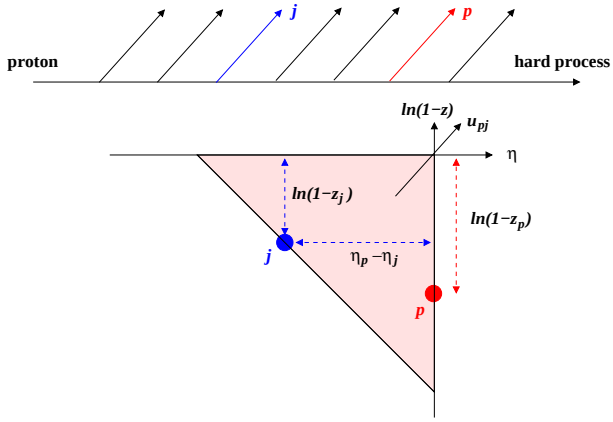

Fig. 1. The kinematics of the two-gluon phase space of the NLO correction. The variable η is rapidity of the emitted gluon and z is the conventional lightcone variable of the emitter quark.

of this phase space gluon density in Fig. 2, we use a set of variables $\ln(1-z_p)$ and u_{pj}^2 in order to have a flat plateau representing manifestly the leading LO Sudakov singularity. The LHS of Fig. 2 shows this Sudakov LO plateau.

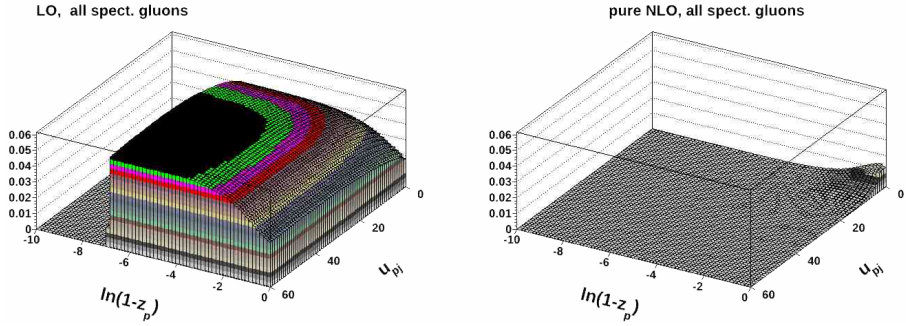

Fig. 2. The inclusive distribution of gluons according to LO distribution (left) and due to the two-real NLO contribution (right).

On the other hand, the NLO contribution plotted in the RHS of Fig. 2 clearly concentrates in the corner $z_p \simeq 0, u_{pj} \simeq 0$, and is manifestly free of any singularities (it is integrable to a finite value). This is quite similar as in the single-gluon phase space of the hard process shown in Fig. 5 in Ref. [8].

In the next step, let us order spectator gluons (indexed by j) and split the LO distribution (similarly as in Fig. 6 of Ref. [8]) into the hardest in the variable u_{pj} and the rest[6]. The resulting two components are shown

[6] We cannot order in $\ln(1-z_p)$ because the head gluon p is just one.

in Fig. 3. The hardest gluon distribution differs from the one in Fig. 6 of Ref. [8], nevertheless it has the same property needed for NLO completeness — it reproduces well the inclusive LO distribution (LHS plot in Fig. 2) in the region where the NLO contribution (RHS plot in Fig. 2) is nonzero.

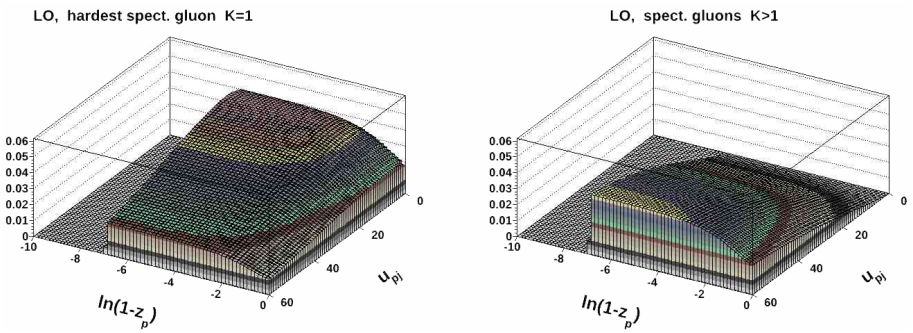

Fig. 3. The inclusive LO distribution of gluons of the left plot in Fig. 2 split into the hardest (in u_{pj}) gluon (left) and the rest (right).

In view of the above, we expect that preserving only one term in the sums over j in Eq. (1), from the gluon with the maximum u_{pj}, will effectively lead to NLO result within a good numerical approximation (formally up to NNLO terms). We shall check this conjecture in the following.

3. Numerical results

In the following, we shall check numerically that taking only one or two hardest (in u-variable) spectator gluons in the NLO MC weight of Eq. (1) does not significantly disturb the NLO result of the QCD evolution. This is the principal result of this work.

As a warm-up exercise, we reproduce the result of Ref. [10], in which we use Eq. (1) with summation over all spectator gluons j_1 and j_2. In Fig. 4, the total (LO+NLO) quark distribution evolved with single and double NLO insertion is compared with the benchmark inclusive calculation. The two are indistinguishable, and to see the difference one should look at the lower plot in Fig. 4, where the ratios of the exclusive and inclusive results are plotted for the single and double NLO insertions separately. They agree perfectly within the statistical errors.

Next, in the calculation presented in Fig. 5, we replace the sums over spectators with the one or two terms from the hardest gluons in the variable u_{pj}, for the single gluon insertion component. As we see, this truncated result reproduces very well the previous single NLO insertion component in the evolved quark distribution. The actual difference is better seen in the

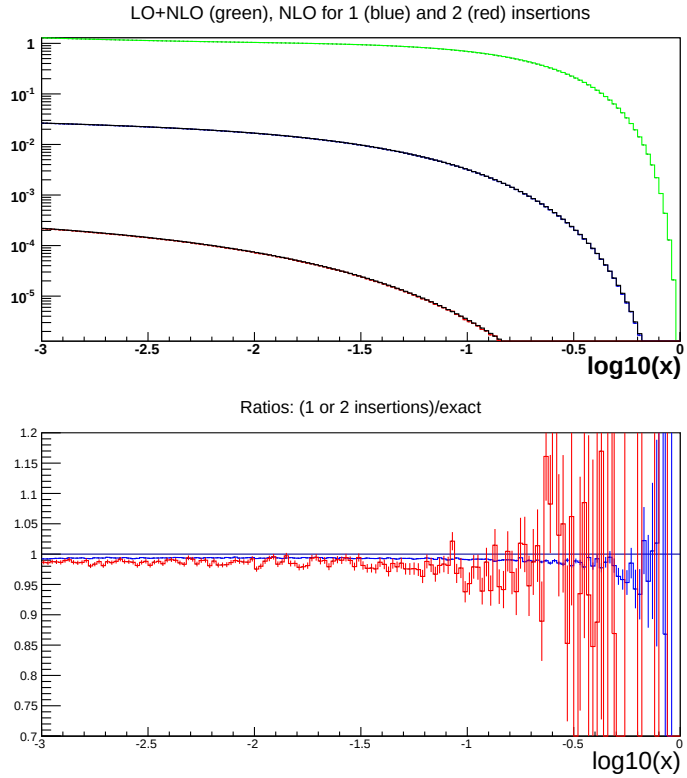

Fig. 4. The distribution of the quark evolved from $Q = 100$ GeV to $Q = 10$ TeV. Upper plot: the upper line (green) represents LO+NLO quark distribution and two lines below are the components due to 1 (middle/blue) and 2 (bottom/red) NLO insertions in Eq. (1). The corresponding inclusive benchmark results are also plotted (black), but they are almost indistinguishable, hence the corresponding ratios (exclusive/inclusive) are provided in the lower plot.

lower plot of Fig. 5 representing the ratios of the truncated and complete sums over spectator gluons. Of course, the case with two hardest spectator gluons looks better, but the single hardest gluon would be sufficient. It should be added that in the above result we have adjusted $\lambda = 2$ in the definition of u_{pj}. For $\lambda = 1$, the ratio for single spectator gluon would be ~ 0.7 at the low x limit (remaining formally all the time correct modulo NNLO corrections).

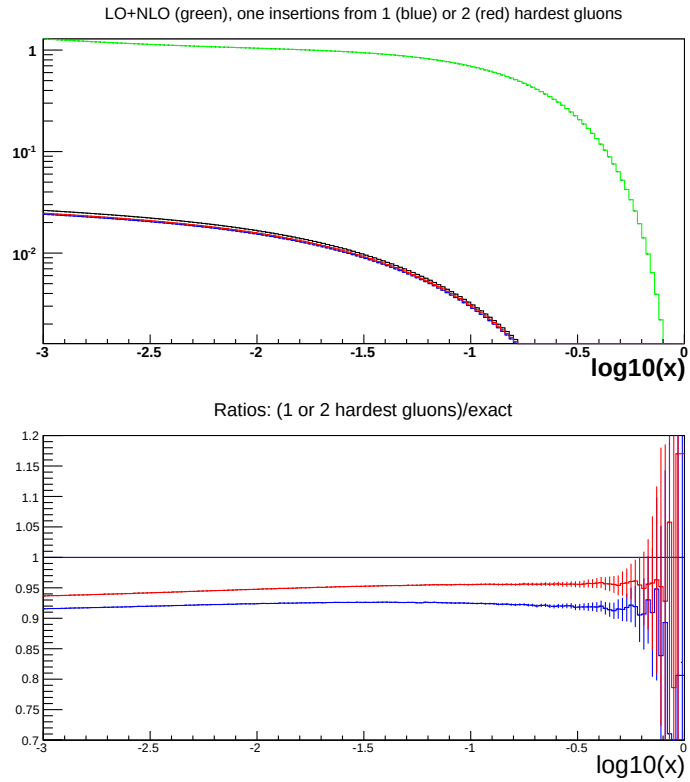

Fig. 5. Upper plot: the upper line (green) represents LO+NLO quark distribution and three lines below represent a single NLO insertion component with the complete sum (black) over spectator gluons as in Eq. (1), and two other versions with the sum truncated to one (blue) and two (red) hardest spectator gluons. The two corresponding ratios (truncated/complete) are shown in the lower plot.

4. Summary and outlook

A new methodology of adding the QCD NLO corrections to the NLO initial state Monte Carlo parton shower is refined and tested numerically, albeit for a limited set of the NLO diagrams and in the simplified MC model. This result presents another important step towards realistic implementation of the NLO parton shower MC, to be combined with the NNLO-corrected hard process.

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